"LEONARDO'S WATERY CHAOS"

Institute for Advanced Study, April 21, 1993

(first presented in a dialog with the physicist Tullio Regge, as the sixth episode in a television series "Dietro lo Specchio," broadcast by RAI 2, in Turin, Italy, October 21, 1981)

(click here for first page)

LEONARDO'S WATERY CHAOS (With Two Appendixes) Irving Lavin Institute for Advanced Study, April 21, 1993

In order to explain my idea it seems appropriate to tell you how my interest in the subject came about. Shortly after I came to Princeton in the Fall of 1974 I was invited to join a small group of people from a variety of fields in giving some account of what was new—recent developments and promising directions for future research—in our respective disciplines. Nothing ever came of the publication project the organizer of the meeting envisaged, but I was left scarred forever by the presentation of one of the participants in that little game of intellectual show and tell, the late Arthur Wightman, professor of mathematical physics at Princeton University. Wightman spoke about the new and interesting studies of what he called catastrophe theory—that was the first time I had heard the phrase. In order to explain the subject to the laymen amongst us he gave the example of an object, like a stick placed in a stream of flowing water whose velocity is gradually increasing (which he did not illustrate, nor did he mention Leonardo). The pattern of ripples caused by the obstacle develop and change according to certain regular, mathematically definable rules, up to a certain point, literally an instant in time, after which the water becomes turbulent in the specific sense that it can no longer be described mathematically. The flow of water becomes unpredictable at the moment of what he called catastrophe. The problem of interest to the scientist was to try to push back (or

forward) as far as possible the moment of catastrophe, reducing to a minimum the limit of predictability. The scientist then spoke of catastrophe, turbulence and predictability—all of which I, as an art historian used to thinking in biblical and Christian eschatological terms, translated mentally into the term "chaos," which, as far as I can recall, Professor Wightman did not then use.

His presentation struck me, as an art historian, like a bolt of lightning because it gave me a completely new sense-insight, I like to think-into what might be described as one of the major categories of the mind of Leonardo da Vinci. In fact, stunning parallels can be found among Leonardo's studies of water and hydrodynamics, not for just one or two points, but for practically every detail of Wightman's account. No less important than these details, however, some of which I will show you presently, was the light this realization in turn shed on another heretofore unposed, let alone unanswered question, namely, why was Leonardo so inordinately obsessed with water in the first place? Of course, as we all know, Leonardo was fascinated by virtually everything he saw, and it could be argued that, speaking relatively, perhaps even absolutely, he saw a great deal more than anyone else either before or since. But within the vast range and depth of Leonardo's perceptual and intellectual ken, recorded in innumerable drawings and verbal observations in his great legacy of notebooks and study sheets, it is abundantly clear that water preoccupied him much more than any of the many other subjects to which he turned his attention. This is evident for several reasons. One is that water was one of his lifelong interests. He studied water practically without interruption from his youth until his death.

Secondly, the study of water constituted for him a coherent theme, ranging in scope from minute observations to abstract theory. Already as a young man he planned to write a great treatise on water, and in his notes throughout his life he continued to speak of and prepare this comprehensive work. Like so many of his dreams, it was never realized; the surviving notes give no hint of how he planned to organize the work, and no indication of its line of argument, and I doubt whether it ever reached that level of conceptualization. Even so, and this is a third point that indicates the special role of water in Leonardo's thought—the sheer volume of his studies of water, hundreds of drawings and notes, exceeds by far that of his work on any other single theme. Leonardo was truly enthralled, not to say obsessed by water—so much so that this fact in itself calls for explanation.

Many of his observations and theories, including some of his most spectacular and famous drawings, were of a practical nature. Some of his most fabulous and grandiose ideas were intended to achieve, sometimes quite unrealistically, concrete and useful results. He wanted to redirect the River Arno (**Fig. 1**) and introduce a system of canals (**Fig. 2**). He planned great projects and invented ingenious devices putting water to work in a variety of ways. However, the great majority of water studies were not of this kind. They are not practical but theoretical studies of what we would now call the science of hydrodynamics. Professor Wightman's explanation of turbulence and catastrophe struck me like a bolt of lightning because it suggested a reason why Leonardo had such a passion for water and its movements.

I may illustrate how Leonardo confronted the problem of water by considering a few among the hundreds—more than 1000—of preserved water studies. On one sheet he

3

studies various kinds of what might be called contrary currents (he calls them movements): above, he indicates currents that cross one another and create countercurrents, noting that the lower wave will pass under the higher one (**Fig. 3**). The second sketch shows water in descent ("in discendenza") over an irregular bottom, the point being that the water will flow faster at the surface than down below. The next demonstration below concerns water in "percussion" (in percussione) and in "resurgence" (in risaltazione): water falls into water and Leonardo studies the effects of the impact; here the point is that a light object caught between the two movements will remain in the same place as long as the velocity of the stream is constant. At the bottom of the sheet, he studies water running horizontally in a channel with an irregular bottom, illustrating the point that in this case the water will flow more slowly at the surface than underneath.¹

Another sheet shows the same kind of study, that is, the crossings of various wave currents, and he seeks to classify each type of wave and each type of current (**Fig. 4**). In another group of studies Leonardo confronted precisely the problem described by Professor Wightman, water running against and around an obstacle. On one particularly important sheet such a study appears at the upper right, illustrating the effects of shifting the angle of the immersed impediment (**Fig. 5**). Below is an enlarged illustration and detailed description of water in percussion: that is, water is itself the obstacle to water, and in this case the contrast is between the resulting currents on the surface, under the surface, and surging upward carrying bubbles of entrapped air. The relationship between air and water, both in combination and as analogous media, was also a subject that

¹ L'onde che per contari moti si sconterranno insieme, quella che fia in pit basso sito seguira suo corso sotto la pit alta—L'acqua che con declinante movimento discendera sopra grobbuloso fondo, fia di pit veloce corso in superfizie che di sotto—Quel corpo di lieve quality, it quale si trovera infra la percussione e la risaltazione dell'acque, mai mutera sito stando it flume d'equale movimento—L'acqua di globbuloso fondo e di piana superfizie si movers con pit velocity in fondo che di sopra" (MS A, fol. 24v, Brizio 1980, p. 281)

greatly preoccupied Leonardo and played a critical role in the development of his thought that concerns me here.

As frequently happens with Leonardo, his observations are truly amazing. I once showed a group of the drawings to a hydraulic engineer here in Princeton, who having studied them for several weeks, was also astonished by them, reporting that Leonardo evidently saw with the naked eye, as it were, some actions of water that have been confirmed only in modern times with sophisticated methods of observation, measurement and control. Such observations are popularly attributed to a superior physiological apparatus with which Leonardo was endowed, but I doubt it. I suspect that his studies are just as much the product of cerebration as of observation, deductions or inductions from what can be seen to take place to what must be assumed to take place. It should be said that the reverse may also occur. There are, for example, certain anatomical studies based on actual dissections, where details that Leonard must inevitably have seen are recorded in accordance with accepted tradition, rather than direct observation.

In this case, although the effects of falling water in percussione and risaltazione are discretely analyzed and lucidly displayed, the complications seem on the verge of getting out of control, of reaching the boiling point, as it were. It was from this effect, I believe, that derived toward the end of Leonardo's life a coherent group of drawings that are among the most spectacular and mysterious works in the whole history of western art. These are the so-called Deluge drawings, preserved in the Royal Library at Windsor, in which Leonardo seems to record an apocalyptic vision of some ultimate, global catastrophe (**Figs. 6, 7**). In the drawings a landscape is seen far below—technically a bird's eye but here more aptly described as a cosmonaut's view—about to be overwhelmed by a great mass of turbulent, boiling water. In the work that seems to be the apogee of the series, only the vast storm is shown, in which rushing currents of air, clouds, rain and water are discernible but also merge imperceptibly with one another (**Fig. 8**). The drawings cannot in any literal sense represent the biblical flood, if only because that event was traditionally identified in terms of the destruction of man and beast, or their salvation in Noah's ark. Nor can the drawings actually portray the end of the world which, as was universally assumed in the Christian tradition, would come by fire. On a metaphorical level, however, the Deluge and the End of the World were closely related concepts; the End of the World was often defined as a second Deluge.

What fascinates me in all this is the possibility that through his studies of wave motions and currents, and especially his preoccupation with the effects of moving water encountering obstacles, including water itself, and the surrounding air, Leonardo may have done something more, or at least something other than anticipate modern observations of hydrodynamic phenomena. Leonardo may have arrived not simply at an idea of turbulence and catastrophe as narrowly defined but at a concept of uncontrolled universal turbulence, akin to the ancient ideas of an original, primeval chaos—like that recorded in the second verse of Genesis: "And the earth was without form, and void; and the darkness was upon the face of the deep"—from which the world was formed and to which it will one day inevitably return. I suspect Leonardo had in mind something like a cognitive definition of reality, identifying rationality with the world we know and irrationality with the protogenic world we cannot know. It amounts to a kind of epistemological philosophy, or indeed an epistemological metaphysic in which order and knowledge inhabit the world that exists, disorder and ignorance that which came before and will come after.

I must emphasize that this definition of a coherent and explicit cosmology is strictly my own. Leonardo nowhere speaks directly of turbulence as an abstract idea, and he has no concepts of predictability or non-predictability as such. He does express many ideas, however, that reveal a sense of inner unity and coherence in nature, ideas that suggest to me, at least, that the water studies and the deluge drawings form part of a single conceptual continuum—or rather discontinuum. The element of continuity underlies the very multiplicity and complexity of water, in which Leonardo obviously saw reflected important principles of nature. The discontinuity between the deluge drawings and the rest of Leonardo's work on water is evident in several ways. The drawings are complete unto themselves, and without the explanatory texts that frequently accompany the studies proper. There is no indication, as far as I know, that the Deluge would have been explored in the treatise on water; the concept seems to have stood apart in this sense, too, as if outside the domain of rational analysis. And they convey a sense of general cataclysm not only by the wildly exploding and gyrating forms they depict but also because the events themselves are shown as vast, all-encompassing land-, sea- and airscapes that fill the whole page. Conversely, Leonardo seems never to have depicted such chaotic disintegrations in isolation and on the mini-scale of his other experimental and theoretical studies. This treatment has an important psychological, not to say metaphysical concomitant by virtue of the fact that in the isolated water studies the source, or cause of the effect, whether it be a rushing stream or a cascading waterfall, was plainly evident, whereas the cause or source of the deluge drawings lies somewhere off

the page, in the universe at large. One might say that In Leonardo's ken chaos appears only in large, complex systems. Perhaps the greatest miracle of all is that Leonardo managed to convey the idea of total and absolute disruption of all natural order with a piece of black chalk on sheets of paper measuring barely 6 by 8 inches.

Some scholars have suggested that the drawings may refer to specific, particularly violent storms, earthquakes and the like, that occurred in Italy and elsewhere during Leonardo's lifetime, of which he took careful note. But no specific site has ever been identified, and even if they are related to actual events, the drawings give an almost surreal sense that the very cosmos itself is unraveling. On the other hand, they are certainly not the inchoate scribbles of a feverish imagination spinning out of control. Although the drawings themselves are not accompanied by discursive texts, we do have many comparable verbal descriptions by Leonardo of such disastrous events, and even the most horrid and pathetic of these are presented in the sharp, precise language of an objective, if impassioned observer, as subjects for representation.

OF THE DELUGE AND HOW TO REPRESENT IT IN A

PICTURE. Let the dark and gloomy air be seen buffeted by the rush of contrary winds and dense from the continued rain mingled with hail and bearing hither and thither an infinite number of branches torn from the trees and mixed with numberless leaves. All round may be seen venerable trees, uprooted and stripped by the fury of the winds; and fragments of mountains, already scoured bare by the torrents, falling into those torrents and choking their valleys till the swollen rivers overflow and submerge the wide lands and their inhabitants.²

Clearly, both the descriptions and the drawings are the products of careful, detailed analysis and reflection on the theme of universal cataclysm.

Hints of the nature of the underlying relationship between the water studies and the Deluge drawings are suggested by many kinds of observations in which Leonardo connects, sometimes quite unexpectedly, widely divergent aspects of nature. In one of the drawings of water passing an obstacle, for example, the wake forms patterns that strikingly resemble tresses of braided hair (**Fig. 9**). The similarity is not accidental, for in the accompanying text Leonardo explicitly describes the relationship.

Observe the motion of the surface of the water, which resembles that of hair, which has two motions, one of which depends on the weight of the hair, the other on the direction of the curls; thus the water forms turning eddies, one of which follows the impetus of the main course while the other follows that of incidence and reflection.³

² W. 12665v; Richter 1970, I, p. 306-7, No. 608: Vedeasi la oscura e nvolosa aria essere comnattuta dal corso di diversi veti e avvilupati dalla continua pioggia e misti colla gragnuola, le quali or qua ora làportauanoninfinita amificatione della stracciate piante, miste con ifinite foglie; d'intorno vedeasi le antiche piante diradicate e stracciate dal furor de' venti, vedevasi le ruine de' moti, già scalzati dal corso de' lor fiumi, ruinare sopra i medesimi fiumi e chiudere le loro valli; li quali fiumi rigorgati allagauano e somergieuano le moltissime terre colli lor popoli .

³ W. 12579r; Richter 1970, I, p. 200, No. 389: Nota il moto del liuello dell' acqua, il quale fa a vso de' capelli, che ànno due moti, de' quali l'uno attēde al peso del uello, l'altro al liniamento delle volte; così

He thus saw an underlying homology between liquids and solids under certain conditions. He drew the analogy between moving water and the movements of air between liquids and gasses, or between hydra- and aerodynamics, we would say; and between the movements of fishes and birds in their respective media. He observed the similarity of the network of channels formed by a river and its tributaries, to the network of blood vessels in the body; that is, he saw a dynamic mechanism common to the drainage system of the earth and the human circulatory system.

All this shows that with astonishing breadth and depth of observation and conceptualization Leonardo passed from one domain of nature to another, and he was able to do so precisely because he perceived the underlying community between aspects of the world that seem superficially quite unrelated. What he arrived at, finally, was an independent, abstract notion of pure force or movement, and I believe this universal dynamic is the global idea embodied in the deluge drawings. Considered in this light, it is no accident that precisely in his studies of water Leonardo defined two basic principles that remain central to our own understanding of the inner workings of nature. One of these is now called the principle of continuity, which he illustrates in two, complementary drawings, one concerning the width the other concerning the depth of a river. "A river of uniform depth will have a more rapid flow at the narrower section than at the wider, to the extent that the greater width surpasses the lesser." "Every movement of water of equal width and surface will run as much more swiftly in one place or another

l'acqua à le sue volte revertiginose, delle quali vna parte attende al inpeto del corso principale, l'altro attēde al moto incidēte e reflesso.

as it is less deep in the one place or the other" (**Fig**. **10**). ⁴ In other words, the quantity of water passing a certain point remains constant, despite variations in the shape of the channel.

The second principle is embodied in Leonardo's illustration of two balls (corpo sperico), one thrown a long distance, the other thrown toward the ground so that it bounces (**Figs. 11, 12**).⁵ In the accompanying text Leonardo notes that the two balls travel the same distance in the same amount of time; he assumes that the two trajectories are identical with respect to time and distance. Of course, Leonardo disregards the effects of friction, but the underlying assumption is akin to our idea of the conservation energy. The same principle governs the action of water passing an obstacle. He notes that the water moves the same distance despite any intervening obstacle. In fact, there are two movements: that moving around the obstacle and that forming the eddies of the wake, and the water in the broken eddy reaches the same point at the same time as the smoothly flowing water. He states the idea as follows:

The force of movement created by any force is a product of the proportion between that which is moved and that which moves; and if the force meets any resistance, it will complete its proper distance by going around or

⁴ "Il fiume d'eguale profondità, avrà tanto più di fuga nella minore larghezza, che nella maggiore, quanta la maggiore larghezza avanza la minore."(MS A, fol. 57r; Gombrich 1969, p. 180 n. 24) "Ogni movimento d'acqua d'eguale larghezza e superfizie correra tanto piu forte 'n uno loco che nell'altro, quanta fia men profondo nell'uno che nell'altro." (MS A, fol. 57r; Gombrich 1976, pp. 43, 139 n. 24)

⁵ Ogni corpo sperico di densa e resistente superfizie, mosso da pari potenzia farà tanto movimento con sua balzi causati da duro en solio smalto, quanto a gittarlo libero per l'aria. O mirabile giustizia di te, primo motore! Tu non ài voluto mancare a nessuna potenzia l'ordini e cqualità de' sua neciessari effetti. Richter 1970, II, p. 285, No. 1134 (MS A, fol. 24r; Gombrich 1976, p. 45, fig. 93)

jumping above so that computing the time and distance, the course would be the same as if there had been no obstacle.⁶

The formulation is incredibly simple, beautiful and complete, especially considering that it was far from obvious to the naked eye, and scarcely the product of intuition. Leonardo must have reasoned it through. The ideas of continuity and conservation taken together in turn underlie Leonardo's wonderful studies of vortices and whirlpools created in a widening channel when the swiftly moving water encounters the slower water ahead and compensates by turning back upon itself (**Figs. 13, 14**). To my mind, the coincidence of opposites embodied in these notions reflects a rational conception of the universe based on the assumption of an underlying symmetry in nature. Indeed, one might say that the almost Einsteinian ideas of a single force and an ultimate symmetry constitute the twin peaks of Leonardo's studies of water.

One of Leonardo's most moving apostrophes to the beauty of creation refers to this very combination of force and symmetry:

O, wonderful justice of you, the Prime Mover. You have chosen not to deprive any movement of the order, and quality of its necessary effects.⁷

The idea of the Prime Mover is ultimately derived from Aristotle, of course, except that here the nature of the prime mover is defined in terms of the complementarity of

⁶ MS A, fol. 60v (no illustration); Gombrich 1976, p. 139 n. 32.

⁷ Cf. n. 5 above.

movement and its effect, that is, the symmetry of forces. In turn, the combined ideas of force and symmetry became the basis for distinguishing between being and non-being, rationality and chaos (**Figs. 15, 16**).⁸ To indicate the consistency, profundity, and completeness of Leonardo's thought, I shall read another passage in which he speaks of the Book of the Waters. To understand the statement fully one must realize that throughout his life Leonardo pondered how to begin the work. There are many passages written at various stages in which he says that in the first book of the treatise on water we will speak of this or that point, and in one of the last of these observations, made at the end of his life, he proposes to begin the Book of the Waters as follows (**Fig. 17**):

The first book of the waters.

Nothing shares a surface with something and something shares a surface with nothingness. And the surface of something is not part of that thing, whence it follows that the surface of nothingness is part of nothingness, whence it follows that a single surface is the limit between two things that are in contact. Since the surface of water is not part of the water, and hence is not part of the air or of other bodies placed between them, what is it then that divides the air from the water?⁹

⁸ Windsor 12385; Pedretti-Clark 1968, I, p.55, II, pl. 12385; I, p. 56, II, pl. 12387.

⁹ Primo libro delle acque. Il nulla ha superfizie colla cosa e la cosa ha superfizie col nulla; e la superfizie della cosa non e parte d'essa cosa: seguita che la superfizie del nulla non è parte di tal nulla, onde è necessario che una superfizie sola sia termine comune di due cose che sieno in contatto: come la superfizie dell'acqua non è parte d'acqua e per conseguenza non è parte dell'aria, nè altri corpi infra lora s'interpone. Che è quel ch'adunque che divide l'aria dall'acqua?...Brizio 1980, pp. 545f., Cod. Arundel (Brit. Mus. MS 263), fol. 159a/b (my translation).

Thus, at the end of his life, at the same time he was making the deluge drawings, he would have begun the treatise on water with a question, a question that arises from his perception of water as the junction between nothingness and existence. That question is the beginning and the end.—

APPENDIXES A AND B

APPENDIX A: Letter from Frank Wilczek commenting on this paper

The "principle of continuity" in hydrodynamics as formulated in the quotations from Leonardo is consistent with modern hydrodynamics. In fact his formulation suggests the modern "equation of continuity" which embodies the conservation of mass. On the other hand the idea that equal "forces" or "impetus" will lead to equal distance travelled per unit time is not correct (that is, it is in contradiction with modern mechanics), whether friction is neglected or not. I believe it is taken from the Medieval <u>impetus</u> theory. According to that doctrine (roughly speaking), motion is a quantity initiated by an external agent which produces a given displacement per unit time. This is related to the modern idea of <u>momentum</u>, which is conserved in the absence of external forces and, where constant, produces another motion.

However in the example of a bouncing ball, and still more in the case of water flow, the ball (or water molecules) are definitely subject to external forces – gravity in one case, the pressure of neighboring water molecules in the other. Under those conditions, it is not at all true that particles travel equal distances in equal times.

Unfortunately the second principle you quote, which is essentially the same as the previous ballistic principle (but not the <u>correct</u> principle of continuity) is just wrong. So I think referring to it as "incredibly beautiful, complete and persuasive" is misguided. I am also not sure how all this points to any underlying "symmetry of nature". It is remarkable how in the case of water, as apparently in his creations, Leonardo created compelling illustrations of a <u>conceptual</u> Universe which convincingly resembles, but is definitely not, ours.

Best,

Frank W.

APPENDIX B

M E M 0 RAN DUM

TO: Frank Wilczek

FROM: Irving Lavin

DATE: May 14, 1993

Dear Frank,

Many thanks for your comments. My problem, I think, is that I think I understand you, but I'm not sure I understand Leonardo! You are absolutely right that he believed in "impetus," along with the Aristotelian theory that all motion tended to come to a halt (a sort of gravity without direction, I suppose). Therefore while eliminating friction he was also implicitly eliminating gravity in favor of an inherent inertia (rather than an external force) presumably affecting both balls in the same way and degree. Impetus and the absence of external force perhaps also explain why he thought that water passing an obstacle would, by going round and jumping over, reach the same point in the same time as if there were no obstacle. I realize it's a silly question, but given these assumptions, would he still be wrong? If not, would it relate to conservation of momentum or energy? The really critical point, I agree, is the one you stated in your last sentence: a conceptual universe that convincingly resembles (perhaps more so than any before), but is definitely not, ours. The same description applies to his greatest paintings--though I don't know anyone who has said it so beautifully!

I hope you will talk to me some more when I return in August. Meanwhile, warm thanks again.

Leonardo's Watery Chaos—Bibliography of Works Cited

Brizio, Anna Maria, ed., Scritti scelti di Leonardo da Vinci, Turin, 1980.

Gombrich, Ernst Hans, "The Form of Movement in Water and Air," in O'Malley, Charles Donald, *Leonardo'sLegacy. An International Symposium*, Berkeley and Los Angeles, 1969, 171-204

The Heritage of Apelles. Studies in the Art of the Renaissance, Ithaca, N.Y., 1976.

Pedretti, Carlo, ed., Kenneth Clark, The drawings of Leonardo da Vinci in the collection of Her Majesty the Queen at Windsor Castle, 3 vols., London, 1968.

Pedretti, Carlo, ed., Leonardo da Vinci. Nature Studies from the Royal Library at Windsor Castle, exhib. cat., Florence, 1981.

Ravaisson-Mollien, Charles, *Les manuscrits de Léonard de Vinci*, (vol. 4), *Le Manuscrit A de la Bibliothèque de l'Institut*, Paris, 1881.

Richter, Jean Paul, The Notebooks of Leonardo da Vinci, 2 vols., New York, 1970.

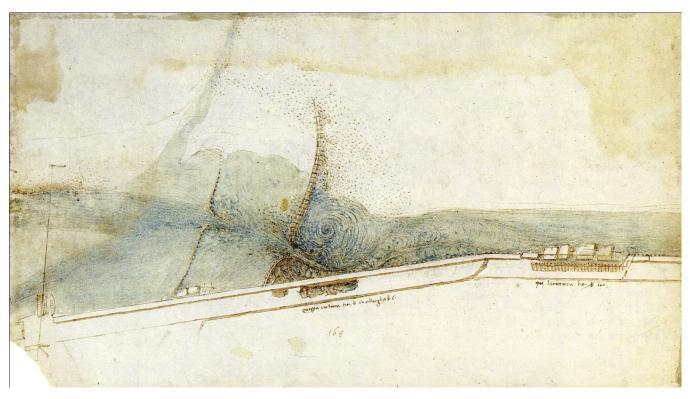


Fig. 1 .. Leonardo da Vinci, plan for diverting the Arno, ca. 1504. Windsor, Royal Library, 12680.

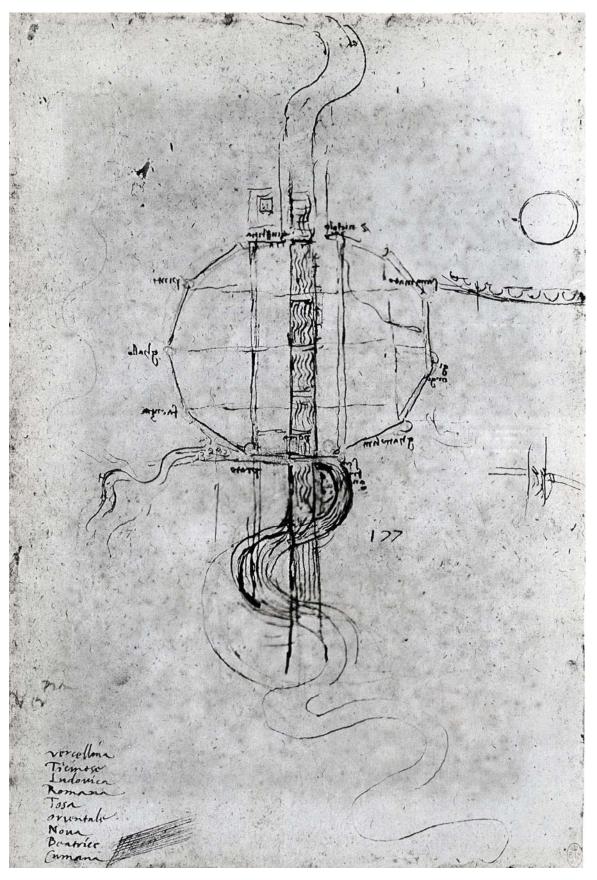


Fig. 2 Leonardo da Vinci, plan for straightening the Arno, ca. 1515. Windsor, Royal Library, 12681.

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Fig. 3 Leonardo da Vinci, movement of water, ca. 1492. Paris, MS. A, fol. 24v.



Fig. 4 Leonardo da Vinci, movement of water, ca. 1508-9. Paris MS. F, fol. 92v, detail.



Fig. 5 Leonardo da Vinci, movement of water, 1507-9. Windsor, Royal Library, 12660v.

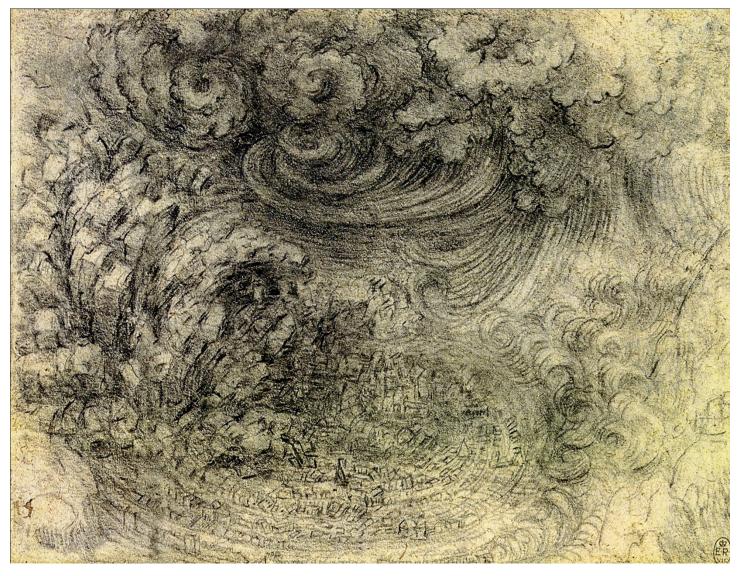


Fig. 6 Leonardo da Vinci, Deluge, ca. 1512-16, Windsor, Royal Library, 12378.



Fig. 7 Leonardo da Vinci, Deluge. Windsor, Royal Library, 12384.

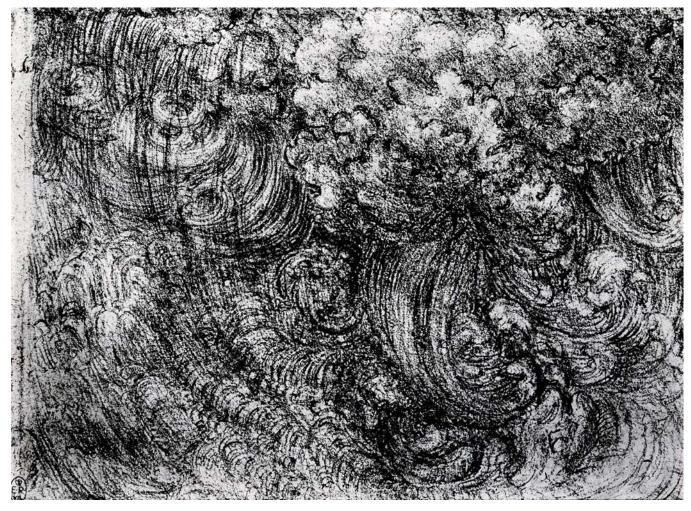


Fig. 8 Leonardo da Vinci, Deluge. Windsor, Royal Library, 12383.

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Fig. 9 Leonardo da Vinci, movement of water, ca. 1513. Windsor, Royal Library, 12579r, detail.

bole townare allow comine commercialing . might fatt amore come fine for fish tern and the Carge while De Bern - grade elle argive girmin potrolin the milto בפיל באגנייי אירדי כיו יכותי לע לבלי את כיוז לאואים אובצה אבלי יזיל ווה לי באילי זינהי אתוצווה איצו. [NA. PRIMANALARA (prostog of votos the for the we set or po poterol Dis Gip: Secution the day of the far of cupiterer duige of the ilegun for Qui fre prushe from . 202. Bullet from . opicon no-figh for agoit. לינהיותי כ האמה אל כ יוויאו יוקם פט - י איזיאו יוקם פט - י איזיאו י כ קעוקם או ליכפס כ אהייה דים אוריודי להי לידי לילי לי הבנם פלי לעת איירי אי לפצ חשירי נין יאולי את. אייחיון [pingie]]. אתרהם ולפינים כמישר דרו יצי כקומילה שמלשותי פי prinne : - יישריטר יניר ל אחריו ידיום ציטלת למיר כי ונידי דני יצו לחיר היאר ירי עבוידי the man ment of the ment of the second of th)- (cilitation mode - homen - iltriger Ciferman - efoly magion frighter the mergion of Marians . Amonghin Baman Devidenno. So : 11. Ano ed an internet and inter be fulled a fulled as for eliadul. c.or. tucit an faire callant care square. Der whom is (sou Medulen. chun: whole offer . Lubro escore of oly est ul wurder: עייליות אונות שווא שווא עושאי הדעורי אין איי איוויי אין איי אייוויא איין איי איין איי ofine of inf in where i light of the state o may: mi UNTIT כצייות: (חרוןאת. ניףועיניוניוי ובי (האיייה Oudal . New Sor Do . con to we we we do lo be we bole of con bo by de up לי זיין ארדה (האדה בשט המואה שי אדיהן האדה שניוני ויה איז ני (הי הנק אה י לאיר וי למי לנידים . E | . ולאי לם א. אין ה לתומה שואהוה (שאי להאידה אי we wanty . chow and . . (we we dre to wante we bed . whon of the אינגיאר ברי אולינדי איזאר יאנה לי אחלי אייזיי יחיש בירה אוני אחי אאידות while chede we fine esolo bolun preserve version and we will be Unished & Churchy etobe offrame fra . Cor besi ilmore belesonen velleforene gibudul bon fring 220.3. milder. civigen verybury - moulue. you want and and one were we wend you but in nut Ingenne your ill to ou Binette magoron . drunger - (undrove - (undre vounder quad non .srad febrig. On: (4. propo inone . Apricon . & comment . 20 formander . D. Parone . Golerman an of what we are we Aver: B. A. B. an what is anuto bi downer anno mider so candes but Port Agen & who port the dauge to get alle the ountil in holme S. ohn with mut in got. nul: -web. as AN THE HE WE AND ON EN Im Ji - wangelind - 1/4 A . my Gara wile bile date

Fig. 10 Leonardo da Vinci, movement of water. Paris, MS. A fol. 57r. (click here to return to text)

AT virised invary off our . with in light will an burned a shere as in a flow Ogm. cfor form. make makerning a hansel of an ball of an Simulo. No. Harin Ommer bill gib han & wpime motor ... In non to bo we magne . AN. Www. barren . Carton guilde forment dole pin duci cedarige u: (the offighe trace min fibo ar orguns cirge bound by Licky fi. wope . we put in . I dave . by make a first of the law we founds . and the practice practice 40 - My num vis bund . Just a set in sugar is gard . House is gurd . www. in suge gund . En what fur . which ar unever the worder was fare , our frank ale the Berie Wind aning when gouger burne a generate pagent as in the UN: 010 A pr. H.V v3:14:4 להקצי הציום בקטיוור חוזי משבר לתלקותום להיות אם יולהלבי תכלי בלסלטיו כי לה לתה לא אותוום זוח זות הסביחן להלבי איילה להיבה אי להן בייאי איל מב כב ון להיא אווי ויווי ואווי קאוויים ולה לבי אם א אירוי "I and in survey of an analla et. the for pit in the forman Buckape out it לם לד אילי לי לא לא ייייון ילם אימוים מצעוטי לי לראי יואי אקו א אילא ייי איל יי לא ניצי אייים לאלבי לולחכרו אווי בי איזר איוקטלו אי ויוומנרוט לאלחלא כחפולכיא שי חדיאר לו ואוטינינס סיבי יווי בלום איי הם אישותי לגלחיקי לי שבלו אי לבלאי יוסטן להאימףי לתרקעילה אווי אין ידידות קאי לה למאה זף לעלם והראים ני אומיה ולפרים שילי שרבטני ניאון מייח im our relation of the provide of a special of the stands At pin A pip big Anna An Butan: course that or the biling ally two gatan Within wind wither . Warning . Waluny . Willing

Fig. 11 Leonardo da Vinci, diagram of the law of rebound, ca. 1492. Paris, MS. A, fol. 24r whole page.

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Fig. 12 Leonardo da Vinci, diagram of the law of rebound, ca. 1492. Paris, MS. A, fol. 24r, detail.

vite uptime bivue uttento tiv alle e Casif to po que le famolia allugur ogenb. adag : b. elly anoun wended be valuer . . . Bod. . were of in gaogener, efter alguan. A tola. as no da. quale maired dolar foil no gane rifater indto com דופא שוות הלדי לם דיולת ולויחון להחור ביול ניפס וילה חושה שלה הולו שול ויול אי לאל עול (a summer interes Servers Server and all fur more for moter fut manuer lano ch. laftere G. Gola Gable 1 NN. 1: Woli I luinto ciminto fingle wello mor meto Acquello . Sicher Auto www. un with summun our stige soles in municipal of pro כנידבים יות אוויואנרי ול שבפיילים וניפטים וה שטוראות אי לאי (אי כיאקוסחו יו outiled o und outs איי איווכרי לה למחקב באינ בסווק באושי כאפירוי שופטו או באירו בי א לאוייר (מבקומה - בציבון אדי יאדה למכבצם. שייי לי כל המחקד יצי האמיות יויותי הייותי הצימרו. לי יציה שיי אי לאיו of an under succession of succession of the succession of an under succession of the ון אי העי עוג זיין מסריי יא קיער לשי כקה (הלקוע ל שירה עי הואי הלקי ני אפטרי אי לא אי איי את מבאמת . קמונ (בחיצו לי ואיזוה לתלורה היידים שתני נספי יי אי פוי לי כלי כי אייוה איצייר באמינה ושא שניהיה אישמטני אי איאאיי ע איני נצייר אישו איוא לאוי הניון ופתר לו לסלות זהל מינידם לביקעוות לה יולטיושים mindom aning atogut Andar a aning ato an aning atom aning י ליוחן (רוב הרצטוקה ולעם אין יאיינים אם פני נא אי ווז אי זייטןי חטוייטטא לי חטו איי ז לחוי דם שלבי זה אין יואר כרט לה מעתני (הכרס התקווה לני מטלאי לאווט יה לה אויט יה לה לאחר ניאור ביקער את משכתלמוטיה: להעותי לילה שוט לחדה כור לצעלחר: ידי טולעווטאר - לה אות ite bis. Cande c. bis Governew. Des. nomas begenter (: w. f. m. g. f. w. c. du. לט יוים (כידו אנ בם (אזוותאי כפי שנידו אוני או עו אני שי (י כ will not fut time tord. Do be (no women all as by gull a ste during terminis williame vedine inge basin funde more there for still been toff an e femero

Fig. 13 Leonardo da Vinci, vortices of water turning back, ca. 1492. Paris, MS. A, fol. 60r.



Fig. 14 Leonardo da Vinci, vortices of water turning back, ca. 1492. Paris, MS. A, fol. 60r, detail.



Fig. 15 Leonardo da Vinci, Deluge. Windsor, Royal Library, 12385.

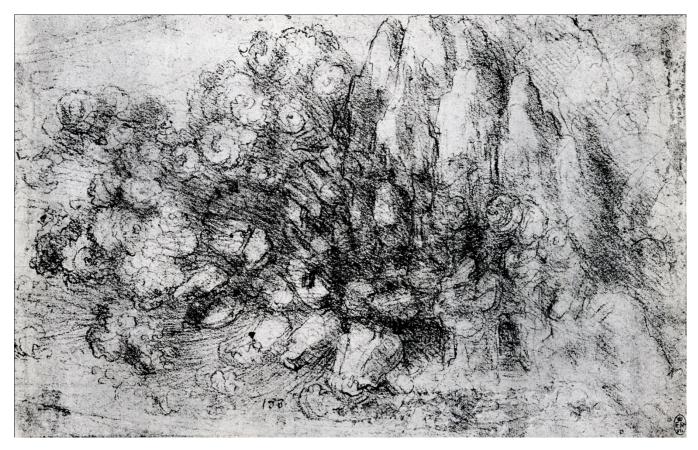


Fig. 16 Leonardo da Vinci, Deluge. Windsor, Royal Library, 12387.

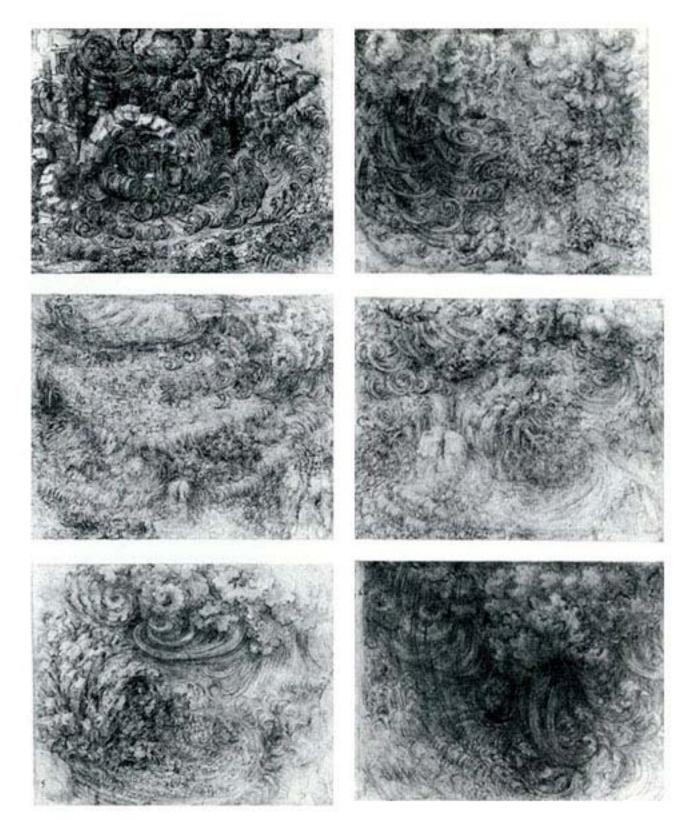


Fig. 17 Leonardo da Vinci, Deluge Drawings. Windsor, Royal Library, left: 12380, 12385, 12387; right: 12384, 12386, 12383