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Every paper on technical mathematics offered for publication should be submitted with return postage to the Chairman of the appropriate Committee, or to a Committee member whom the Chairman may designate to examine it, after being requested to do so by the writer. If approved for publication, the Committee will forward it to the Editor and Manager at Baton Rouge, who will notify the writer of its acceptance for publication. If the paper is not approved the Committee will so notify the Editor and Manager, who will inform the writer accordingly.

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This Journal is dedicated to the following aims:

1. Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values.

2. To supply an additional medium for the publication of expository mathematical articles.

3. To promote more scientific methods of teaching mathematics.

4. To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Humanism and History of Mathematics

Edited by
G. WALDO DUNNINGTON

Biographical Sketch

OTTO NEUGEBAUER

Professor Otto Neugebauer was born May 26, 1899, at Insbruck in Tyrol, the son of a railway construction engineer, Rudolph Neugebauer. His parents died when he was quite young; his boyhood was spent at Graz in Styria, where he grew up and attended the secondary school, graduating in March, 1917. From October 1917 until November, 1918, he was in military service on the field, with an Austrian mountain battery principally on the Italian front. At the signing of the armistice he was taken prisoner by the Italians. Returning in the fall of 1919, he studied mathematical physics at the University of Graz under Michael Radaković and Roland Weitzenböck. While studying at the University of Munich in 1921-1922 under Arthur Rosenthal and Arnold Johannes Wilhelm Sommerfeld, Neugebauer was so stimulated by their lectures that he decided to devote his life to mathematics. Moving on to Göttingen the following year, he studied mathematics under Professors Richard Courant, Edmund Georg Hermann Landau, and the late Emmy Noether, Egyptian under Professors Hermann Kees and the late Kurt Sethe.

At the University of Göttingen, Neugebauer became an assistant in the department of mathematics in the fall of 1923, the following October (1924) special assistant to Courant, at that time head of the department. Göttingen conferred the Ph.D. on him in 1926; the doctoral thesis is a study of Egyptian fractions. He received the "venia legendi" for the history of mathematics (December 17, 1927) and began lecturing several months later. Further promotion came in 1930 to "Oberassistent" and on January 26, 1932, from Privatdozent to associate professor. Neugebauer was granted at his own request a leave of absence from Göttingen on ~~June 4~~, 1934, and went to the University of Copenhagen where he has since remained. He is married and has two children. For purposes of scientific research he had previously (spring, 1924) spent some time with Harald Bohr in Copenhagen, with Father Deimel in Rome on Sumerian, and in the fall of 1928 with W. W. Struve and B. A. Turajeff in Leningrad.

← Jan. 18,

As part of his manifold activities Prof. Neugebauer edits two important periodicals "*Zentralblatt für Mathematik und ihre Grenzgebiete*" and the "*Zentralblatt für Mechanik*"; in addition, he edits the two valuable series of monographs, the "*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*" and the "*Ergebnisse der Mathematik und ihrer Grenzgebiete*". In 1932 appeared no less than six distinct contributions from his pen dealing with the history of ancient algebra, the sexagesimal system and Babylonian fractions, Apollonius, Babylonain series, square root approximations, and siege calculations. A later paper covers formulas for the volume of a truncated pyramid in pre-Grecian mathematics (1933), followed by monographs on the origin of the sexagesimal system, the geometry of the circle,and the application of astronomy to chronology in Babylon.

Professor Neugebauer has announced a series of three volumes on the history of ancient astronomy and mathematics. The first volume is "*Pre-Grecian Mathematics*" (1934). Volume two will treat "Grecian Mathematics" and volume three "Babylonian and Grecian Astronomy". The author has given us in volume one our first complete presentation of Babylonian and Grecian mathematics. It is gratifying to find a scholar who lives up to the high standards he sets in our accompanying article. He does not lose his way in a maze of details, but portrays for us the evolution of ancient mathematical thought and exhibits the foundations on which our present knowledge of ancient mathematics is based. The volume closes with a discussion of Babylonian geometry and algebra; not until recent years has much been known about this subject, and Neugebauer has done more than anyone else to clear it up. Within a few years he has become one of the leading historians of mathematics and probably the most eminent living authority on ancient mathematics.

There remains the pleasant duty of mentioning Neugebauer's monumental edition "*Mathematical Cuneiform Texts*" which appeared in 1935, in two volumes. Space does not permit a full discussion of them here, suffice it to say they have been heaped with praise in all journals. Much of the material offered is interesting to the Assyriologist, the general historian, and the philologist. Architectural, engineering, economic and legal problems are touched on. The texts cover nearly two centuries and deal with the most diverse mathematical problems. One finds here a very complete bibliography of Babylonian mathematics.

G. W. D.

OTTO NEUGEBAUER

The History of Mathematics

By OTTO NEUGEBAUER
University of Copenhagen

Translation by Departmental Editor
G. WALDO DUNNINGTON

The historiography of a science usually does not enjoy any too high an esteem among its productive representatives. The reasons for this are several and they are not difficult to recognize, especially in a science like mathematics, which can distinguish with such precision between secured possession and unsolved problem. Mathematical problems and methods are indeed, like every other element of existence, historically conditioned, but that portion of the way already traversed—which for their continuation one must know and as such survey—is a relatively short one. Probably long centuries worked only in closest connection with antiquity, but the great rise of modern science begins with the appearance on the scene of entirely *new* ideas whose import lies in the opening up of hitherto unknown questions rather than in the settlement of old ones. In all natural sciences definitive results may have an absolute value—in mathematics a settled theory (the technical term for this is a "classical theory") is something dead, which cannot captivate fresh forces.

With this set-up the non-historical character of mathematics is not to be wondered at. In addition however, or rather connected with the above is the fact that the existing historical presentations of this science cannot interest the professional because their authors, as outsiders, with all their formal knowledge of subject matter, do not touch the real essence and interest of the problems. Instead one finds things treated such as the subject "Geometric forms that were in existence before the advent of life on the planet" (in a "History of Mathematics" appearing in 1923), which have nothing at all to do with mathematics. And only too often an anecdote collection, or even worse, an endless chain of priority questions must replace *history*.

It would not be worth while to write about such things, if one had to regard them as irrevocably united with the substance of the history of mathematics. Indeed I do not believe that the above mentioned purely objective relationship between progressing research and its history can be changed; but I regard it as an absolutely attainable goal, so to re-cast the history of mathematics *in itself*, that, like the history of philosophy, it will become an integral member in the series of modern sciences and not lead to an entirely meaningless existence untouched by mathematical as well as historical spirit. Then one will again dare to hope that even the purely professional mathematician, from this broader viewpoint, can profit by occupation with it.

I have already touched on the point where according to my opinion the chief deficiency in the present condition lies: in the lack of an *historical-problem* attitude. I should like to explain somewhat more in detail by a very special question the way I desire this to be understood: What is the treasure of mathematical knowledge which the Greeks took over from the Orient?

Immediately an objection: What interest at all does such a question have? Indeed it is quite immaterial to know whether some Egyptian or this or that Greek possessed a formula for the volume of a truncated cone or not. And such an objection really exists quite properly, as long as one contemplates such knowledge only on account of its absolute content, but not as points of demarcation on a scale which one needs in order to be able to draw any sort of historical comparisons at all. However immaterial it may be in itself, whether these points of the scale are constituted by propositions of "elementary mathematics" or by propositions of an optional brand of modern analysis—that we have to deal with the one or the other is historically accidental—nevertheless the gaining of reliable factual material becomes important as a *preliminary* labor in order to have safe ground under our feet, and not to go to seed by merely attitudinizing esthetically.

Here I must interpolate a purely methodical remark which refers to the securing of the foundation for answering the question asked above. Our entire tradition from the ancient Orient exhibits one great advantage: we scarcely know *one* name of an artist or scholar. The entire cultural evolution of those periods is from the very beginning most closely united with the national unit, its picture is presented in much more tranquil lines before a larger background, than in an epoch when the struggle for "master or school" or for "genuine or false" distorts the perspectives. The chronological arrangement necessary for every historical comprehension must of itself ensue in a much broader framework: *in the framework of general history*. Thus with all the scantiness of our knowledge the history of Egyptian art, religion, and indeed of linguistic history forms with the "pragmatic" history a much more closely knit unit than is the case for analogous Grecian conditions. Directly therefrom arises however the demand to connect the utterances of mathematical thought with these general viewpoints. Not until the investigation of purely objective questions takes place

strictly on the basis of the history of civilization can one expect to attain a relatively correct evaluation of the separate problems. Then too such an investigation obtains, on the other hand, a much more general meaning because it is able to bring to light in a quite precisely comprehensible manner very charactertistic features of a people.

The disappearance of the individual in the history of Egyptian civilization has not always been regarded as an advantage. Thus the poor scribe "Ahmes" who immortalized himself as the copyist of the mathematical Rhind papyrus (the most important monument of Egyptian mathematics), has had to assume all possible titles from "king" or "teacher in an agricultural school" down to unskilled "pupil". Or however one has taken refuge in an intentional concealment of the individual behind the very popular "priest castes", although they do not exist, at least in the genesis period of all Egyptian sciences, thus it is maintained: "Mathematics as a science was in Egypt the exclusive possession of the priest caste and was carried on in the priesthood as an occult science and concealed from the people,"—with such success indeed, that not the slightest vestige of this "occult science" has been handed down to us!

To the demand for arrangement of historico-mathematical investigations in the scheme of general history of civilization is joined a sphere of questions much more difficult of access, as soon as it has to do with ancient history: the connection with philology. Such a fundamental investigation as *Sethe's*[1] book "On Numbers and Numeral Words among the ancient Egyptians" shows how much can still be summoned from these things for the beginnings of mathematical thought. One must not carelessly pass by these things, as soon as one asks questions about the historical development of the most important categories of thought, especially in a period of mathematical research like the present, in which the question about the logical foundations of mathematics assumes a central position. Here is really one of the points where the most uncognate sciences encroach quite directly on each other, so that it is not pertinent to grant philosophical speculation the only decision.

If one turns to the actual content of pre-Grecian mathematics, the first impression is that it has an exclusively "elementary" character, and is in itself rather homogeneous: simple problems of calculation,

[1] Ths distinguished Egyptologist, Prof. Kurt Sethe (1866-1934), liked to observe the phenomena of Egyptian culture and civilization from the viewpoint of general history and by comparison with other cultures, to assign them their place in the evolution of civilization. His work Über Zahlen und Zahlworte bei den alten Ägyptern und was für andere Völker und Sprachen daraus zu lernen ist (1916), as well as a number of his other papers, definitely enriched the history of mathematics.—G. W. D.

executed in part with the help of numerical tables, or problems involving the calculation of areas and volumes of geometrical figures such as those demanded by agriculture or at the most stonemasonry. But if one observes the role which these things played in their own cultures, this picture is quite essentially changed and offers problems which are by all means worthy of historical investigation. The deep-reaching distinction between the two great cultural units Egypt and Babylonia, even in simple numerical notation and application of the first calculation exercises, asserts itself here quite essentially. The beginnings are indeed in both districts the same: Hieroglyphics with special signs for the most important numerical values 1, 10, $^1/_2$, $^1/_3$ etc. and a purely additive counting foundation of all calculation. Now however the individual development sets in. The fundamental problem of Egyptian mathematics (which bears a purely "arithmetical" character) can be bluntly formulated: to provide those oldest *additive* methods with as sizable a domain of operations as possible. That which we today call "multiplication" is in Egypt repeated addition (by continued doubling and suitable collecting); indeed the entire fraction calculation, which in this form extended its influence over all of later antiquity far into our Middle Ages, owes its externally very intricate methodology only to the consistent execution of the same fundamental principle. We have here in abstract pure culture so to speak the same tenacious adherence to old traditional forms which characterizes all other portions of Egyptian life, which has made its theological systems a chaos so difficult to unravel. The inevitable transformation of religious concepts does not ensue by the formation of clear, new systems but by artificial interpretation of the oldest texts, whereby their simple meaning is distorted and (from *our* viewpoint) the most contradictory statements are entangled, rather than give up the fiction that everything has been thus from olden times.

Quite different in Mesopotamia with its much more stirring history. Even the further development of the hieroglyphic system created by the Sumerians ensues in an essentially different direction. While the hieroglyphic system in Egypt, at least as a system for inscriptions, was preserved to the latest period and while the hieratic writing represents only a levelling-off of it, nevertheless in Babylonia the hieroglyphic symbols, already of a marked linear style, were finally replaced by a number of pure "cuneiform symbols" which give up every conscious connection with the old hieroglyphics. Of course this is also tied up with external influences such as the inconvenience of clay as a writing material and the rise of new elements in the population. Consequently for numerical notation a system of figures very

deficient in symbols is formed, which approximate closely our present notation by "local value".[2] However the latter is again conditioned by a very early creation of independent multiplication (as is shown by multiplication tables and tables of squares preserved from a very ancient period) and a strong influence on the entire system of notation due to the standardizing of weights and measures. The details of this process are of course much too complicated for me to discuss here. As to general history it is stimulating to remark that the Babylonian system of *writing* ran into a cul de sac by preventing the gradual transition from hieroglyphics to "uniconsonantal symbols" and finally to "letters", that mathematics however from the beginning on pointed in a direction which in its consistent expansion by means of the Hindu local value system (with which our present notation by digits is identical) has furnished one of the most important supports for modern further development.

I hope one will be able to see even from these hasty references that the elementary, indeed frequently primitive character of the mathematical problems and methods of this early period, which has never been calmly admitted as such and efforts made to conceal it by phrases like "primeval Egyptian wisdom", results in quite the opposite by granting us an especially clear insight into the historical beginnings of mathematical thought. To be sure, one must waive the desire to build up a linear chain of mathematical knowledge from earliest antiquity to our time, but one must learn to place independent cultures like separate personalities beside each other. Then it will be recognized that every people and every epoch seeks in its own manner to meet the problems presented to it and the *comparison* of these various phases receives a new meaning. Not until Oriental mathematics in its singularities is really known, will one know how to evaluate properly those of Grecian mathematics and look for *those* points where specifically Grecian problem-attitudes set in.

But historical processes do not originate only by the encampment of various cultural types beside each other, for beside this "horizontal" articulating of cultures a "vertical" stratification plays an ever recurring role. The desire to regard such a complex structure as Grecian civilization as a unity would be a very fundamental mistake. Between the influences from outside enter the great differences between groups of the most varied intellectual tendencies. Pythagoreans, the Academy and the Sophists cannot be brought into one line of development, but stand in their mutual influence as independent simultaneous factors

[2] "Principle of position", or value due to position of the digits.—G. W. D.

beside each other. Thus one cannot co-ordinate the attitude of all these tendencies on mathemarical problems, even though the external result of occupation with mathematical questions is a permanent increase of objective knowledge. The really essential thing however is not the question whether one multiplies the square of the radius by 3.16 like the Egyptians or by 3.14 in order to determine the area of a circle, but to fathom the collective attitude on the problem of measuring the area of figures bounded by curves and the meaning of infinite processes to which this leads. And in the answering of *these* questions one will again have groups quite separated by principle to differentiate, groups whose transformations are to be pursued in detail, in order to gain a true-to-life picture of the entire process. And when the Arabs or West European civilization later tie on to the Grecian acquisitions, this occurs every time in a special manner and with preference for quite a definite method among these various currents, in spite of all continuity with reference to mere content. Thus the history of mathematics suddenly reaches out far beyond its narrower framework and offers no end of the most interesting questions, which reward the effort to review steps long archaic in mathematical thought.

And beside this general historical significance here comes to the front yet another which refers to mathematics in the narrower sense, but on that very account must not be forgotten. I mean, viz., that only *historical* thinking can possibly form a balance to the much deplored specialization. The last phase of our science inaugurated in grand style at the passage of the eighteenth into the nineteenth century, of whose universal character the great French scholars and the Humboldt brothers may serve as an example, has not been able to maintain this initial level. It is clear that a rigorous establishment of the newly unlocked sciences is to be accomplished only by the greatest division of labor in careful separate investigation. A consequence thereof was however not only a separation of the single sciences from each other, but also a crumbling of these disciplines themselves into divisions scarcely understandable or interesting to each other. There is no doubt that a serious reaction must be set in against this condition and in part already has set in in a very perceptible manner. The question about the whence and whither of a science, about its place in the broader sphere of our entire civilization, is being asked more and more decidedly. In all fields it is being shown that only in the *synthesis* of modern research methods with the less hampered perspectives of a deeper intellectual content can a guarantee for restoration of the unity

of all sciences be found. The work[3] by Felix Klein *"Lectures on the Development of Mathematics in the Nineteenth Century"* shows as does none other what the historical view in this sense can mean for mathematics. Truly historical thinking united with the most intimate research activity speaks to us here, reminding each one to understand and evaluate his own research tendency as an element of a great historical process.

It will not be vouchsafed to many to write the history of a science in this sense. However every single historical investigation can count as a usable *preliminary* performance toward further synthesis only if it is guided by two viewpoints: to see the history of mathematics in the framework of general history and to understand mathematics itself not as a collection of formulas to be continually increased, but as a living unity.

[3] Professors Neugebauer, Richard Courant, and Erich Bessel-Hagen (Bonn) prepared this volume for publication. It is entitled *Volesungen über die Entwicklung der Mathematik im 19. Jahrhundert* and appeared with the imprimatur of the publishing firm Julius Springer in Berlin, 1926, a year after Klein's death. These lectures cover a period of approximately 1914-1919 and were delivered by Klein to a small circle of students in his home. His death in 1925 brought to nought his plan to issue a more voluminous work on the subject, although this presentation fills nearly 400 pages. These lectures are especially charming because they are published just as he gave them, and never received literary "finishing touches". The book is a history of mathematical ideas rather than mathematicians, and an amazing example of his power of presentation; he portrays the train of thought leading to some discovery and as Professor Neugebauer has so well said above, he views isolated pieces of research within a larger framework. Klein's main thesis, running through the entire book, is that regardless of this or that genius a fixed and definite evolution of mathematical ideas exists. The science moves and must move ahead continually on this stream of development. The appearance of a genius merely hastens the current.—G. W. D.

As the name implies, the authors in their ten years of development of the material in their text, have had the student's viewpoint uppermost in mind. Brief introductory paragraphs are given before new topics are taken up, giving the reader the meat of the subject in a few words. The book appears to be thoroughly teachable.

Agnes Scott College. HENRY A. ROBINSON.