MATHEMATICS—FROM SERVANT TO PARTNER

Phillip A. Griffiths

INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY

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A NOTE TO THE READER

The Institute for Advanced Study hosted "A Celebration of Mathematics 1933–1993" on April 2 and 3, 1993, to mark the dedication of its new mathematics building. The celebration included lectures of particular interest to mathematicians and scientists on Friday, April 2, and presentations of more general interest concerning science and science policy on Saturday, April 3.

On April 3, Dr. Phillip A. Griffiths, director of the Institute for Advanced Study, spoke on "Mathematics — From Servant to Partner." The text of his lecture follows.

MATHEMATICS — FROM SERVANT TO PARTNER

Phillip A. Griffiths

April 3, 1993

Good morning. I am delighted to welcome you all here for this dedication, and to tell you why this is such a significant event for us. As director of the Institute, I am aware of how much the new mathematics building will benefit both our visiting members and our permanent faculty.

Since the founding of the School of Mathematics in 1930, more than 3,000 mathematicians from many nations have come here as Members — our word for fellows. This includes three-quarters of the mathematicians who have won the Fields Medal. I believe that this facility will strengthen what is already considered a leading center for the field.

Of course, I am officially neutral when it comes to supporting the different kinds of scholarship we do at the Institute. But on this particular day I shall confess that I am by training and instinct a mathematician. Perhaps it takes one to know how one feels, but I shall presume to speak for the mathematics community in expressing a feeling of great excitement.

Yesterday we heard evidence of this excitement in some fine talks about mathematics itself. Today I would like to broaden the focus to the relationship between mathematics and the sciences.

PROBLEMS OF NOMENCLATURE

I said I am a mathematician, and many of you know that means I am trapped, like other mathematicians, between the devil and the deep blue sea. The devil is trying to explain mathematics with metaphors, at the risk of oversimplification. The deep blue sea is that swamp of mathematical terms incomprehensible to other law-abiding citizens, including most of our scientific colleagues. This is both a small difficulty for today and a much larger problem for the math community and our society, where mathematics is poorly appreciated. For today, I've decided to use vernacular English — if you'll allow me a couple of asides to my colleagues.

Let me begin with two points that seem very important. The first is that mathematics is fundamentally different from the sciences. And the second is that the relationship between mathematics and other disciplines has changed substantially in recent years — mostly for the good.

One way to think about this change is to look at nomenclature. Mathematics has been called the Queen of Sciences. In the eyes of some, she was somewhat superior, unto herself. There is a suggestion of smugness, as if to say that mathematicians don't need anyone else. There is also a feeling of blue sky; of airy exercises performed for their own sake. Indeed, the mathematician G.H. Hardy once said that the practice of mathematics can best be justified as an art form.

Mathematics has also been called the Servant of Science — that is, the quantitative handyman who supplies the tools and often the framework for the sciences. And provosts and deans sometimes think of mathematics departments as "service departments."

At any rate, neither Queen nor Servant seems adequate today. Mathematics functions not so much above or below other disciplines, but within and around them. It is growing into the status of a full and interactive partner. It will be my challenge to convince you of this and to show that is healthy — not only for mathematics but also for fields traditionally close to mathematics like physics as well as fields as seemingly far removed as business and psychology and health policy analysis.

LANGUAGE

Another way to think about mathematics is that it is also a language, one that we study every year in school, just as we do English. And it is a language on which the sciences depend whenever they need to quantify what they are doing. But it is more than that. Richard Feynman, an American physicist who shared the Nobel Prize in 1965 for work in quantum electrodynamics, has said that the universe seems to be indescribable *except* in the language of mathematics.

Perhaps the most famous illustration of this involved Sir Isaac Newton, who wanted a theoretical framework to express the motion of objects under the influence of gravity, including Kepler's laws of planetary motion. This desire drove him to develop his universal law of gravitational attraction as well as the calculus, one of the great achievements in the history of science.

Another example of using mathematics as the appropriate language is part of the folklore of Albert Einstein. He had spent years attempting to formulate the possibility that gravitation is really just a reflection of the curvature of spacetime, but he didn't know how to express this mathematically. He turned one day to his close friend Marcel Grossman and said, "Grossman, you have to help me or I'll go crazy." Grossman told him about Bernhard Riemann's work with curved space, which had followed earlier work of Gauss, Bolyai, and Lobachevsky. A vast body of basic research had already been done and was ready for use. Einstein, a physicist first and a mathematician only by necessity, breathed a sigh of relief and went ahead with the general theory of relativity.

But the connection between mathematics and physics has grown still deeper. Another Nobel laureate in physics, Steven Weinberg, has spoken of "spooky" coincidences. He said it is spooky for the physicist to get to an idea and then discover that the mathematicians have already been there before him. A famous example relates to group theory. Group theory was invented by the French mathematician Evariste Galois in the early 19th century in order to solve a problem purely internal to mathematics. The problem was how to find the roots of polynomial equations, which is a very abstract pursuit. Galois made one of the most original leaps in the history of mathematics and introduced the concept of a group. A group is a mathematical way of expressing the concept of symmetry, and since the latter part of the last century the subject of group theory has become highly developed.

When the physicists discovered group theory in the first half of this century, they found that it was just what they needed to unify the great conservation laws of energy, momentum, spin, charge, and so on. These laws were reflections of symmetries in the world around us, and this subtle principle is one of the fundamental concepts in science today. For example, it turns out for rather complicated reasons that the most basic question you can ask about an elementary particle is which symmetry group it reflects. The important point here is that Galois' motivation was one internal to mathematics, an effort to solve an interesting problem in algebra. We can only guess what more this genius might have accomplished had he not been killed in a duel at the age of 21.

THE POWER OF MATHEMATICS

In the past, mathematicians have been accused of living in ivory towers, of losing themselves in the abstract beauty of their own conjectures. Indeed, we must plead guilty to abstraction. We all know that in geometry, for example, we are studying points infinitely small, lines infinitely narrow, and circles perfectly round — that is, ideal objects. The concept of the ideal is as old as Plato, and in popular conception of little relevance to the world we see around us.

But I would submit that abstraction is not always a bad thing, that in fact it is a realm of great potential and utility. Another way to look at this is to say that mathematicians concern themselves with internal consistency. That is, they are absolutely faithful to the rules of their own game. Again, they are criticized because their game concerns not just the world as it is but also the world as it *might* be. Modern theoreticians make up models of the natural world on an idealized mathematical basis.

But I would submit also that the world is *not* what it appears to be. Who would ever imagine, for example, looking out the window, that mass becomes infinite as we near the speed of light? Indeed, who would care? The mathematicians would care, of course, and, if history is any guide, others would eventually care, too — often years or decades after the mathematics was done. Riemannian geometry, the fundamental tool in studying relativity,

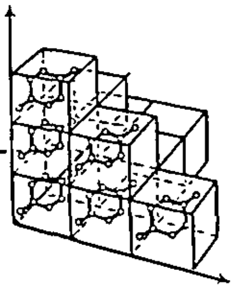


Figure 1

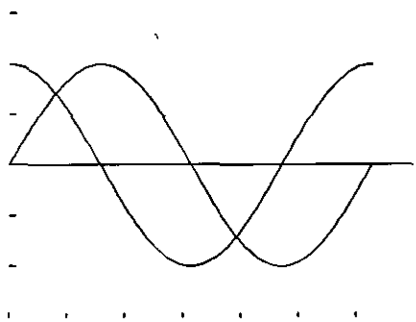
was worked out about 60 years before Einstein needed it. Lie groups predated their application to particle physics by at least 30 years.

One beautiful example of mathematics turning out to be "realer than real" was the discovery of the positron. Paul Dirac, the British physicist and mathematician who discovered the correct equation of motion of the electron, did this based largely on considerations of symmetry. But something totally unexpected came as a result: Dirac's equation predicted the existence of a particle identical to the electron in every respect but charge. No one had ever observed this hypothetical "anti-particle," but quite naturally experimentalists set out to look for it. The discovery of the positron soon thereafter was one of the triumphs of physics. I would say it was equally a triumph of mathematics.

I'd like to describe a more modern example of this phenomenon in more detail, because I think it shows how powerfully and unexpectedly mathematics and the sciences can resonate in productive partnerships. Around 1950, a scientist named Herbert Hauptman, who was trained as a mathematician, became interested in a puzzle regarding the structure of crystals. Chemists had known since early in the century that x-rays passing through crystals are scattered, or diffracted, when they strike atoms within the crystal. When they placed x-ray film behind the crystal, the x-rays would darken the film in diffraction patterns which varied according to the location of the atoms (Figure 1).

The puzzle for the chemists was that they couldn't pinpoint the position of the atoms in the crystal. This is because x-rays, like other electromagnetic radiation, may be thought of as

Figure 2



waves — thus they have both amplitude and phase (Figure 2). The diffraction patterns detect only the amplitudes — and not the phases — of the x-ray waves. This problem had baffled chemists for over 40 years.

It was Hauptman's genius to realize that this issue could be formulated purely as a mathematical problem, and one that had a very elegant solution.

Here I must offer an aside to the mathematicians in the audience, if the rest of you will bear with me. The electron density function is triply periodic and, roughly speaking, from measurement we can determine the absolute values of its Fourier coefficients. Hauptman's insight was that electron density function is special in that it is a non-negative function with very small support. From this he was able to deduce the phases from the intensities.

The point here for all of us is that Hauptman's insight pointed the way to extracting enough information to determine the phase information; then he could go on to determine the crystal geometry. The crystallographers had seen only the shadow of a physical phenomenon, but he showed it was possible to reconstruct the actual phenomenon from its shadow using classical mathematics that had been available for about a century. Dr. Hauptman recalled in a speech a few years ago that prior to 1950, his work was regarded as ridiculous, and he himself as rather foolish. Indeed, he had had only one chemistry course in his life — freshman chemistry. Nonetheless, in 1985 he was awarded the Nobel Prize in Chemistry for his use of classical mathematics to solve a puzzle that had defeated generations of chemists.

THE RELEVANCE OF MATHEMATICS IS UNPREDICTABLE

Professor Armand Borel of the Institute has said that mathematics resembles an iceberg: beneath the surface is the realm of pure mathematics, hidden from the public view, if you will. Above the water is the tip, the visible part which we call applied mathematics. Most people only see the tip, but they don't realize that this would not exist without the much larger portion below.

A good illustration came to surface several decades ago when an engineer named Allan M. Cormack was searching for a way to pinpoint the location and density of an object in the body without surgery. At that time only x-rays were available, and they gave information in only two dimensions.

You will see below (Figure 3) an object of varying density in a plane. We can't see inside the shaded region, but by shooting rays through it and seeing how much comes out the other side we can measure the total amount of matter along a line. The problem is to reconstruct from this information the densities of the interior of the object.

As it turned out, a mathematical solution to this problem had been around for many years, dating from the work of a mathematician named Radon. By Radon's solution, Cormack saw that by taking x-rays from many different angles you can determine the location and shape of objects in the body. Thus was born the CAT scan, a three-dimensional picture of the body's organs. This same principle has been extended to magnetic resonance imaging, MRI, which is even more discriminating. In both techniques we

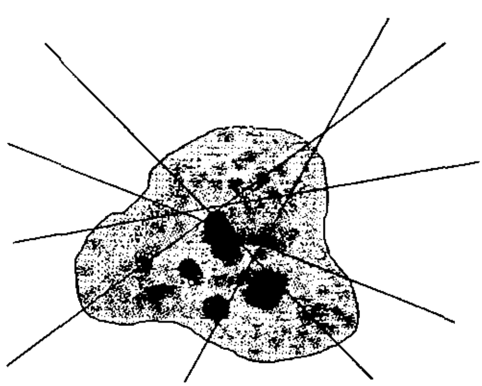


Figure 3

are making large numbers of essentially one-dimensional measurements and using a mathematical technique to reconstruct three-dimensional images. Most recently the PET scan — positron emission tomography — has been developed from the same technique. It measures metabolism as well as anatomy, this time making use of Dirac's anti-particle.

By the way, the mathematics of emission tomography uses an algorithm that arose in the 1960s during an analysis of Soviet communication codes.

Dr. Cormack's application of Radon's transform has moved far beyond medicine. As he himself recently noted, in the field of paleoanthropology, it has been applied to a certain Mrs. Pless, a woman who walked the earth about 2.9 million years ago. A CAT scan of Mrs. Pless's ear chamber revealed what had been suspected by other means — that our great-great-great-to-the-Nth grandmothers walked around in an upright position.

Radon's technique has also been applied to oceanography, where it is used to determine ocean temperature. This is not just an academic matter, since the temperature of the ocean greatly influences weather patterns on land. The traditional way to measure a large area, say 300 square kilometers, is to throw a thermometer overboard from a ship and then steam back and forth in a grid pattern. Ships are so slow that temperatures change before the measurement is complete. Using sound emitters and detectors, however, oceanographers can measure the velocity of sound pulses which are dependent on water temperature. By doing this for a number of sources and detectors, they quickly have enough measurements to use Radon's technique. To accomplish this feat the old-fashioned way, they would need a ship that could steam at nearly 3,000 knots.

There are many more techniques that make use of Radon's problem, but I'll mention just two. In 1938, an Armenian astronomer used it to measure the velocity distribution of stars near the sun, and, in 1958, an Australian astronomer used it to produce a picture of the brightness distribution of the moon.

By the way, Dr. Cormack won the Nobel Prize in Medicine in 1979.

BREAKING DOWN INTERNAL BARRIERS

Now I come to one of the reasons mathematics is healthy today — the breakdown of internal barriers. Following World War II, a notable feature of mathematics was a powerful trend toward specialization into rather narrow subfields. One consequence of this was that a number of areas in mathematics were explored very deeply. Another consequence was that for many years mathematicians had trouble communicating even with their own colleagues in different subdisciplines. This fragmentation still exists, but is complemented by another trend where interesting problems are addressed in an overarching manner. Although this trend began within mathematics itself, it is nowhere better illustrated than by the study of Yang-Mills equations. These are a set of differential equations that seek to extend the famous equations of James Clerk Maxwell, who unified the phenomena of electricity and magnetism under a single theory of electromagnetism.

Maxwell's equations and their extensions turn out to have a beautiful geometry and their study has attracted mathematicians from such disparate specialties as topology, algebra, differential geometry, algebraic geometry, and partial differential equations. This illustrates one of the basic features of contemporary mathematics — the formerly rigid organization in separate "silos" has given way to a much more fluid structure. Our subject is beginning to organize around interesting problems as well as subdisciplinary titles.

THE RESONANCE BETWEEN MATHEMATICS AND SCIENCE

Now I come to one of the most interesting of these problems — string theory. Here we see one of the liveliest synergies between mathematics and science. I have tried hard not to use the word synergy because it is a politically correct "beltway" term, but I have to admit it fits what I am trying to say.

You could also think of the relationships between mathematics and science as a kind of resonance. Imagine a child on a swing. As the swing reaches the back of its arc, he pulls on the ropes and this propels him farther than gravity alone. In the same way, new scientific problems are inspiring new mathematical work, and new discoveries in mathematics are finding application in science. Just like the swing and the child, fields are resonating together with greater combined effect than the sum of the separate parts.

String theory gets its name from the notion that the fundamental units of matter are shaped more like tiny, vibrating strings than like particles. One reason for all the attention to string theory is that for the first time it seems to incorporate gravity into a broad description of matter on a microscopic scale. The general theory of relativity explains gravity, but breaks down at extremely small distances. It has long been suspected that the modifications needed to make a quantum theory of gravity could help cure the spacetime singularities which detract from the relativistic model.

Several papers published recently by scientists working at the Institute show that in string theory, spacetime can evolve smoothly from one topology to another without encountering any kind of singularity. This provides important new evidence that string theory may indeed achieve the first consistent quantum theory of gravity.

In terms of my theme for today, I find it fascinating to see how string theory has led a group of theoretical physicists deep into mathematics, beyond even traditional mathematical physics. One result is that string theory is yielding insights with as much importance for mathematics as for physics. Indeed, the physicists have made a series of spectacular predictions about mathematics and these predictions are already beginning to be verified.

NEW APPLICATIONS

Until now, I have talked mostly about the deep interactions between mathematics and the physical sciences. Here I want to expand this point into new areas. Mathematics is making numerous contributions to a whole range of other disciplines, and in doing so is changing almost everything we do. At the same time, some of these disciplines are challenging mathematicians with interesting new types of problems which in turn lead to new applications. And the more fundamental the mathematics, the wider the application.

A good example of an extremely rich and challenging field is fluid dynamics. Underlying much of this field is a set of equations called the Navier-Stokes equations, which describe the flow of fluids. Fluids may mean air, liquids, even some solids. Mathematicians are using the Navier-Stokes equations to study an incredible range of phenomena: hurricanes, blood flow through the heart, oil moving through porous ground, fuel mixing in carburetors, aircraft flying through air, crystals forming from liquids, plasma in a fusion reactor, the motion of galaxies, clouds, winds, currents. You can imagine why many kinds of scientists are interested in Navier-Stokes equations today.

The movement of fluids is still too complex for complete theoretical understanding or full computer simulation. And some phenomena in fluid flow can't be measured. So researchers combine theoretical modeling, computer simulations, and experiment. Such techniques, made possible by the rapid development of high-speed computers, are new and make use of methods from many subfields across mathematics. Of special scientific and practical interest is the attempt — not yet successful — to understand turbulence, as well as chaotic behavior. Chaotic behavior — where a small change produces a large effect, such as the ability of relatively few molecules of chlorofluorocarbons to precipitate the large ozone hole in our atmosphere — is one aspect of mathematics that has seized popular attention. Another example of chaos is the intricate and beautiful patterns we see when oil is added to water. A little oil, that is.

Another example of a powerful application of mathematics to whole new areas is seen in control theory, a branch of the theory of dynamical systems. To take one case, high-performance aircraft used to be designed primarily by wind tunnel experiments and the like, after which a model was built and some adventurous test pilot was asked to see how it would fly. With modern control theory, the integration of design and performance is much more efficient, and more test pilots live to become grandparents.

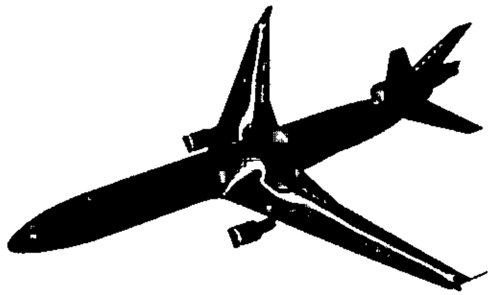


Figure 4

In view of our discussion of nomenclature, I think you'll agree that the distinction between pure and applied research is no longer helpful. Nor, perhaps, is the distinction between goal-oriented and curiosity-driven research. Instead, a good deal of interesting mathematical work that we used to call "pure" has its origin in very practical research. And some day the results of that pure work will likely return to the practical research — with interest.

COMPUTERS

I have mentioned computers, and here I want to emphasize that today they do far more than crunch numbers; they allow scientists to do things inconceivable a generation ago. One powerful new use is computer modeling, which is revolutionizing many areas of science and engineering. What modeling does is to replace costly experiments with computer simulations. In aircraft design, as we said, the wind tunnel is going the way of the horse and buggy; the testing of shapes is done by computer. These models (Figure 4), coupled with algorithmic approaches, have opened doors to areas too complex for actual testing of theory, such as how proteins fold and unfold, or how petroleum flows through porous rock deep beneath the ground. And once again there is benefit from many synergies as knowledge flows quickly back and forth between mathematics and computational science.

I have an aside not for mathematicians this time but for sailors — maybe there are some here today. In recent years the hull design of yachts is increasingly done by computer modeling. One specifies the desired characteristics — usually increased speed — and the computer generates the best design. But other characteristics such as seaworthiness and sea-kindliness are less easily designed by computer. An experienced sailor can intuitively sense these from the beauty of the boat, so you need both — the computer and an aesthetic sense — to design the best hulls.

LIFE SCIENCES

I mentioned that mathematics is traditionally used in the physical sciences, where today it provides both theoretical frameworks and quantitative tools. A few areas of mathematics have in the past been useful in the life sciences, especially statistics, but generally not at fundamental levels. Today this is changing. Thanks to new techniques and computers, mathematics is finally beginning to be able to deal with the complexities of biological organisms. In particular, the unique capability of mathematics to discern patterns and organize information is starting to penetrate such basic systems as neural networks.

We have already mentioned the importance of mathematics to CAT scans and MRI. Also, fluid dynamics has led to computational models of the kidney, pancreas, ear, and other organs. Computer models of the human heart have even led to improved design of artificial heart valves. As provost at Duke, I observed that grateful patients generously supported the Medical Center. This has yet to happen for us in mathematics, but perhaps it will.

In yet another partnership, mathematicians and biologists together are exploring the mechanisms of DNA replication. It is known that DNA exists in tangled and knotted form, and must untangle during replication. It so happens that mathematics contains a subfield called knot theory which along with probability theory and combinatorics, is helping biologists understand the complex three-dimensional mechanics of DNA sequencing.

An area that has surged in sophistication with the new power of computers is epidemiology. There is a major effort to mathematically model the AIDS epidemic. The model has shown that

HIV does not spread like the agents of most other epidemics. The complexity of these models is so great it has surpassed the ability of our fastest computers, so mathematicians are challenged to apply the simplifying power of their discipline.

ECONOMICS

One of the most valuable contributions of mathematics to economics is the general equilibrium model, which seeks to predict the behavior of free markets. The power of this model, work on which won Kenneth Arrow the Nobel Prize, has encouraged a general mathematicization of the whole field of economics. Henry Rosovsky relates an amusing story that illustrates several points. When Arrow won the Nobel he was a faculty member at Harvard, and Henry, who was then the Harvard Dean, mentioned this to a distinguished colleague in the math department. This individual asked for a copy of Arrow's work, and after seeing it he said that the mathematics was elementary and could have been done by a Harvard freshman. Of course, he was partly right: the mathematics was not advanced. But this is not the point. As in so many breakthroughs, Arrow's achievement was to unite two fields in a product more powerful than the sum of its parts.

I want to say a little more about computer modeling in the context of industrial uses. This change is so revolutionary that industries which do not use it fall quickly behind. It was made possible by huge advances in mathematical modeling, computer hardware, and mathematical algorithms, and the development of all these is extremely rapid and competitive.

One example is the design of microprocessor chips, which is done by mathematical methods, particularly those of discrete mathematics. A common task in testing circuit boards is to move a tool to hundreds or thousands of points and perform some action at each point, such as drilling or probing. Minimizing the time required is often a version of the so-called travelling salesman problem, visiting all the vertices of a graph with a path of minimal length. Advances in graph theory and computational complexity have yielded simple ways to do this. New algorithms give quick solutions within 1% of optimal to problems involving tens of thousands of points.

Another area of basic mathematics — the study of finite fields — has led to applications in computing and communications. For example, a recent challenge to telephone companies is to build telephone systems that are both efficient and robust — that is, that use the smallest possible number of lines to carry phone calls, but can quickly reroute calls when the system breaks down. Mathematically this can be calculated with a graph consisting of vertices (in this case the telephone exchanges) connected by edges (in this case the telephone lines connecting the exchanges). Mathematicians at the Institute and elsewhere have led the way in applying deep basic mathematics to practical challenges like this. If history is any guide, the most powerful applications, however, lie in the future, in areas yet to be imagined.

Many kinds of partnerships with industry are forming rapidly. For example, other developments in discrete optimization are revolutionizing how products are manufactured, ordered, stored, and delivered. The consequences can be striking. You are probably familiar with the recent troubles of the shoe industry. Almost all the shoes sold in this country were manufactured abroad — until a few years ago. Thanks in large part to computerized techniques for restocking inventories, the balance has now shifted in favor of the domestic industry.

Similarly, the City of New York has made use of combinatorial optimization techniques to rework its sanitation crew schedules. This is now saving $25 million a year, as well as providing better service and more convenient work schedules.

U.S. airlines, thanks to advanced combinatorial algorithms for scheduling, need fewer planes and personnel to cover the same number of flights. They are also better able to respond to weather disruptions, although on both of these points some might have to agree with what my wife said when the weatherman announced spring a few days ago, "You could have fooled me."

WAVELETS

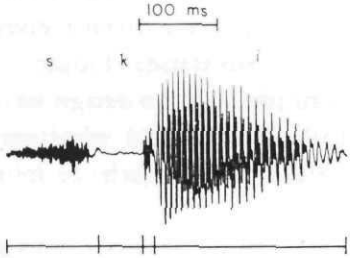
One of the most interesting aspects of mathematics is how often we hear about a breakthrough in technology — and then learn how old are its mathematical roots. I'm thinking of the powerful data compression trick of using wavelets, which come from the venerable field of harmonic analysis. Researchers at Yale have found they can compress and restore virtually any kind of image or sound by using mathematically generated shapes that look like tiny waves. They have found they can reduce the FBI's collection of 300 million fingerprints by a factor of 20, and also reduce the time required to transmit a fingerprint by telephone from 20 minutes to one minute. This will save taxpayers about $25 million in optical storage disks alone. Scotland Yard plans to use wavelets for the same purpose. Maybe even the IRS will discover them.

PATTERNS

I want to conclude with one of the most intriguing fields where mathematics has formed a new partnership: pattern theory. The term "pattern theory" was first introduced in the 1970s as a subfield of mathematics relating to computer vision, speech recognition, signal processing, and parts of artificial intelligence. Its proponents now think that it might contain the germ of a universal theory of thought itself, so you can see that mathematics is running with some pretty fast company.

From a practical point of view, a typical challenge to pattern theorists is to design a machine that can transcribe speech into printed words. If we all pronounced words in exactly the same way — producing identical patterns of sound waves — this would be easy. But we don't and it isn't. When I say the word "ski,"

Figure 5



(Figure 5) it doesn't produce the same sonogram as when you say the word "ski." Our sound machine has to be robust enough to recognize the essential parts of the sounds you and I make when we say "ski."

In the past, theorists have tried to model the brain as an enormously complex computer, but this approach has failed in the real world. A computer is superb at responding to logical input. But researchers like David Mumford of Harvard now believe that the brain is fundamentally different from a computer. What our brain perceives, writes Mumford, is not the raw sensory signal, which is usually fuzzy and ambiguous, but a clever reconstruction of that signal (Figure 6). And the reconstruction makes

Figure 6

extensive use of memories, expectations, and logic. To Mumford, we do our thinking by pattern recognition, and pattern recognition is far more than standard logic.

I am reminded of the design of yacht hulls — the computer allows us to go far beyond what we could do otherwise, but it has yet to discern the subtle coupling of aesthetic quality and utility.

Mumford admits that pattern theory is a long way from a full theory of cognition, but he thinks that already it is more successful than any other. So we find mathematics, in the midst of biological and behavioral company, propelling a broad and intriguing theory that is only in its infancy.

PROBLEMS

One of the points I have tried to make is that mathematics is extremely useful to our society. If this is true, one would think that we as a society would vigorously support the research that leads to new uses and that student interest would be at an all time high. Today that is not the case. The mathematics community has yet to effectively demonstrate to the public and their elected representatives that our subject is different from the sciences. We do not design widgets or cure diseases, yet our impact on engineering and medicine is enabling and significant. But the community has dwelled so long in splendid isolation that the public poorly understands what we do.

In addition, the training of a mathematician is too compressed to allow one to develop both deep knowledge and broad interests — both of which are necessary for the kind of work I've described today. Instead, there is immense pressure to specialize early, to apply for grants in that specialty, and to get tenure.

The synergies and partnerships I have described present new opportunities. The intellectual trends both within and without the field are very positive, and there is emerging a balance between our looking inward and outward that has not always been there. But we are not accustomed to explaining to others — much less marketing — our subject.

If we are to hope for better and more support, then we as a community and we as a society must do better. In particular, we have to produce better mathematics teachers. I can honestly say that the most important person in my own career, in forming my decision to become a mathematician, was Lottie Wilson, my high school mathematics teacher of long ago. Mrs. Wilson had two essential qualities for getting her subject across: she understood the majesty and the mystery of mathematics, and she knew there is no substitute for getting the right answer.

I remember when I was teaching at Harvard how upset I was by the issue of partial credit. Students in a freshman calculus course expected partial credit on an exam just for having gotten the problem started or arriving at some reasonable — but incorrect — answer. I asked them one day, "Suppose you end up being a doctor. Are your patients going to be satisfied if your diagnosis is partially correct?" This comment was not well received. But the point is a serious one: mathematics is a subject in which there are definite answers and a student can take real satisfaction in "getting it right." But we teachers must better communicate the beauty as well as the utility of our subject.

THE INSTITUTE FOR ADVANCED STUDY

The challenge for us here at the Institute is to do the very best research we can, to communicate it whenever possible to the outside world, and to serve as exemplars. As we dedicate this new building, it is important to point out that we're not building an ivory tower, or a cloister, but a forum for the interchange of ideas. Mathematics is still very much a small science, an individual activity done with a pencil and paper and a lively imagination. But it is also highly interactive. The most productive working environment seems to be one in which an individual can share ideas with others when he chooses, then retire into privacy for quiet reflection. Increasingly, he may also need access to a computer. This allows the total enterprise to span a great many topics, remain flexible, and respond quickly to the rest of science.

We think that the new building is well designed to nourish the healthy practice of mathematics. One of our strengths is the ability to do fundamental, nondirected research whose outcome is unknown but whose power can be enormous. I hope I've made a case this morning for the value of such work.

It is a well-kept secret that doing mathematics really is fun — at least for mathematicians — and I am amazed at how often we use the word "beautiful" to describe work that satisfies us. I am reminded of a remark by a mathematician named Jacques Tits, who was talking with some anthropologists about early human experiments with fire. One anthropologist suggested that these humans were motivated by a desire for better cooking; another thought they were after a dependable source of heat. Tits said he believed fire came under human control because of their fascination with the flame.

I believe that the best mathematicians are fascinated by the flame, and that this is a good thing. Because, fortunately for society, their fascination has in the end provided the good cooking and reliable heat we all need.

Thank you very much.

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