

Chern, S.S.

October 6, 1981

Professor S. S. Chern  
Department of Mathematics  
University of California  
Berkeley, California 94720

Dear Professor Chern:

This is to confirm that we have reserved a room for you at Marquand House, 150 Stockton Street, Princeton. It is the Institute's guest house. And since there is a housekeeper there, Mrs. Moriarty, it would be helpful to her if you can let her or me know the approximate time of your arrival on October 16. She will expect you to stay through the night of the 18th. Breakfast will be provided each morning.

I have cancelled the Nassau Inn reservation which I told you about in my previous letter.

Sincerely yours,

School Administrative Officer

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

SHORT-TERM HOUSING REQUEST FORM

*Chern*

DATE: September 1, 1981

TO: MARY WISNOVSKY

FROM: SCHOOL OF MATHEMATICS

NAME OF VISITOR: Professor S. S. Chern

ADDRESS: Department of Mathematics, University of California

Berkeley, CA 94720

INVITED BY: Professor Borel

DATES OF VISIT: October 16-19, 1981

ACCOUNT TO BE CHARGED: Professor Chern

arrival date and time if known: October 16, 1981

departure date: October 19, 1981

number of persons: 1

APARTMENT ASSIGNED: MARQUAND HOUSE @ \$25 per night

*9/1/81  
notified  
also Nass  
921-7580  
Cancelled  
10/6/81*

Rates: \$20 per day; \$5 for each additional person

CHECK OUT TIME: BEFORE NOON

INSTRUCTIONS FOR PICKING UP KEYS: All apartments will be locked. Keys may be obtained from the switchboard area in Fuld Hall. If visitors arrive before or after normal working hours (9-5) and there is no security person at the switchboard, the visitor should dial 9-452-3134 using the telephone located next to the staircase in the front lobby of Fuld Hall. The security officer will notify the Institute guard who will then come to Fuld Hall and give the key to the visitor and give him directions to his apartment.

RETURNING KEYS: When the visitor departs he/she must put the key in the envelope and return it to Charles Greb at 96 Einstein Drive or if Mr. Greb is not there, they can slide the key under the office door at 96 Einstein Drive.

cc: School, Sabina, Glenda, ~~Glenda~~, Housing Office

*Mrs. M.*

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, N.J, USA

SCHOOL OF  
MATHEMATICS

I.A.S.

Memo to: Professor Borel

Memo from: CDU

Date: 9/1/81

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We have a reservation for Professor Chern for October 16, 17, 18 nights at Marquand as well as the Nassau Inn.

If all looks well with the Marquand arrangement sometime early in October

I will cancel the Inn. Am I to let him know?

*Yes; also, give the rates*  
*Caroline* *AB*

September 4, 1981

Professor S. S. Chern  
Department of Mathematics  
University of California  
Berkeley, California 94720

Dear Professor Chern:

I have been asked to let you know that we have made reservations for you at the Institute and at the Nassau Inn for the nights of October 16, 17, and 18. The Nassau Inn reservation is a backup in case the Institute guest house is not yet in operation. This is considered unlikely, but we felt it safer to make both.

The rate at Marquand House (at the Institute) is \$25 per night and this includes breakfast. At the Nassau Inn a single rate runs from \$57 through \$64 and \$67. They could not tell me which rate is for yours because they will not know which room is to be assigned until your check-in time.

Looking forward to seeing you then,

Sincerely yours,

School Administrative Officer

(Sm) S.S. Chern

INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

REQUISITION FOR PAYMENT

Date **March 4, 1968**

Pay to **S. S. Chern**

Address **c/o C. D. Underwood**

Approved by (Signature) ..... Amount **\$ 150**

To be charged to **Account 1208**

In payment of (Itemize)

**Honorarium.**

*App. 3/13/68*

Check No. ....

Batch No. ....

Extensions Chkd .....

Entered By .....

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

S. S. Chern  
CDU

February 26, 1968

To: Professors Beurling, Borel, Harish-Chandra, Hörmander,  
Montgomery, Selberg, Whitney

Dear Colleagues:

Professor S. S. Chern will be visiting here on Wednesday and Thursday, March thirteenth and fourteenth. I have invited him to give a lecture on one of these days. I propose that we give him an honorarium of \$150.

Sincerely yours,

AW:MMM

A. Weil

	Approve	Disapprove
Beurling	_____	_____
Borel	_____/_____ ✓	_____
Harish-Chandra	_____/_____ ✓	_____
Hörmander	_____/_____ ✓	_____
Montgomery	_____/_____ ✓	_____
Selberg	_____/_____ ✓	_____
Whitney	_____/_____ ✓	_____

Please return this vote to Miss Underwood.

UNIVERSITY OF CALIFORNIA, BERKELEY

BERKELEY · DAVIS · IRVINE · LOS ANGELES · RIVERSIDE · SAN DIEGO · SAN FRANCISCO



SANTA BARBARA · SANTA CRUZ

A Tribute to the People of California

DEPARTMENT OF MATHEMATICS

BERKELEY, CALIFORNIA 94720

February 21, 1968

Dear André:

I believe when I last wrote you, you were about to go to Bombay. The Zariskis said Eveline went to Paris because of the illness of Mme Gillet. I hope everything went well.

The purpose of this letter is to tell you that I expect to be in Princeton March 13,14. Last fall you asked me to speak about my results with Nirenberg and I could not find the time. If you still wish and Nirenberg has not talked about it in Princeton, I will be glad to speak about it on one of these two days. My title: An intrinsic seminorm in the homology groups of a complex manifold. Needless to say, I will not mind if this cannot be arranged.

I wonder whether you could make a reservation for me at the Nassau Inn for the nights of March 13,14.

We got the card about Sylvie's marriage. Our best wishes. Seeing you soon,

As ever,

*Chen*

→ Th. March 14  
3 p. m.

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chem, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

cc: Mr. Morgan  
Mrs. Barnett  
Miss Underwood ✓

5 October 1964

Dear Dr. Chern:

On the recommendation of the Faculty in the School of Mathematics, I am pleased formally to offer you a membership in the Institute for Advanced Study for the first term of the academic year 1964-1965. We will make available to you a grant-in-aid of \$9,500. to supplement your half-year salary and your National Science Foundation support. We would be very pleased to have you stay on in January, 1965, but we cannot be certain of having housing available for you for that month. We shall, of course, do our best. This supersedes my letter to you of November 6, 1963.

We all look forward with pleasure to having you with us again for a visit.

Sincerely yours,

Robert Oppenheimer

Dr. S. S. Chern  
The Institute for Advanced Study



September 29, 1964

Verna:

Professor Chern now has straightened out his finances among the various sources of income for this term. Instead of the \$13,000 that we have reserved for him, his grant from us will be \$9,500. Will you be sending him a revised invitation?

Thank you,

26 March 1964

Dear Dr. Chern:

I am pleased to inform you that we are reserving the sum  
of \$13,000 for your stay at the Institute next year.

With warm good wishes,

Sincerely,

Robert Oppenheimer

Dr. S. S. Chern  
Department of Mathematics  
University of California  
~~Berkeley, California~~

cc C.D.U.

UNIVERSITY OF CALIFORNIA

Department of Mathematics  
Berkeley 4, California

March 17, 1964

Dear André:

I just filled in the application forms for housing in the project and it occurs to me that I should perhaps write you about the financial aspect, in order to avoid misunderstanding. Oppenheimer mentioned a grant-in-aid and Borel assured me that the Institute could pay my half-year salary plus the pension, if necessary.

My half-year salary is \$12,150. The pension, being 8.25 per cent of it, is \$1,002, so the total is \$13,152. I think it is very likely that I can get \$4,000 from my NSF contract, but it will be some time before I can be really sure about this.

You and several friends expressed the wish for me to stay as long as I can. After discussions with the family, it now looks that I will only stay till the end of the first term, leaving a little before Xmas.

I will be in Bonn and Oberwohlfach in June and I am planning to spend a few days in Paris in the beginning of June. Will you be there at that time?

As ever,

/s/ Chern

March 24, 1964

Verna,

Professor Selberg asked me to let you know that \$13,000 can be reserved for Professor Chern's stay here next year.

Peggy

cc Mr. Morgan  
Mrs. Barnett  
✓ Miss Underwood

6 November 1963

*acc to SO*

Dear Dr. Chern:

On the recommendation of the Faculty in the School of Mathematics, I am pleased formally to offer you a membership in the Institute for Advanced Study for the first term of the academic year 1964-1965. We will make available to you a supplementary grant-in-aid to make up the difference between the amount of support you receive from the National Science Foundation and your half-year salary. We would be very pleased to have you stay on in January, 1965, but we cannot be certain of having housing available for you for that month. We shall, of course, do our best.

We all look forward with pleasure to having you with us again for a visit.

Sincerely yours,

Robert Oppenheimer

Dr. S. S. Chern  
Department of Mathematics  
University of California  
Berkeley 4, California

NOTE TO MR. MORGAN: Please reserve \$13,000.

6. It was agreed to offer S. S. Chern membership for the first term of the academic year 1964-65 and to let him know that we will supply the part of his half-year salary that he cannot get from the National Science Foundation. A reservation of \$13,000 was made for this purpose. He should be told that housing cannot be guaranteed for the part of January 1965 that he wishes to remain here. [The Director will write.]

School Meeting

J.412

10/29/63

(sm) S.S. Chern

September 16, 1963

Mr. Jay Wilson  
Princeton University Press  
William Street  
Princeton, N. J.

Dear Mr. Wilson:

Enclosed is the revision of Professor Chern's paper about which I talked with you today. Thank you for making the necessary arrangements to have it entered in the paper which is to appear in the Morse Volume.

Very truly yours,

Secretary  
School of Mathematics

CC: Professor Chern  
Enclosure

Professor Chern: Thank you very much for the copy of the letter of recommendation for Resnikoff.

UNIVERSITY OF CALIFORNIA

DEPARTMENT OF MATHEMATICS  
BERKELEY 4, CALIFORNIA

September 11, 1963

Miss Caroline D. Underwood  
Institute for Advanced Study  
Princeton, N.J.

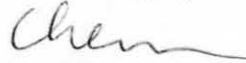
Dear Miss Underwood:

Thank you for your note of September 6. I have kept a copy of my letter to Professor Selberg on Dr. Resnikoff, which I enclose herewith. I hope you will find it readable.

I would like to take this opportunity in asking you to do me a favor. I wish to make a slight change of my paper which is due to appear in the Morse Jubilee Volume. As Professor Cairns is not at the Institute this year, I do not know to whom this revision should be sent. I am enclosing it herewith and I will be grateful if you could forward it to the appropriate person in charge so as to reach the printer with the least delay.

With best regards and thanks,

Sincerely yours,



S.S. Chern

Enclosures:

Copy of letter to Professor A. Selberg  
Revision of Chern's manuscript.



10/8/64

Professor Weil:

With all of the discussion about our not duplicating with NSF funds the NSF funds a member receives from his home institution I just checked on Professor Chern's. It has been recommended that he be paid his grant from the new, as yet not received, NSF grant in Analysis and Algebra. This should probably be changed??? To IAS grants???

CDh

Chen

a) Salary for one semester = \$12,150

b) Regents share of contribution to Retirement System  
=  $12,150 \times 8.25\% = \$1,002.37$

To be paid before Dec 31, 1964.

c) Salary to be paid from NSF contract with Berkeley  
= \$3,727.

d) Salary expected from IAS  
=  $12,150 - 3,727 = \$8,423$

1,002.37

$9,425.37 = 9500.$

(Sm) S.S. Chern

September 2, 1955

Mr. Minot C. Morgan, Jr.  
Institute for Advanced Study

Dear Mr. Morgan:

I am sending in the Final Report on the National  
Science Foundation contract of Professor S. S. Chern  
(NSF-G778) today.

Sincerely yours,

Secretary  
School of Mathematics

September 2, 1955

Mathematical Sciences Division  
National Science Foundation  
Washington 25, D. C.

Gentlemen:

I am enclosing herewith the Final Report on the  
Research Grant NSF-G778 held by Professor S. S. Chern  
during the academic year 1954-55.

Sincerely yours,

Secretary  
School of Mathematics

Enclosure

FINAL REPORT OF RESEARCH WORK ON  
NATIONAL SCIENCE FOUNDATION  
RESEARCH GRANT NSF-0778

by

Shiing-shen Chern

My work during the year 1954-55 was concerned with the following three topics:

1) Characteristic classes of differentiable and complex manifolds. It is shown that the characteristic classes of a complex manifold are defined up to a more strict equivalence relation than cohomology. Also some applications are found, in particular, to the problem of differentiable imbedding of manifolds.

2) Infinite continuous pseudo-groups. Further work is done on the foundation of the theory. More complete results are achieved in the case which generalizes Kahler geometry.

3) Theory of surfaces in Euclidean space. A simple proof is found of the Hopf-Hartman-Wintner theorem on closed special W-surfaces. Work has also been done on the extension of inequalities of the Brunn-Minkowski type to arbitrary closed surfaces.

The following papers are under publication or preparation:

1. An elementary proof of the existence of isothermal parameters on a surface, to appear in Proceedings of the American Mathematical Society.

2. On special W-surfaces, to appear in Proceedings of the A.M.S.

3. On curvature and characteristic classes of a Riemann manifold, to appear in Abh. of Hamburg University.

4. Infinite continuous pseudo-groups, under preparation.

5. On closed surfaces in Euclidean space, under preparation.

THE UNIVERSITY OF CHICAGO  
CHICAGO 37 • ILLINOIS  
DEPARTMENT OF MATHEMATICS

August 15, 1955

Miss C. Underwood  
Institute for Advanced Study  
Princeton, N.J.

Dear Miss Underwood:

Thanks for your letter of August 10. I must apologize in not having sent you my final report as promptly as I should.

I am enclosing now a copy of this report, and I wonder whether it is suitable for the purpose.

With best thanks and greetings,

Sincerely yours

*S. S. Chern*

Report by Shiing-shen Chern

My work during the year 1954-55 is concerned with the following three topics:

1) Characteristic classes of differentiable and complex manifolds. It is shown that the characteristic classes of a complex manifold are defined up to a more strict equivalence relation than cohomology. Also some applications are found, in particular, to the problem of differentiable imbedding of manifolds.

2) Infinite continuous pseudo-groups. Further work is done on the foundation of the theory. More complete results are achieved <sup>in</sup> ~~to~~ the case which generalizes Kahler geometry.

3) Theory of surfaces in Euclidean space. A simple proof is found of the Hopf-Hartman-Wintner theorem on closed special W-surfaces. Work has also been done on the extension of inequalities of the Brunn-Minkowski type to arbitrary closed surfaces.

The following papers are under publication or preparation:

1. An elementary proof of the existence of isothermal parameters on a surface, to appear in Proc. of Amer. Math. Soc.
2. On special W-surfaces, to appear in Proc. AMS.
3. On curvature and characteristic classes of a Riemann manifold, to appear in Abh. of Hamburg University.
4. Infinite continuous pseudo-groups, under preparation.
5. On closed surfaces in Euclidean space, under preparation.

August 10, 1955

Professor S. S. Chern  
Eckhart Hall  
University of Chicago  
Chicago 37, Illinois

Dear Professor Chern:

On behalf of Professor Montgomery and the rest of the Faculty of the School of Mathematics I should like to acknowledge receipt of, and thank you for the letter you have written in support of the application of Dr. W. Klingenberg.

I have been wondering whether you have yet written the final report on your work at the Institute for the end of your contract with the National Science Foundation. Mr. Morgan will wish to know when this is done so he can close the books in the financial work on your contract.

Thank you for the above information.

Sincerely yours,

Secretary  
School of Mathematics



Chern  
granted \$3450  
4/15/54

PROPOSAL FOR NATIONAL SCIENCE FOUNDATION CONTRACT

This proposal is submitted by the Institute for Advanced Study, Princeton, New Jersey, on behalf of Professor Shiing-shen Chern for the academic year 1954-1955.

Summary of mathematical work by S. S. Chern: I got my B.S. degree from Nankai University, Tientsin, China, in 1930. In the next four years I started some work, of a rather modest nature (1, 2, 3)\*, on differential geometry. Since then differential geometry has been the field of my major interest, although some of my later works can be said to belong to it only in a broad sense. In 1934 I was given a Fellowship by the Chinese Government and I went to the University of Hamburg, Germany, to work with Professor W. Blaschke. I received my D.Sc. degree in 1936 with a thesis on the geometry of webs (4, 5). The next year I was working with the great French mathematician Elie Cartan in Paris. It was a most profitable sejour during which my general outlook of mathematics was greatly widened. I returned to China in 1937 to teach at the National Tsing Hua University. In the years 1937-43 I worked on integral geometry, projective connections, geometry of paths, etc. (9 - 25). My main interest was to attach a connection to a geometric being, although the problem at that time was studied only in its various special cases.

A turning point in my mathematical career came when I was granted a stipend by the Institute for Advanced Study in 1943. I stayed in Princeton from 1943 to 1946 and had the benefit of contact with various mathematicians. During this period I was able to combine my training in classical mathematics with the more modern developments. In particular, I saw the role that the theory of fiber bundles was to play in differential geometry and established it in a conclusive way (27, 28, 32, 34, 35). My works in these three or four years are perhaps among the best that I have done.

After I joined the University of Chicago in 1949, I have been interested in the homology theory of fiber bundles (45, 49, 54). In particular, I was able to lay a satisfactory foundation of the theory of characteristic classes on an algebraic variety.

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\* Numbers refer to the papers in the Bibliography.

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

My latest interest is in complex manifolds and infinite continuous pseudo-groups, which seem to form a natural a far-reaching generalization of complex manifolds (55).

Plans for Research: It is proposed to make a study of the different aspects of infinite continuous pseudo-groups with emphasis to the particular cases of complex and symplectic manifolds. The subject can be said to have originated from the Norwegian mathematician Sophus Lie in the last part of the 19th century, as a natural extension of his work on transformation groups. In the beginning of this century Elie Cartan devoted several extensive papers on the subject. While the theory of Lie groups has deservedly received considerable attention in the mathematical researches of the last two decades, the theory of infinite continuous pseudo-groups, which can be considered as its generalization, remains unnoticed. Recently, another of its particular cases, the theory of complex manifolds, has become the object of extensive researches.

Perhaps two of the main reasons that the subject has not received sufficient attention are the unsatisfactory status of its foundations and a lack of problems due to the fact that most of the outstanding local problems were solved by Cartan. This situation has now changed. For, in the first place, the development of notions in modern mathematics has permitted an entirely satisfactory understanding of most of the results in existence, as shown in a forthcoming paper of mine in the "Colloque de géométrie différentielle, Paris". In the second place, this clarification of the foundations opens the way to the study of various global problems, which have so far hardly been touched. Among such problems is the question whether a manifold has a structure corresponding to a given pseudo-group. This contains as particular cases the questions whether an even-dimensional manifold can be given a complex structure or can be a symplectic manifold in the sense of Ehresmann. Recent achievements in fiber bundles have scored great progress on the question of existence or non-existence of tensor fields satisfying algebraic conditions, such as that of continuous linearly independent vector fields. In our problems the tensor field will be subjected not

only to algebraic conditions, but also to conditions in the form of partial differential equations. In a sense these should be more interesting and deeper problems in differential geometry.

More specifically, the plan of research will consist of two parts:

- 1) Further clarification of the work of Cartan. This will perhaps involve more work than it seems. In the first place, it has been found necessary to supply details of Cartan's proofs of his statements and to replace, when possible, his proofs based on computation by more conceptual ones. In the second place, Cartan dealt exclusively with the complex analytic case while our interest will be in the real case and possibly only under differentiability assumptions. Considering the fact that the Cauchy-Kowalewski theorem is not true without analyticity assumptions, a critical study of Cartan's work from this viewpoint seems to be of interest and is by no means trivial.
- 2) Study of some global problems. Two of the most urgent questions are: a) Does the six-dimensional sphere have a complex structure? b) Does the complex projective plane have more than one complex structure? In the case that the answers to these questions are negative, it is my opinion that progress should be achieved by analytic rather than geometrical methods. A more elaborate study of the corresponding differential systems than hitherto undertaken is in order. Similar questions, of different degrees of difficulty and generality, can be proposed and studied.

List of mathematical papers by Shiing-shen Chern:

1. Pairs of plane curves with points in one-to-one correspondence, Sci. Repts. Tsing Hua Univ. 1 (1932) 145-153
2. Triads of rectilinear congruences with generators in correspondence, Tohoku Math. J. 40 (1935) 179-188
3. Associate quadratic complexes of a rectilinear congruence, Tohoku Math. J. 40 (1935) 293-316
4. Abzählungen für Gewebe, Abh. math. Sem. Hamburg 11 (1935) 163-170
5. Eine Invariantentheorie der Dreigewebe aus  $n$ -dimensionalen Mannigfaltigkeiten in  $2n$ -dimensionalen Räumen, Abh. math. Sem. Hamburg 11 (1936) 333-358
6. Sur la géométrie d'une équation différentielle du troisième ordre, C. R. Acad. Sci. Paris 204 (1937) 1227-1229
7. Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif, Bull. Sci. Math. 61 (1937) 234-243

8. On projective normal coordinates, *Ann. of Math.* 39 (1938) 165-171
9. On two affine connections, *J. Univ. Yunnan* 1 (1938) 1-18
10. Sur la géométrie d'un système d'équations différentielles du second ordre, *Bull. Sci. Math.* 63 (1939) 206-212
11. The geometry of higher path-spaces, *J. Chin. Math. Soc.* 2 (1940) 247-276
12. Sur les invariants intégraux en géométrie, *Sci. Repts. Tsing Hua Univ.* 4 (1940) 85-95
13. The geometry of a differential equation  $y'''' = F(x, y, y', y'')$ , *Sci. Repts. Tsing Hua Univ.* 4 (1940) 97-111
14. Sur une généralisation d'une formule de Crofton, *C. R. Acad. Sci. Paris* 210 (1940) 757-758
15. (With Chih-ta Yen) Formula principale cinematica dello spazio ad  $n$  dimensioni, *Boll. Un. Mat. Ital.* 2 (1940) 434-437
16. Generalization of a formula of Crofton, *Tsuhun Univ. J. Sci.* 7 (1940) 1-16
17. On integral geometry in Klein spaces, *Ann. of Math.* 43 (1942) 178-179
18. (With Hsien-chung Wang) Differential geometry in symplectic space, *Sci. Repts. Tsing Hua Univ.* 4 (1947) 453-477
19. Sur une classe remarquable de variétés dans l'espace projectif à  $n$  dimensions, *Sci. Repts. Tsing Hua Univ.* 4 (1947) 328-336
20. Sur les invariants de contact en géométrie projective différentielle, *Acta Pontif. Acad. Sci.* 5 (1941) 123-140
21. On the invariants of contact of curves in a projective space of  $n$  dimensions and their geometrical interpretation, *Acad. Sinica Sci. Record* 1 (1942) 11-15
22. The geometry of isotropic surfaces, *Ann. of Math.* 43 (1942) 545-559
23. On a Weyl geometry defined from an  $(n-1)$ -parameter family of hypersurfaces in a space of  $n$  dimensions, *Acad. Sinica Sci. Record* 1 (1942) 7-10
24. On the Euclidean connections in a Finsler space, *Proc. Nat. Acad. Sci. USA*, 29 (1943) 33-37
25. A generalization of the projective geometry of linear spaces, *Proc. Nat. Acad. Sci. USA*, 29 (1943) 38-43
26. Laplace transforms of a class of higher dimensional varieties in a projective space of  $n$  dimensions, *Proc. Nat. Acad. Sci. USA*, 30 (1944) 95-97
27. A simple intrinsic proof of the Gauss-Bonnet formula for closed Riemannian manifolds, *Ann. of Math.* 54 (1944) 747-752
28. Integral formulas for the characteristic classes of sphere bundles, *Proc. Nat. Acad. Sci. USA*, 30 (1944) 269-273
29. On a theorem of algebra and its geometrical application, *J. Indian Math. Soc.* 8 (1944) 29-36
30. On Grassmann and differential rings and their relations to the theory of multiple integrals, *Sankhya* 7 (1945) 2-8
31. Some new characterizations of the Euclidean sphere, *Duke Math. J.* 12 (1945) 279-290
32. On the curvatura integra in a Riemannian manifold, *Ann. of Math.* 46 (1945) 674-684
33. On Riemannian manifolds of four dimensions, *Bull. Amer. Math. Soc.* 51 (1945) 964-971
34. Some new viewpoints in the differential geometry in the large, *Bull. Amer. Math. Soc.* 52 (1946) 1-30
35. Characteristic classes of Hermitian manifolds, *Ann. of Math.* 47 (1946) 85-121
36. On the characteristic classes of Riemannian manifolds, *Proc. Nat. Acad. Sci. USA*, 33 (1947) 78-82
37. Note on affinely connected manifolds, *Bull. Amer. Math. Soc.* 53 (1947) 820-823; correction, *ibid* 54 (1948) 985-986
38. On the characteristic ring of a differentiable manifold, *Acad. Sinica Sci. Record* 2 (1947) 1-5
39. On the multiplication in the characteristic ring of a sphere bundle, *Ann. of Math.* 49 (1948) 362-372

- 40. Note on projective differential line geometry, Acad. Sinica Sci. Record 2 (1948) 137-139
- 41. (With Yuh-lin Jou) On the orientability of differentiable manifolds, Sci. Repts. Tsing Hua Univ. 5 (1948) 13-17
- 42. (With Ye-fon Sun) The imbedding theorem for fibre bundles, Trans. Amer. Math. Soc. 67 (1949) 286-303
- 43. (With Sze-tsen Hu) Parallelisability of principal fibre bundles, Trans. Amer. Math. Soc. 67 (1949) 304-309
- 44. Local equivalence and Euclidean connections in Finslerian spaces, Sci. Repts. Tsing Hua Univ. 5 (1948) 95-121
- 45. (With E. H. Spanier) The homology structure of fiber bundles, Proc. Nat. Acad. Sci. USA, 36 (1950) 248-255
- 46. Differential geometry of fiber bundles, Proc. Int. Congr. Math. II (1950) 397-411
- 47. Topics in differential geometry, mimeographed notes, Princeton 1951.
- 48. On the kinematic formula in the Euclidean space of n dimensions, Amer. J. Math. 74 (1952) 227-236
- 49. (With E. H. Spanier) A theorem on closed orientable surfaces in four-dimensional space, Comm. Math. Helv. 25 (1951) 205-209
- 50. Some formulas in the theory of surfaces, to appear in Boll. Soc. Mat. Mexicana
- 51. (With C. Chevalley) Elie Cartan and his mathematical work, Bull. Amer. Math. Soc. 58 (1952) 217-250
- 52. Some relations between Riemannian and Hermitian geometries, to appear in Duke Math. J.
- 53. (With N. H. Kuiper) Some theorems on the isometric imbedding of compact Riemannian manifolds in Euclidean space, Ann. of Math. 56 (1952) 422-430
- 54. On the characteristic classes of complex sphere bundles and algebraic varieties, Amer. J. Math. 75 (1953) 565-597
- 55. Infinite continuous pseudo-groups, to appear in Colloque Géom. Diff. Paris
- 56. La géométrie des sous-variétés d'un espace euclidien à plusieurs dimensions, to appear in l'Ens. Math.

Financial Statement:

Salary for Professor Chern (full time)		\$3000
Overhead (at 15 percent)		<u>450</u>
Total		\$3450

FACULTY SPONSOR:

\_\_\_\_\_  
Deane Montgomery, Professor of Mathematics

APPROVED FOR  
THE INSTITUTE FOR ADVANCED STUDY:

DATED Nov. 17, 1953

(Sm) S.S. Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

Copy to Mr. Morgan  
Mrs. Barnett  
✓ Miss Underwood

OFFICE OF THE DIRECTOR

11 May 1954

Dear Professor Chern:

On the recommendation of the Faculty of the School of Mathematics, I am pleased formally to offer you a membership in the Institute for Advanced Study for the academic year 1954-55, during the period of your National Research Foundation and Guggenheim Foundation grants. We can make available to you a grant-in-aid of \$500 to supplement your NRF and Guggenheim funds in defraying your expenses during your sojourn in Princeton. Since your NRF grant includes \$3,000 to be paid to you in salary, there is a possibility that part or all of your financial support may be subject to income tax.

*Science  
Notified Y. Holm  
5/12/54*

We all look forward with pleasure to having you with us for a visit.

Sincerely yours,

Robert Oppenheimer

Professor Shiing-Shen Chern  
Department of Mathematics  
University of Chicago  
Chicago, Illinois

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

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NATIONAL SCIENCE FOUNDATION

Washington 25, D.C.

Copy to: Miss Underwood

April 14, 1954

Dr. Robert Oppenheimer, Director  
The Institute for Advanced Study  
Princeton, New Jersey

Research Grant NSF-G778

Dear Dr. Oppenheimer:

I am pleased to inform you that the sum of \$3,450 is hereby granted by the National Science Foundation to The Institute for Advanced Study, for the support of research entitled "Pseudo-Groups with Emphasis on Complex and Symplectic Manifolds," under the direction of Shing-Shen Chern, Department of Mathematics, for a period of approximately one year. This grant will be paid on or about April 15, 1954, in full.

It is a condition of this grant that it may be revoked in whole or in part by the Foundation after consultation with the principal investigator and the grantee, except that a revocation shall not affect any commitment which, in the judgment of the Foundation and the grantee, had become firm prior to the effective date of the revocation; and that funds not committed by the grantee prior to the conclusion of the work contemplated under this grant shall be returned to the Foundation.

It is a further condition of this grant that disposition of domestic patent and other rights in any inventions or discoveries made or conceived during the research shall be the responsibility of the grantee; that disposition of foreign patent and other rights to any such invention or discovery shall be determined by the United States Government; that the grantee shall give the Foundation reasonable notice of application by the grantee or other person or institution for a foreign or domestic patent on any such invention or discovery; and that upon issue of a domestic patent on any such invention or discovery, the patentee shall grant the Government an irrevocable, royalty-free, nonexclusive license for use of such invention or discovery for governmental purposes.

The Foundation desires that this grant be administered in general accordance with the Foundation's policies for research grants as stated in "Grants for Scientific Research," December 1951, and in conformity with the other understandings reached between the Foundation and the grantee relating to this grant.

Please acknowledge receipt at your earliest convenience.

Sincerely yours,

Signed/Alan T. Waterman  
Alan T. Waterman  
Director

THE UNIVERSITY OF CHICAGO

CHICAGO 37 · ILLINOIS

DEPARTMENT OF MATHEMATICS

April 21, 1954

Dear Deane:

I have been informed by the Guggenheim Foundation that I am awarded a Fellowship for 1954-55, with a grant of \$3,000.00. Some time ago Professor Morse wrote me that my application for a grant of \$3,000.00 from the National Science Foundation will most likely come through. I wonder whether the Institute has now received official information to that effect.

The Guggenheim grant is \$1,000 short of the expectation on which the University Administration has based the financial estimate for my leave of absence. My annual salary for 1954-55 will be \$12,000. Our original estimate to make up this sum is: \$4,000 (Guggenheim) + \$3,000 (NSF) + \$5,000 (Chicago). Besides this the University was planning to pay an extra \$250 for contribution to my annuity fund (The University share should be \$500 or 5% of my salary whichever is lower) and a grant to cover part of my expenses to move my family to Princeton and back. Under the present circumstances the University share of my salary has to be revised upwards. After a talk with MacLane we agreed to find out from you whether it would be possible to get a grant, from the fund reserved for me, of \$500 say, of which \$200 will be for annuity contribution and \$300 for part of my travelling expenses. It is likely that the University will then be able to pay half of my salary, with the other half made up by grants from the NSF and Guggenheim.

With best regards,

Cordially yours

*Chern*



*Copy for Miss Underwood*  
*S.S. Chern*

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

April 23, 1954

Dear Colleagues:

I am enclosing a copy of a letter from Chern and my reply. Chern has now received a grant of \$3000 from the Guggenheim Foundation and a grant of \$3000 from the National Science Foundation. As his letter states, he wishes an additional grant of \$500 from the fund which we have reserved for him. I am very much in favor of giving him this grant.

Sincerely yours,

DM:MM

Deane Montgomery

Encs.

Do you approve?

Yes

No

K. Gödel

yes

—

M. Morse

yes

—

A. Selberg

yes

—

J. von Neumann

yes

—

H. Whitney

yes

—

R. Oppenheimer

yes

—

Please return this letter and its enclosures to Miss Underwood.

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

*orig. sent Dir. off.*  
*4/29/54*

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

April 23, 1954

Professor S. S. Chern  
Department of Mathematics  
The University of Chicago  
Chicago 37, Illinois

Dear Chern:

I have just learned from our General Manager, Mr. Morgan, that, within the last few days, an official letter came from the Science Foundation which makes your grant of \$3000 from them definite. I feel quite sure that we will be able to be helpful in the way you suggest, but it will take a few days for us to consider this in an official way. I will bring up the question immediately and let you know as soon as I can what the outcome is.

As I said before, everyone here is looking forward with great pleasure to your visit.

Sincerely yours,

DM:MMI

Deane Montgomery

THE UNIVERSITY OF CHICAGO  
CHICAGO 37 • ILLINOIS  
DEPARTMENT OF MATHEMATICS

March 11, 1954

Professor M. Morse  
Institute for Advanced Study  
Princeton, N.J.

Dear Professor Morse:

Thank you for your letter of March 6 and for the information that my application for a grant of \$3000 from the National Science Foundation will probably come through in two weeks. According to a preliminary discussion of my plans with the University Administration, we are counting on a sum of \$7000 from outside to make up my next year's salary. As I have applied for a Guggenheim Fellowship, I am not certain whether I would need further financial assistance from the Institute, until the awards of the Guggenheim Fellowship are announced.

I have applied to Mrs. Barnett for housing.

With best thanks and regards,

Cordially yours

*S. S. Chern*

Copy: Mrs. Barnett

March 6, 1954

Professor S. S. Chern  
Feldhart Hall  
University of Chicago  
Chicago 37, Illinois

Dear Chern:

I can tell you confidentially that there seems a good prospect that the grant you sought from the National Science Foundation will come through in about two weeks. The amount was \$3000. I would like to have, if possible, the answers to two questions.

First, would you be able to provide the remainder of money for your expenses from the funds which the University will give you? If not, how much outside money will you need?

Second, would you like us to apply to Mrs. Barnett for housing for you, Mrs. Chern and your two children? Since housing is extremely difficult here and we cannot guarantee anything, it is desirable that such an application be made as soon as possible.

Looking forward to seeing you next year, I am

Cordially yours,

Marston Morse

MM:du

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

OFFICE OF THE DIRECTOR

9 December 1953

Dear Miss Cloud:

In Dr. Oppenheimer's absence, I am writing to acknowledge your letter of November 25th with reference to Professor Shiing-shen Chern's proposal, "Study of Different Aspects of Pseudo-Groups with Emphasis to Particular Cases of Complex and Symplectic Manifolds."

This, or a similar proposal, has not been submitted to any other agency, nor is it our intention to do so.

Sincerely yours,

Rosanna W. Jaffin  
Secretary to the Director

Miss Frances W. Cloud  
Administrative Assistant, Division  
of Mathematical, Physical & Engineering  
Sciences  
National Science Foundation  
Washington 25, D. C.

CC: Miss Underwood

(Sm) S. S. Chern

November 18, 1953

Professor S. S. Chern  
Eckhart Hall  
University of Chicago  
Chicago 37, Illinois

Dear Chern:

We have sent in your application to the National Science Foundation. I imagine there is a very good chance that you will get both the Guggenheim and a grant from the National Science Foundation, and that it is desirable to continue both these applications. However, in order to help you to make definite plans, our department has voted to reserve \$4,000 to pay you if you get no other grant from outside Chicago or to pay you a part of this if the outside grants you receive are insufficient. I think we all understand that if both of your applications are successful that Chicago will probably be able to make up the difference in your salary. Everyone here is very pleased that you will be here. I suggest that you write about housing to Mrs. Ruth Barnett, at the above address, unless you already have some definite plans of your own.

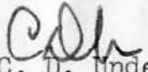
Sincerely yours,

DM:MM

Deane Montgomery

Professor Morse:

What do you say to the proposal made by Professor  
Gödel?

  
C. D. Underwood

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

November 4, 1953

Professor S. S. Chern  
Department of Mathematics  
University of Chicago  
Chicago 37, Illinois

Dear Chern,

I am showing your letter of November 1 to our department. Everyone would like very much to have you here next year. As a first practical step I think it might be desirable to have the Institute apply for a grant in your behalf to the National Science Foundation. I would expect chances for something of this kind to be very good. If you agree I will consult our department about sending in a proposal for you. In this case you could send me the information for an application which would be two or three pages of material giving a summary of your past and intended work and bibliography. Your work is so well known that I presume you could be brief. You would also have to indicate what size the grant should be. With the agreement of all involved the necessary number of copies could be prepared and sent in from here.

Sincerely yours,

Deane Montgomery

DM:js

Faculty  
School of Mathematics

Do you favor having the Institute apply for such a grant for Chern if he wishes to have it done?

Approve

Disapprove

Professors Gödel

Morse

Oppenheimer (absent)

Selberg

von Neumann

Whitney

\*

yes

yes; agree with K.G.\*

yes and?  
agree with \*

yes

Please return to Miss Underwood.

\* I am in favor of writing him officially that funds of such and such an amount will be provided so that he may apply for leave of absence at once

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA



(Sm) S.S. Chern

THE UNIVERSITY OF CHICAGO

CHICAGO 37 • ILLINOIS

DEPARTMENT OF MATHEMATICS

November 1, 1953

Dear Deane:

Thanks for your letter of October 22 and your trouble in sending me Leray's notes, which I have received. Please find enclosed a check of \$3.00 for these notes.

I appreciate very much your consideration in bringing up the financial question concerning my plan next year. As you know, a leave of absence for a Guggenheim Fellowship is in our University the equivalent of a sabbatical leave. In general, the University consents to pay the balance of my salary. I have been given a green light to apply for the Guggenheim, but, until I get it, no official action is expected to be taken by the University. A grant from the Institute or from the National Science Foundation will mean, in the event I get the Guggenheim, a saving ~~of~~ this Department, as the balance of my salary will be correspondingly smaller. It will strengthen my position in getting the leave of absence, for the Department would then be able to bring in a replacement. If I do not get the Guggenheim, such a grant would be necessary for me to apply for the leave of absence.

It seems to me that in both cases it is to my advantage to apply for a grant, as you have suggested. Please let me know if some official forms have to be filled.

I am at the moment inclined to think that I should spend both terms of 1954-55 in Princeton. Unless I write you to the contrary, this will be my plan.

Sincerely yours

S. S. Chern

A summary of mathematical work by S.S.Chern:

I got my B.S. degree from Nankai University, Tientsin, China, in 1930. In the next four years I started some work, of a rather modest nature (1,2,3)<sup>\*</sup>, on differential geometry. Since then differential geometry has been the field of my major interest, although some of my later works can be said to belong to it only in a broad sense. In 1934 I was given a Fellowship by the Chinese Government and I went to the University of Hamburg, Germany, to work with Professor W.Blaschke. I received my D.Sc. degree in 1936 with a thesis on the geometry of webs (4,5). The next year I was working with the great French mathematician Elie Cartan in Paris. It was a most profitable sejour during which my general outlook of mathematics was greatly widened. I returned to China in 1937 to teach at the National Tsing Hua University. In the years 1937-43 I worked on integral geometry, projective connections, geometry of paths, etc. (9-25). My main interest was to attach a connection to a geometric being, although the problem at that time was studied only in its various special cases.

A turning point in my mathematical career came when I was granted a stipend by the Institute for Advanced Study in 1943. I stayed in Princeton <sup>from</sup> ~~in~~ 1943 to 1946 and had the benefit of contact with <sup>various</sup> mathematicians. During this period I was able to combine my training in classical mathematics with the more modern developments. In particular, I saw the role that the theory of fiber bundles was to play in differential geometry and established it in a conclusive way (27,28,32,34,35). My

\* Numbers refer to the papers in the Bibliography.

works in these three or four years are perhaps among the best that I have done.

After I joined the University of Chicago in 1949, I have been interested in the homology theory of fiber bundles (45,49,54). In particular, I was able to lay a satisfactory foundation of the theory of characteristic classes on an algebraic variety. My latest interest is in complex manifolds and infinite continuous pseudo-groups, which seem to form a natural and far-reaching generalization of complex manifolds (55).

## PLANS FOR RESEARCH

It is proposed to make a study of the different aspects of infinite continuous pseudo-groups with emphasis to the particular cases of complex and symplectic manifolds. The subject can be said to have originated from the Norwegian mathematician Sophus Lie in the last part of the 19th century, as a natural extension of his work on transformation groups. In the beginning of this century Elie Cartan devoted several extensive papers on the subject. While the theory of Lie groups has deservedly received considerable attention in the mathematical researches of the last two decades, the theory of infinite continuous pseudo-groups, which can be considered as its generalization, remains unnoticed. Recently, another of its particular cases, the theory of complex manifolds, has become the object of extensive researches.

Perhaps two of the main reasons that the subject has not received sufficient attention are the unsatisfactory status of its foundations and a lack of problems due to the fact that most of the outstanding local problems were solved by Cartan. This situation has now changed. For, in the first place, the development of notions in modern mathematics has permitted an entirely satisfactory understanding of most of the results in existence, as shown in a forthcoming paper of mine in the "Colloque de géométrie différentielle, Paris". In the second place, this clarification of the foundations opens the way to the study of various global problems, which have so far hardly been touched. Among such problems is the question whether a manifold has a structure corresponding to a given pseudo-groups. This contains as particular cases the questions whether an even-dimensional manifold can be given a complex structure or can be a symplectic manifold in the sense of Ehresmann.

Recent achievements in fiber bundles have scored great progress on the question of existence or non-existence of tensor fields satisfying algebraic conditions, such as that of continuous linearly independent vector fields. In our problems the tensor field will be subjected not only to algebraic conditions, but also to conditions in the form of partial differential equations. In a sense these should be more interesting and deeper problems in differential geometry.

More specifically, the plan of research will consist of two parts:

- 1) Further clarification of the work of Cartan. This will perhaps involve more work than it seems. In the first place, it has been found necessary to supply details of Cartan's proofs of his statements and to replace, when possible, his proofs based on computation by more conceptual ones. In the second place, Cartan dealt exclusively with the complex analytic case while our interest will be in the real case and possibly only under differentiability assumptions. Considering the fact that the Cauchy-Kowalewski theorem is not true without analyticity assumptions, a critical study of Cartan's work from this viewpoint seems to be of interest and is by no means trivial.

- 2) Study of some global problems. Two of the most urgent questions are: a) Does the six-dimensional sphere have a complex structure? b) Does the complex projective plane have more than one complex structure? In the case that the answers to these questions are negative, it is my opinion that progress should be achieved by analytic rather than geometrical methods. A more elaborate study of the corresponding differential systems than hitherto undertaken is in order. Similar questions, of different degrees of difficulty and generality, can be proposed and studied.

I have started thinking about these questions about a year ago. It will perhaps take several years before the scope and significance of the subject become more clear. However, it is believed that some notable progress could be achieved after a year's work. Results which will be obtained will be published in standard mathematical journals. As the subject has contact with various branches of mathematics, such as algebraic topology, theory of partial differential equations, differential geometry, etc., it would be advisable to pursue this work in a large center where advice will be available. If a Fellowship will be awarded, it is my intention to spend a major part of the year at the Institute for Advanced Study, Princeton, N. J. If possible, I also plan to visit Paris and Zurich for two or three months in order to discuss my problems with mathematicians in my general field such as H. Cartan, A. Lichnerowicz in Paris, C. Ehresmann in Strassbourg, and H. Hopf, B. Eckmann in Zurich.

I believe in the intrinsic value of knowledge and intend to make my share of contribution in the field of pure mathematics. I expect to devote my life in mathematical research.

Shing-shen Chern

THE UNIVERSITY OF CHICAGO  
CHICAGO 37 • ILLINOIS  
DEPARTMENT OF MATHEMATICS

November 11, 1953

Dear Deane:

Thanks for your letter of November 4. I am enclosing herewith a summary of my work and two copies each of my plan of research and Bibliography. The latter are the ones I submitted to the Guggenheim and the former is its abbreviated version. I wonder whether they make up a suitable application for a grant from the NSF. I shall appreciate any changes you care to make.

As for the size of the grant, it seems to me that it would depend on the fact whether I could get a Guggenheim. In the affirmative case I feel that a sum of \$3,000.00 will be as much as I would expect. I have no idea what I should do if I do not get a Guggenheim. Being inexperienced in such matters, I shall welcome any suggestion from you.

Sincerely yours

*S. S. Chern*

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

November 4, 1953

Professor S. S. Chern  
Department of Mathematics  
University of Chicago  
Chicago 37, Illinois

Dear Chern,

I am showing your letter of November 1 to our department. Everyone would like very much to have you here next year. As a first practical step I think it might be desirable to have the Institute apply for a grant in your behalf to the National Science Foundation. I would expect chances for something of this kind to be very good. If you agree I will consult our department about sending in a proposal for you. In this case you could send me the information for an application which would be two or three pages of material giving a summary of your past and intended work and bibliography. Your work is so well known that I presume you could be brief. You would also have to indicate what size the grant should be. With the agreement of all involved the necessary number of copies could be prepared and sent in from here.

Sincerely yours,

Deane Montgomery

DM:js

Faculty  
School of Mathematics

Do you favor having the Institute apply for such a grant for Chern if he wishes to have it done?

Approve      Disapprove

Professor Gödel  
Morse  
Oppenheimer (absent)  
Selberg  
von Neumann  
Whitney

Please return to Miss Underwood.



November 4, 1953

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Department of Mathematics  
University of Chicago  
Chicago 37, Illinois

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DM:js

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School of Mathematics

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Approve      Disapprove

Professor Gödel  
Morse  
Oppenheimer (absent)  
Selberg  
von Neumann  
Whitney

Please return to Miss Underwood.

PROPOSAL FOR NATIONAL SCIENCE FOUNDATION CONTRACT

This proposal is submitted by the Institute for Advanced Study, Princeton, New Jersey, on behalf of Professor Shiing-shen Chern for the academic year 1954-1955.

Summary of mathematical work by S. S. Chern: I got my B.S. degree from Nankai University, Tientsin, China, in 1930. In the next four years I started some work, of a rather modest nature (1, 2, 3)\*, on differential geometry. Since then differential geometry has been the field of my major interest, although some of my later works can be said to belong to it only in a broad sense. In 1934 I was given a Fellowship by the Chinese Government and I went to the University of Hamburg, Germany, to work with Professor W. Blaschke. I received my D.Sc. degree in 1936 with a thesis on the geometry of webs (4, 5). The next year I was working with the great French mathematician Elie Cartan in Paris. It was a most profitable sejour during which my general outlook of mathematics was greatly widened. I returned to China in 1937 to teach at the National Tsing Hua University. In the years 1937-43 I worked on integral geometry, projective connections, geometry of paths, etc. (9 - 25). My main interest was to attach a connection to a geometric being, although the problem at that time was studied only in its various special cases.

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After I joined the University of Chicago in 1949, I have been interested in the homology theory of fiber bundles (45, 49, 54). In particular, I was able to lay a satisfactory foundation of the theory of characteristic classes on an algebraic variety.

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My latest interest is in complex manifolds and infinite continuous pseudo-groups, which seem to form a natural a far-reaching generalization of complex manifolds (55).

Plans for Research: It is proposed to make a study of the different aspects of infinite continuous pseudo-groups with emphasis to the particular cases of complex and symplectic manifolds. The subject can be said to have originated from the Norwegian mathematician Sophus Lie in the last part of the 19th century, as a natural extension of his work on transformation groups. In the beginning of this century Elie Cartan devoted several extensive papers on the subject. While the theory of Lie groups has deservedly received considerable attention in the mathematical researches of the last two decades, the theory of infinite continuous pseudo-groups, which can be considered as its generalization, remains unnoticed. Recently, another of its particular cases, the theory of complex manifolds, has become the object of extensive researches.

Perhaps two of the main reasons that the subject has not received sufficient attention are the unsatisfactory status of its foundations and a lack of problems due to the fact that most of the outstanding local problems were solved by Cartan. This situation has now changed. For, in the first place, the development of notions in modern mathematics has permitted an entirely satisfactory understanding of most of the results in existence, as shown in a forthcoming paper of mine in the "Colloque de géométrie différentielle, Paris". In the second place, this clarification of the foundations opens the way to the study of various global problems, which have so far hardly been touched. Among such problems is the question whether a manifold has a structure corresponding to a given pseudo-group. This contains as particular cases the questions whether an even-dimensional manifold can be given a complex structure or can be a symplectic manifold in the sense of Ehresmann. Recent achievements in fiber bundles have scored great progress on the question of existence or non-existence of tensor fields satisfying algebraic conditions, such as that of continuous linearly independent vector fields. In our problems the tensor field will be subjected not

only to algebraic conditions, but also to conditions in the form of partial differential equations. In a sense these should be more interesting and deeper problems in differential geometry.

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- 2) Study of some global problems. Two of the most urgent questions are: a) Does the six-dimensional sphere have a complex structure? b) Does the complex projective plane have more than one complex structure? In the case that the answers to these questions are negative, it is my opinion that progress should be achieved by analytic rather than geometrical methods. A more elaborate study of the corresponding differential systems than hitherto undertaken is in order. Similar questions, of different degrees of difficulty and generality, can be proposed and studied.

List of mathematical papers by Shiing-shen Chern:

1. Pairs of plane curves with points in one-to-one correspondence, Sci. Repts. Tsing Hua Univ. 1 (1932) 145-153
2. Triads of rectilinear congruences with generators in correspondence, Tohoku Math. J. 40 (1935) 179-188
3. Associate quadratic complexes of a rectilinear congruence, Tohoku Math. J. 40 (1935) 293-316
4. Abzählungen für Gewebe, Abh. math. Sem. Hamburg 11 (1935) 163-170
5. Eine Invariantentheorie der Dreigewebe aus  $n$ -dimensionalen Mannigfaltigkeiten in  $2n$ -dimensionalen Räumen, Abh. math. Sem. Hamburg 11 (1936) 333-358
6. Sur la géométrie d'une équation différentielle du troisième ordre, C. R. Acad. Sci. Paris 204 (1937) 1227-1229
7. Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif, Bull. Sci. Math. 61 (1937) 234-243

8. On projective normal coordinates, *Ann. of Math.* 39 (1938) 165-171
9. On two affine connections, *J. Univ. Yunnan* 1 (1938) 1-18
10. Sur la géométrie d'un système d'équations différentielles du second ordre, *Bull. Sci. Math.* 63 (1939) 206-212
11. The geometry of higher path-spaces, *J. Chin. Math. Soc.* 2 (1940) 247-276
12. Sur les invariants intégraux en géométrie, *Sci. Repts. Tsing Hua Univ.* 4 (1940) 85-95
13. The geometry of a differential equation  $y''' = F(x, y, y', y'')$ , *Sci. Repts. Tsing Hua Univ.* 4 (1940) 97-111
14. Sur une généralisation d'une formule de Crofton, *C. R. Acad. Sci. Paris* 210 (1940) 757-758
15. (With Chih-ta Yen) Formula principale cinematica dello spazio ad  $n$  dimensioni, *Boll. Un. Mat. Ital.* 2 (1940) 434-437
16. Generalization of a formula of Crofton, *Wuhan Univ. J. Sci.* 7 (1940) 1-16
17. On integral geometry in Klein spaces, *Ann. of Math.* 43 (1942) 178-179
18. (With Hsien-chung Wang) Differential geometry in symplectic space, *Sci. Repts. Tsing Hua Univ.* 4 (1947) 453-477
19. Sur une classe remarquable de variétés dans l'espace projectif à  $n$  dimensions, *Sci. Repts. Tsing Hua Univ.* 4 (1947) 328-336
20. Sur les invariants de contact en géométrie projective différentielle, *Acta Pontif. Acad. Sci.* 5 (1941) 123-140
21. On the invariants of contact of curves in a projective space of  $n$  dimensions and their geometrical interpretation, *Acad. Sinica Sci. Record* 1 (1942) 11-15
22. The geometry of isotropic surfaces, *Ann. of Math.* 43 (1942) 545-559
23. On a Weyl geometry defined from an  $(n-1)$ -parameter family of hypersurfaces in a space of  $n$  dimensions, *Acad. Sinica Sci. Record* 1 (1942) 7-10
24. On the Euclidean connections in a Finsler space, *Proc. Nat. Acad. Sci. USA*, 29 (1943) 33-37
25. A generalization of the projective geometry of linear spaces, *Proc. Nat. Acad. Sci. USA*, 29 (1943) 38-43
26. Laplace transforms of a class of higher dimensional varieties in a projective space of  $n$  dimensions, *Proc. Nat. Acad. Sci. USA*, 30 (1944) 95-97
27. A simple intrinsic proof of the Gauss-Bonnet formula for closed Riemannian manifolds, *Ann. of Math.* 54 (1944) 747-752
28. Integral formulas for the characteristic classes of sphere bundles, *Proc. Nat. Acad. Sci. USA*, 30 (1944) 269-273
29. On a theorem of algebra and its geometrical application, *J. Indian Math. Soc.* 8 (1944) 29-36
30. On Grassmann and differential rings and their relations to the theory of multiple integrals, *Sankhya* 7 (1945) 2-8
31. Some new characterizations of the Euclidean sphere, *Duke Math. J.* 12 (1945) 279-290
32. On the curvatura integra in a Riemannian manifold, *Ann. of Math.* 46 (1945) 674-684
33. On Riemannian manifolds of four dimensions, *Bull. Amer. Math. Soc.* 51 (1945) 964-971
34. Some new viewpoints in the differential geometry in the large, *Bull. Amer. Math. Soc.* 52 (1946) 1-30
35. Characteristic classes of Hermitian manifolds, *Ann. of Math.* 47 (1946) 85-121
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37. Note on affinely connected manifolds, *Bull. Amer. Math. Soc.* 53 (1947) 820-823; correction, *ibid* 54 (1948) 985-986
38. On the characteristic ring of a differentiable manifold, *Acad. Sinica Sci. Record* 2 (1947) 1-5
39. On the multiplication in the characteristic ring of a sphere bundle, *Ann. of Math.* 49 (1948) 362-372

- 40. Note on projective differential line geometry, Acad. Sinica Sci. Record 2 (1948) 137-139
- 41. (With Yuh-lin Jou) On the orientability of differentiable manifolds, Sci. Repts. Tsing Hua Univ. 5 (1948) 13-17
- 42. (With Ye-fon Sun) The imbedding theorem for fibre bundles, Trans. Amer. Math. Soc. 67 (1949) 286-303
- 43. (With Sze-tsen Hu) Parallelisability of principal fibre bundles, Trans. Amer. Math. Soc. 67 (1949) 304-309
- 44. Local equivalence and Euclidean connections in Finslerian spaces, Sci. Repts. Tsing Hua Univ. 5 (1948) 95-121
- 45. (With E. H. Spanier) The homology structure of fiber bundles, Proc. Nat. Acad. Sci. USA, 36 (1950) 248-255
- 46. Differential geometry of fiber bundles, Proc. Int. Congr. Math. II (1950) 397-411
- 47. Topics in differential geometry, mimeographed notes, Princeton 1951.
- 48. On the kinematic formula in the Euclidean space of n dimensions, Amer. J. Math. 74 (1952) 227-236
- 49. (With E. H. Spanier) A theorem on closed orientable surfaces in four-dimensional space, Comm. Math. Helv. 25 (1951) 205-209
- 50. Some formulas in the theory of surfaces, to appear in Boll. Soc. Mat. Mexicana
- 51. (With C. Chevalley) Elie Cartan and his mathematical work, Bull. Amer. Math. Soc. 58 (1952) 217-250
- 52. Some relations between Riemannian and Hermitian geometries, to appear in Duke Math. J.
- 53. (With N. H. Kuiper) Some theorems on the isometric imbedding of compact Riemannian manifolds in Euclidean space, Ann. of Math. 56 (1952) 422-430
- 54. On the characteristic classes of complex sphere bundles and algebraic varieties, Amer. J. Math. 75 (1953) 565-597
- 55. Infinite continuous pseudo-groups, to appear in Colloque Géom. Diff. Paris
- 56. La géométrie des sous-variétés d'un espace euclidien à plusieurs dimensions, to appear in l'Ens. Math.

Financial Statement:

Salary for Professor Chern (full time)	\$3000
Overhead (at 15 percent)	<u>450</u>
Total	\$3450

FACULTY SPONSOR:

Deane Montgomery, Professor of Mathematics

APPROVED FOR  
THE INSTITUTE FOR ADVANCED STUDY:

DATED \_\_\_\_\_

October 22, 1953

Circulated for information:

Professors Gödel

Morse

Oppenheimer

Selberg

von Neumann

Whitney

✓  
✓  
✓  
✓  
✓

Please return to Miss Underwood.

(Sm) S.S. Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

October 22, 1953

Professor S. S. Chern  
Department of Mathematics  
University of Chicago  
Chicago 37, Illinois

Dear Chern:

I find that Leray's notes are for sale and two copies are being sent to you. Some revisions and improvements of these are now in process but I judged from your letter that you would like to have what is available at present.

Everyone here would be enthusiastic to have you here for as long a time as possible. If you would care to have the Institute take up the question of a grant in your behalf either from itself or from some such agency as the National Science Foundation, I know that everyone here would want to be cooperative. If such a thing is advantageous from your point of view I will consult with the people in our department. Certainly all of us would want to do all that we could to make your stay here possible.

Liao is now working on a fixed point theorem for periodic transformations of composite period which is very nice.

Sincerely yours,

Deane Montgomery

DM:js

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA



THE UNIVERSITY OF CHICAGO  
CHICAGO 37 • ILLINOIS  
DEPARTMENT OF MATHEMATICS

October 20, 1953

Dear Dean:

This is to inform you that I have applied for a Guggenheim Fellowship for 1954-55 and have given your name as reference. If it be granted, it is my present plan to spend at least one term in Princeton. I hope I will thus be able to renew my relationship with the Institute, to which I owe so much for my mathematical work.

Meanwhile, I wonder whether it would be possible to get two copies of Leray's notes on partial differential equations. There is a considerable interest here in this material, because Stone is conducting a seminar on it. As I understand that these notes are not yet for sale, I wonder whether your help would enable me to get them in the unfinished form.

With best regards,

Sincerely yours

S. S. Chern

(5m) S.S. Chern

THE UNIVERSITY OF CHICAGO  
CHICAGO 37 • ILLINOIS  
DEPARTMENT OF MATHEMATICS

September 1, 1949

Dear Miss Blake:

There is a friend of mine, Dr. H. C. Wang, who would like to have some application forms for a stipend to the Institute for next year. I shall be much obliged, if you would send me some forms, to be forwarded to him.

Meanwhile, although I have got my mails forwarded from the Institute, it seems that I have not got some printed matters, in particular, the latest issue of the Annals. This may be due to the reason that Miss Iglehart is away, which causes some delay.

everything forwarded

We have got one of the university apartments, and are at the moment busy in procuring furniture and moving. This settles one of the main problems. My visa is still on a temporary basis, But I understand that some action of the Immigration Office here is due soon. The Summer Quarter is just over and I expect to remain here, with the possibility of making a trip to Canada. Both my wife and I find the neighborhood of the University better than expected, although we miss the country life in the Project.

With best wishes and thanks, also from my wife,

Sincerely yours

S. S. Chern

Prof. Wang ✓

sent

Reply Sep. 6/49

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chem, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(52) 55 Chern

May 16, 1949

Dear Richard:

Chern, as you probably know, has accepted a permanent professorship at the University of Chicago. But this cannot be effective unless he succeeds in changing his visa into a non-quota immigration visa. He has applied at the American Consulate in Toronto, but it seems the easiest way for him to get admitted to Canada would be by way of an invitation for a mathematical lecture at Toronto University. I have just written to Robinson asking whether the Department there would be able to extend an invitation to Chern, but I realize that it is pretty late and the term is about to close. If you can help in any way to bring it about that Chern gets an invitation, please do!

Are you in contact with Tuan? I guess it's difficult now to do anything for him.

Cordially yours,

Hermann Weyl

Professor Richard Brauer  
Mathematics Department  
University of Michigan  
Ann Arbor, Michigan  
HW:CB

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(Sm) S.S. Chern

May 16, 1949

Dear Robinson:

I wonder whether it would still be possible before the close of the term for your Department to extend an invitation for a lecture or a talk in the Mathematical Colloquium to Professor Shing-Shen Chern, who is now at our Institute.

Chern, who recently came over with his family from Nanking, is as you probably know a very outstanding man in the field of differential geometry in the large and topology. During the last years he built up the mathematical wing of the Academia Sinica in China. Because it was impossible for him to continue his work there in the present circumstances, we invited him to the Institute for three years, which is exceptional. But since he arrived here he has had several offers from other universities, and recently accepted a permanent professorship at the University of Chicago to begin July 1. I think this is sufficient evidence of the caliber of the man. There is no doubt that he could give you a very interesting talk.

But the reason why this is urgent is his visa problem. He won't be able to teach at Chicago without changing his visa and immigrating into the United States. He applied to the American Consul in Toronto, but the Consul would not take action before Chern shows that he can enter Canada. An invitation to Toronto would be the easiest way to make it possible for him to present his case in person to the Consul.

We are very fond of Chern here, and he is a man who deserves every support.

I remember with great delight the days which I spent in Toronto about a year ago. Let me take this opportunity to thank you for sending me a reprint of your paper "On the disjoint product of irreducible representations of the symmetric group", which is of course of great interest to me. I reciprocate by sending you my paper in the Canadian Journal and another about which I talked in Toronto last year.

With best regards,

Yours sincerely,

Professor G. de B. Robinson  
Mathematics Department  
University of Toronto  
Toronto 5, Ontario, Canada.  
HW:CB

Hermann Weyl

P.S. I just spoke to Chern, and when I asked him he suggested as topic for a lecture, "Differential geometry and topology".

(5m) S S Chern

April 8, 1949

Dear Mr. Mulle:

I should like to write you a few words about Professor Shiing Shen Chern, whom I know quite well both personally and scientifically.

He was at our Institute during the academic years 1943-1945 and through the fall of 1945, and during this time did scientific work of the very highest grade,-- so much so that it is fair to say that he is a mathematician of the first rank as well as being, according to well informed opinion, the leading mathematician of China. He has recently come to the United States again, because the disturbed conditions in China, and particularly at Nanking, make it impossible for him to continue his scientific work there. Several American universities have sought his services, but he now wishes to accept a full professorship at the University of Chicago, which is of course a very distinguished appointment.

I think that on every account his case as an applicant for the necessary permission to do permanent work in the United States can be described as exceptionally meritorious.

Yours sincerely,

Mr. Henry L. Mulle  
Assistant District Commissioner  
Immigration and Naturalization Service  
42 South 15th Street  
Philadelphia, Pennsylvania  
OV:CB

Oswald Veblen

Copy to Mrs. Leary

(3m) 35 Chern

April 13, 1949

The Commissioner of Immigration  
and Naturalization Service  
Department of Justice  
Washington 25, D.C.

Dear Sir:

May I take the liberty of suggesting that Professor Shing Shen Chern be given permission to accept a professorship at the University of Chicago while still in his present status of holding a 3(2) visa? I think that I may best convey the reasons for such exceptional treatment of Professor Chern by giving you a short outline of his position in mathematical science and his relationship to the Institute for Advanced Study.

He first came to our Institute in 1943, largely as a result of the interest which several of my colleagues and I felt in his scientific work. Before that he had already spent several years of study in France and Germany, during which time he had obtained a very thorough training in classical differential geometry. This is a branch of mathematics which deals with the local peculiarities of a surface or of its generalizations to any number of dimensions. These generalizations have been very widely studied during the last twenty or thirty years because of their applications to the theory of relativity. In particular Professor Chern became thoroughly acquainted with the methods of the great French geometer Élie Cartan, who is now over eighty years old and to whom Professor Chern is a worthy successor. Chern's program of work was to combine these purely local methods of studying manifolds with the "global" methods of topology. The latter subject is a branch of mathematics which has been very much cultivated during this same period of twenty or thirty years, here at Princeton. The time is now ripe for a combination of these two mathematical tendencies which should result in a very valuable theory of "differential geometry in the large".

Professor Chern made substantial progress on this program while he was here at the Institute. He returned to China early in 1946 to assume a position of very considerable scientific and administrative importance in the Academia Sinica at Nanking. But a continuation of this work in China has been rendered impossible by the course of the Chinese civil war. When we learned of the conditions under which he was trying to live, our Institute extended him an invitation which made it possible for him to come back to Princeton, where he now is. Since his return several universities have been seeking his services, but it is quite clear that the position which he has been offered at the University of Chicago affords a unique opportunity for him, not only to carry out the scientific program which I have outlined, but also to influence the development of science in this country, and perhaps to become the founder of an important new school of scientific thought. For this Professor Chern is very well qualified, first because he has the temperament of an influential teacher, secondly because his command of English and his knowledge of American life are exceptionally good; but most of all because he has a unique training for a scientific program which is recognized by mathematicians as thoroughly sound and very much to be desired. I think it is fair to say that there is no one else available anywhere who is as well qualified as is Professor Chern to carry out this program.

Very truly yours,

Oswald Veblen

OV:CB

Copy to Mrs. Leary  
Mr. H. L. Zuckler

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

Prof. Neblen is holding  
app. — (before 13) letter copy to  
Mulle

(5m) Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

OFFICE OF THE DIRECTOR

February 22, 1949

To: Miss Trinterud  
From: Mrs. Leary  
Subject: Grant for Professor S. S. Chern (1948-1949)

A grant of \$4000. should be charged against the School of Mathematics for Professor S. S. Chern for the 1948-1949 term. The sum advanced to him to cover traveling costs from China should be deducted from the grant; the balance is to be paid monthly for the five months February through June 1949.

Copy: Prof. Veblen

The monthly payments will be about \$360. Prof. Chern at the time of his arrival said he did not need cash immediately; approximately \$2200. was advanced to him for travel expenses. His rent, including utilities, coal, etc. comes to a little over \$100. a month. He has one of the three-bedroom apartments.



Publications of Shiing-shen Chern

- Abzählungen für Gewebe, Abh.d.Hamb.Univ. 11 (1935), 163-170  
Eine Invariantentheorie der Dreigewebe aus  $r$ -dimensionalen Mannigfaltigkeiten im  $R_{2r}$ ,  
Abh.d.Hamb.Univ. 11 (1936), 333-358  
Sur la géométrie d'une équation différentielle du troisième ordre, Comptes Rendus  
Acad.Paris 204 (1937), 1227-1229  
Sur la possibilité de plonger un espace à connexion projective donné dans un espace  
projectif, Bull.Sc.Math., 2 série, 61 (1937), 234-243  
On projective normal coordinates, Annals Math. 39 (1938), 165-171  
The geometry of higher path-spaces, Jour.Chin.Math.Soc. 2 (1940), 247-276  
On a Weyl geometry defined from an  $(n-1)$ -parameter family of hypersurfaces in a  
space of  $n$  dimensions, Science Record 1 (1941), 5-11  
On the invariants of contact of curves in a projective space of  $n$  dimensions  
and their geometrical interpretation, Science Record 1 (1941), 13-17  
On integral geometry in Klein spaces, Ann.Math. 43 (1942), 178-189  
The geometry of isotropic surfaces, Ann.Math. 43 (1942), 545-559  
On the euclidean connections in a Finsler space, N.A.S.Proc, 20 (1943), 33-37  
A generalization of the projective geometry of linear spaces, NAS Proc. 20  
(1943), 38-43  
Laplace transforms of a class of higher dimensional varieties in a projective  
space of  $n$  dimensions, NAS Proc. 30 (1944), 95-97  
Integral formulas for the characteristic classes of sphere bundles, NAS Proc.  
30 (1944), 269-273  
A simple intrinsic proof of the Gauss-Bonnet formula for closed Riemannian mani-  
folds, Ann.Math. 45 (1944), 747-752  
Some new characterizations of the Euclidean sphere, Duke Math.Jour. 12 (1945),  
279-290  
On the curvatura integra in a Riemannian manifold, Ann.Math. 46 (1945), 674-684  
On Riemannian manifolds of four dimensions, AMS Bull. 51 (1945), 964-971  
Some new viewpoints in differential geometry in the large, AMS Bull. 52 (1946), 1-30  
Characteristic classes of Hermitian manifolds, Ann.Math. 47 (1946), 85-121

*For complete*

Chern, Shiing-shen

1942-Dec.18(?) /45 (Sailed from  
San Francisco Feb.27/46  
Feb.5/49-

B.S. 1930 Nankai Univ.

M.S. 1934 Tsing Hua Univ., Peiping, China

D.Sc. 1936 Hamburg Univ., Germany

Born Oct.26,1911, Kashing, Chekiang Province, China

Citizenship - Chinese

Married (1st trip- without family); 2 children

Entered USA:

- 1) At Miami, Florida, Aug.11/43 under Official\*Visa No. 332 issued May 25/43 at  
Chinking, China. Visa and Entry permit unlimited. \* Official's(?)
- 2) At Honolulu Jan.1/49 by Pan-Amer. World Airways under Visitor's  
Visa No. 598, issued Shanghai, China, Dec.28/48

Princeton home address:

- 1) 47 Wiggins Street
- 2) 3-D Goodman Road

Permanent address - Institute of Mathematics, 320 Yoyang Road, Shanghai, China

In case of emergency notify Mr. S.C.Chang, 148 Governor Street, Providence, R.I.

Honors and societies - Ting Wen-kiang Prize of Academia Sinica 1943; member, German  
Math. Soc., AMS

Positions held:

Research Fellowship of Natl. Tsing Hua Univ. 1934-36

" " " China Foundation 1936-37

Natl. Tsing Hua Univ. - Prof. Math. 1937-

IAS stipend and Chinese Govt. grant 1943-Dec./45

Academia Sinica Inst. of Math., 1946-

IAS stipend Feb.5/49-

(On leave)

(On leave)

Field - Math.

(52) S.S. Chern

INSTITUTE FOR ADVANCED STUDY  
Princeton, New Jersey

School of **Mathematics**

Arrival date: **Feb. 5. 1949**

Full name: **Shiing-Shen Chern**

~~Departure date:~~

Princeton address: **3 D Goodman Road**

✓ Telephone:

Permanent address: **Institute of Mathematics, 320 Yoyang Road, Shanghai, China**

Date and place of birth:

Married or single: **M**

Citizenship: **Chinese**

Number of children accompanying you: **2**

✓ If foreign, under what kind of visa did you enter the United States? (6 -) <sup>visita</sup>

→ Visa No: **598**

→ Place and date of issue: **Shanghai, Dec. 29, 1948**

→ Expiration date of visa: **Dec 28, 1949**

Date and port of entry to United States: **Jan 1, 1949, Honolulu**

Name of ship:

→ Name of airline: **Pan-American World Airways**

→ Expiration date of entry permit:

✓ Name and address of person, outside of Princeton, to be notified in case of emergency:

**Mr. S.C. Cheng, 148 Governor Street, Providence, R.I.**

✓ ~~Project or~~ field of work at Institute: **Mathematics**

Academic degrees: (Give name of college or university and year conferred)

Fellowships: (Give full name, dates and place of study)

All positions held: (Please give dates and state if on leave of absence)

Honors and Societies:

Publications: (Please give title and reference in full, with Vol. No, year and page numbers. Use other side of sheet.)

~~(Please fill out in duplicate and send copy to Director's Office.)~~

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(Su) Chern

February 4, 1949

Dear Chern:

I am delighted to have your letter and to hear that you will cooperate with our little seminar. I decided that it would be best to have Kuiper make his report next Monday, and not to start your series until the following Monday, February 14.

It has always been a source of regret to me that I didn't have the energy during the war years to take advantage of your presence here in Princeton. I hope it will be somewhat better this time.

Hoping to see you on Monday morning at 10:20,

Yours sincerely,

Professor S. S. Chern  
Kindness of Mrs. W. Y. Chang  
OV:GB

Oswald Veblen

(Sm) S. S. Chern

Air Mail

January 31, 1949

My dear Chern:

While I was away on my vacation I was delighted to hear that you had arrived in this country, and now on returning to Princeton I hear that you will probably get here on Saturday. I myself may be out of reach at that time and I am therefore sending you this word. There is a group of mathematicians here who are very much interested in your field and who have, some of them, done related work. We are therefore hoping very much that you will take an active, and indeed a leading, part in our little seminar. The seminar meets regularly on Monday morning at 10:40. Just today we had a report from Dr. Santaló on "Integral formulas for intersection of surfaces", and would be quite ready to hear from you on Monday the 7th, if you were inclined to speak. On the other hand, if you would prefer to wait, the substitute program will be a paper by Dr. Nicolaas H. Kuiper in which he gives a classification of compact conformally Euclidean spaces.

Of course it is obvious that both I and the others to whom I have referred would be very anxious to have you cooperate in our seminar. On the other hand I realize that it is quite possible that you may have other plans, and indeed might prefer not to make any plans at all until you have been here for a while. Therefore please don't feel that this letter is as insistent as it sounds.

Sincerely yours,

Oswald Veblen

Professor S. S. Chern  
Care Professor André Weil  
Eckhart Hall  
University of Chicago  
Chicago 37, Illinois  
OV:CB

(S.M.) Chern

Air Mail

January 17, 1949

Dear Professor Stone:

Professor Veblen from Florida asks me to let you know that Professor S. S. Chern is now in California, Care Professor H. W. Bohnenblust, 4595 Thorndale Road, Pasadena 4, where he expects to stay until January 30. After that he can be reached in care of Professor Andre Weil until February 4, when he will leave Chicago for Princeton.

Sincerely yours,

Secretary, School of Mathematics

Professor Marshall H. Stone  
Eckhart Hall  
University of Chicago  
Chicago 37, Ill.

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

Profs. Weyl ✓  
Morse ✓  
v. Neumann ✓ Berkeley  
Veblen ✓  
Siegel ✓  
January 10, 1949  
(32m) Chem

Dear Miss Blake:

9 Blake

I am glad to inform you that I have arrived San Francisco, with my family. Although my wife and children were seasick, the air trip was wonderful. In order to visit a few institutions and friends, we are going to stop at a few places on the way. If you do not hear from me to the contrary, we shall be in Princeton on February 5, which will probably be the time for the Institute to reopen.

Please hold my mails if there is any. If there is anything important, I think I can be reached by the following addresses:

Before Jan 30, c/o Professor F. Bohnenbust, 3595 Therman Ave Road, Pasadena 10, Cal.

Before Feb 4, c/o Professor A. Weil, Eckhart Hall, University of Chicago, Chicago 37, Ill.

I shall appreciate very much, if you communicate my exact date of arrival to Mrs. J. D. Leary, so as to have my apartment ready.

Looking forward with great pleasure in seeing you and my friends in Princeton soon,

Sincerely yours  
S. S. Chern

copy sent Mrs. Leary  
Jan 15

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

OFFICE OF THE DIRECTOR

December 23, 1948

Dear Professor Chern:

Dr. Oppenheimer has given me your letter of December 14th, and I am writing to say that we are arranging to have one of the Institute apartments ready for you and your family. So far as we can see now, it will be one of our three-bedroom units. The apartments are on the Institute grounds, very near Fuld Hall. They are furnished, and we can supply you with some china, cooking utensils, etc., in the event you will not have them. The attached rental schedule will give you more details.

Since we have to do some moving in order to make one of the apartments available for you, it will be very helpful if you can let me know just as soon as the date of your arrival is definite. We realize how difficult it must be to get passage but we hope you will be successful.

We will be very glad to welcome you and your family.

Yours sincerely,

(Mrs. John D. Leary)  
Aide to the Director

Professor S. S. Chern  
Institute of Mathematics  
Academia Sinica  
320 Yoyang Road  
Shanghai, China

✓ Copy: School of Mathematics

Mrs. Stephens - Hold 3D Goodman; am trying to get something else for month of February for Stiefels; arrival of Cherns uncertain - latest advice is arriving by air about Jan. 8. Prof. Chern, wife, boy 8, girl 1.



930 ✓  
COPY

December 23, 1948

Dear Professor Chern:

Your letter which I received yesterday I handed at once to Mrs. Leary who is now in charge of finding and assigning housing for the Institute members. By the end of the day I understand that she had arranged an apartment for you in the group of apartments near the Institute building. The apartment would be furnished unless you should chance to wish otherwise, which seems unlikely. You will hear from Mrs. Leary in more detail, but I hasten to get this assurance to you before you leave.

It is good news that you have got as far as Shanghai, and that you have a reservation, if you wish it, to fly so soon. We look forward to your arrival with the greatest pleasure.

Sincerely yours,

93

LETTERHEAD OF

INSTITUTE OF MATHEMATICS, ACADEMIA SINICA  
320 Yoyang Road, Shanghai, China

December 21, 1948

Dr. Robert Oppenheimer  
Institute for Advanced Study  
Princeton, New Jersey  
U. S. A.

Dear Dr. Oppenheimer:

Enclosed please find a calculation memo for the travelling expenses of myself and my family (2½ persons) from Shanghai to Princeton, amounting to \$2,171.14. As my trip has been arranged by the China Travel Service, may I request you to deposit the same amount to the following account:

China Travel Service, Shanghai Office  
Wells Fargo Bank and Union Trust Company  
Market at Montgomery Street  
San Francisco

I am scheduled to leave Shanghai by air on December 29th. I expect to spend about two weeks in the West Coast and to be in Princeton in the end of January. In San Francisco I think I can be reached by the following address: c/o Professor Y. R. Chao, Department of Chinese, University of California, Berkeley, California.

With my thanks in advance,

Sincerely yours

(signed S. S. Chern)

Copy: Prof. Veblen  
Prof. Weyl ✓  
Miss Blake - School of Mathematics file

NOTE; The requested sum has been sent to the Wells Fargo Bank with request it be deposited to China Travel Service account.

EWL

C O P Y

INSTITUTE OF MATHEMATICS

Academia Sinica

320 Yoyang Road

Shanghai, China

December 14, 1948

Dear Dr. Oppenheimer:

I am glad to inform you that I have got our passport and am now applying for the visa. If things come out smoothly, the visa will probably be granted in ten days. The transportation, however, poses a more serious problem. Because of the West Coast strike and the evacuation, there is a great backlog of people on the waiting list for ships. Perhaps we have to go by air.

As my chance of coming to Princeton increases, I am thinking of the housing problem after our arrival. I understand that there is a number of apartments built on the Institute ground. It would be a most satisfactory arrangement, if some vacancy can be found there. I shall be much obliged, if you could give some thought on this problem. Meanwhile, I am writing to my Chinese friends in Princeton to help me in this matter.

I have yet no idea about the date of our departure. An air reservation has been made on January 5, 1949, but we shall go by plane only after all hopes to go by sea are abandoned.

Please notice that my new address is in Shanghai.

With best regards,

Sincerely yours

(Signed) S. S. CHERN

Prof. Morse ✓  
v. Neumann ✓  
Siegel ✓  
Veblen ✓  
Weyl ✓

G. Blake

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

OFFICE OF THE DIRECTOR


December 10, 1948

TO WHOM IT MAY CONCERN:

This will certify that Professor S. S. Chern has been appointed a member of the Institute for Advanced Study in Princeton, New Jersey from January 1, 1949 to December 31, 1951, and that funds will be made available to defray the cost of travel for him and his family from China to the United States and for the maintenance of Professor Chern and his family while in the United States under this appointment.

Robert Oppenheimer  
Director

Subscribed and sworn to before me  
this 10th Day of December, 1948.

  
*Katherine Russell*  
Notary Public of New Jersey

COPY

*Frags. Einstein ✓  
Morse ✓  
v. Zimmann ✓  
Siegel ✓  
Veltan ✓  
Weyl ✓  
G. Blake*

*(S.S.) Chern*

December 2, 1948

Dr. Robert Oppenheimer  
Institute for Advanced Study  
Princeton, New Jersey  
U. S. A.

Dear Dr. Oppenheimer:

I received with thanks your cable of November 29, which confirms my appointment at the Institute. My procedure of securing the passport has been ready and, if things come out smoothly, I shall be able to get the passport in a week.

I talked to Mr. George Harris of the American Embassy and the United States Educational Foundation in China. He promised his help in visa and transportation. He suggested that in getting the visa it would be necessary to present to the American authorities a formal document of my appointment and travelling allowances, the cable being not considered as valid. May I therefore request you to send me such a document, one copy each, to the following addresses:

Professor S.S. Chern  
Institute of Mathematics  
Academia Sinica, 320 Yoyang Road  
Shanghai, China

Professor S.S. Chern  
Institute of Mathematics  
Academia Sinica, 1 Jeou Hwa Shan  
Nanking, China

(This does not mean that there are two institutes. Our Institute used to be in Shanghai and was moved to Nanking last spring. The building is now being used by other institutes of the Academy).

According to Mr. Harris it would be much easier to get a plane reservation than ship. As the situation here may develop very quickly, I think it is advisable to depart as early as possible. However, travel by air will make the total expenses a pretty large sum, and I do not want to put such a drain on the already fixed budget of the Institute. I believe I can pull together some resources of my own. In case the sum becomes too large, I shall be completely agreeable to pay a part of it myself. I have no doubt that this can be settled after I arrive in Princeton.

Please send further communications to my Shanghai address.

With best regards,

(Signed) S.S. Chern  
Shiing-shen Chern

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

COPY

*After copy*

December 2, 1948

Dr. Robert Oppenheimer  
Institute for Advanced Study  
Princeton, New Jersey  
U. S. A.

Dear Dr. Oppenheimer:

I received with thanks your cable of November 29, which confirms my appointment at the Institute. My procedure of securing the passport has been ready and, if things come out smoothly, I shall be able to get the passport in a week.

I talked to Mr. George Harris of the American Embassy and the United States Educational Foundation in China. He promised his help in visa and transportation. He suggested that in getting the visa it would be necessary to present to the American authorities a formal document of my appointment and travelling allowances, the cable being not considered as valid. May I therefore request you to send me such a document, one copy each, to the following addresses:

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Institute of Mathematics  
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Please send further communications to my Shanghai address.  
With best regards,

(Signed) S.S. Chern  
Shiing-shen Chern

(Sinc) Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

OFFICE OF THE DIRECTOR

December 1, 1948

Certified cable

American Embassy  
Nanking  
China

This is to certify that three year appointment starting January 1, 1949 with an annual grant of \$4,000 is offered to Professor S. S. Chern, Academia Sinica, Nanking, by the Institute for Advanced Study.

Robert Oppenheimer

Subscribed and sworn to before me  
this 1st Day of December, 1948.

Notary Public of New Jersey

Copy: Professor Veblen  
Charge: School of Mathematics

12/2/48

Professor Veblen:  
The Western Union office did not  
know how to send this off - had to get instruc-  
tions. It finally went last evening about 7 p.m.  
Cost \$17.70.

EWL

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(Sm) S.S. Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

Prof. Morse ✓  
✓ Newman ✓  
Siegel ✓  
Veblen ✓  
Kauf ✓

OFFICE OF THE DIRECTOR

J. D. Leary

November 30, 1948

Special Delivery

Mr. P. Yen  
Chinese Consulate General  
1250 Sixth Avenue  
New York City

Dear Mr. Yen:

In confirmation of my telephone conversation with you today, I am sending you this written request that the Consul General cable the Nanking Foreign Ministry to the following effect:

The Institute for Advanced Study has offered Professor S. S. Chern, Institute of Mathematics, Academia Sinica, Nanking, a three year membership starting January 1, 1949 with an annual grant of \$4,000. Funds can be made available for transporting Professor Chern and his family to this country.

The School of Mathematics here is most anxious to facilitate the coming of Professor Chern to this country, and in present circumstances, believes that prompt action is necessary. We would appreciate your sending a certified cable embodying this appointment to membership at the Institute for Advanced Study immediately. Our check for \$20. is enclosed. It is our understanding that this amount is requested to cover the cable charges and that refund will be made to us if this is an over-payment.

Thank you very much for your courteous help.

Yours sincerely,

(Mrs. John D. Leary)  
Aide to the Director

✓ Copy: Prof. Veblen  
Enc. \$20. check - #2585

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chem, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA



Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
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DAY LETTER	NIGHT LETTER

# WESTERN UNION

1206

INTERNATIONAL SERVICE	
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FULL RATE	DEFERRED
CODE	NIGHT LETTER

JOSEPH L. EGAN, PRESIDENT

NO. WDS.-CL. OF SVC.	PD. OR COLL.	CASH NO.	CHARGE TO THE ACCOUNT OF	TIME FILED

Send the following message, subject to the terms on back hereof, which are hereby agreed to

Deferred Cablegram  
November 29, 1948

Professor S. S. Chern  
Institute of Mathematics  
Academia Sinica  
Nanking 5, China

Glad to offer you membership Institute for Advanced Study for three years starting January 1, 1949 with annual grant \$4,000. Can make available funds for your passage. Shall communicate State Department to facilitate your entry.

Robert Oppenheimer

Charge: School of Mathematics  
Copy to: School of Mathematics

**A NEW  
CURTIS  
SERVICE**

Telegraph your order for America's favorite magazines—HOLIDAY, 1 yr., \$5 • the Post, 1 yr., \$5 • LADIES' HOME JOURNAL, 1 yr., \$3. All prices U. S. only. No charge for wire. Pay Western Union clerk for subscription or when billed by publisher.

Publisher will, on subscriber's request, refund full amount paid for copies not previously mailed. Prices subject to change without notice.

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(32m) 53 Chern

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DAY LETTER	NIGHT LETTER

# WESTERN UNION

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JOSEPH L. EGAN, PRESIDENT

NO. WDS.-CL. OF SVC.	PD. OR COLL.	CASH NO.	CHARGE TO THE ACCOUNT OF	TIME FILED

Send the following message, subject to the terms on back hereof, which are hereby agreed to

Prop. Einstein ✓  
 Morse ✓  
 15. Krummholz ✓  
 Siegel ✓  
 Valden ✓  
 TO J. R. Oppenheimer  
 Weyl ✓

#20.

Nanking, China  
November 29, 1948

Would you request Chinese Consular General to confirm Nanking Foreign Ministry by cable three year appointment with family accommodations.

Copy to School Mathematics

Chern

<b>A NEW CURTIS SERVICE</b>	<b>Telegraph your order for America's favorite magazines—HOLIDAY, 1 yr., \$5 • the Post, 1 yr., \$5 • LADIES' HOME JOURNAL, 1 yr., \$3. All prices U. S. only. No charge for wire. Pay Western Union clerk for subscription or when billed by publisher.</b>	Publisher will, on subscriber's request, refund full amount paid for copies not previously mailed. Prices subject to change without notice.
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FULL RATE TELEGRAM	SERIAL
DAY LETTER	NIGHT LETTER

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CODE	NIGHT LETTER

JOSEPH L. EGAN, PRESIDENT

NO. WDS.-CL. OF SVC.	PD. OR COLL.	CASH NO.	CHARGE TO THE ACCOUNT OF	TIME FILED

Send the following message, subject to the terms on back hereof, which are hereby agreed to

Nanking, China  
November 29, 1948

TO: J. R. Oppenheimer

Would you send me confirmation for three year appointment with family accommodations. Please cable reply.

Chern

Copy to School Mathematics

**A NEW  
CURTIS  
SERVICE**

Telegraph your order for America's favorite magazines—HOLIDAY, 1 yr., \$5 • the Post, 1 yr., \$5 • LADIES' HOME JOURNAL, 1 yr., \$3. All prices U. S. only. No charge for wire. Pay Western Union clerk for subscription or when billed by publisher.

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PRINCETON UNIVERSITY  
PRINCETON NEW JERSEY

Department of  
MATHEMATICS

Address reply to  
FINE HALL  
BOX 708  
PRINCETON, N.J.

November 24, 1948

Dear

I have just received word from Professor S. S. Chern, Institute of Mathematics, Academic Sinica, 1 Jeou Hwa Shan, Nanking 5, China, to the effect that conditions in China are now so disastrous for scientific work that he is eager to leave the country to come to the United States on a permanent or temporary basis. As Chern was notoriously a "last ditch fighter" the situation must be exceedingly bad indeed.

The information about Chern is as follows. He is 37 years old, married and the father of two children. He studied in Germany and France and took his doctorate at Hamburg in 1936. He spent two years at the Institute for Advanced Study, 1943-1945. His early work was in differential geometry, but he soon switched to topology, working mainly in a combination of topology and differential geometry. There is no question that Chern has acquired in recent years a front rank position as a topologist. He has an exceedingly pleasant personality, speaks excellent English and is a forceful and interesting lecturer. His success as a teacher at all levels is a certainty. Indeed no Chinese mathematician known to the undersigned personally could fit in better in any one of our institutions.

While it might take him a little time to get ready, I believe that he could probably be available as early as next Spring. In case you so desire, I should be glad to provide any further information that I may have.

Sincerely yours,

S. Lefschetz

國立中央研究院數學研究所  
INSTITUTE OF MATHEMATICS

ACADEMIA SINICA  
~~220 / NANKING / R/O/D~~  
SHANGHAI, CHINA  
1 Jeou Hwa Shan, Nanking 5

November 22, 1948

Dear Professor Weyl:

Yesterday I received a cable from Dr. Oppenheimer, offering his help to bring me to the US. I imagine that you and my other friends of the School of Mathematics must be the motive for this action, and it is needless to say that I am deeply grateful for your kindness.

The situation is such that Nanking itself is in danger of falling into communist hands, and I think it would be my last choice to be behind the iron curtain. I am therefore starting preparations to make the trip. As I believe I should bring my family along, the undertaking would probably involve a considerable amount of difficulties, particularly owing to the rapidly changing situation.

I have written to Dr. Oppenheimer, suggesting the steps he would care <sup>to take</sup> to help me. Among these I believe the most important thing is to get the permission and help from the State Department, because it would then solve the difficulties of both visa and transportation.

The general attitude of the intellectuals here is that of resign, as there are so few other alternatives. However, nobody seems to have any idea of the communists' attitude toward the educated people, particularly those educated in the West. It is commonly believed that it would be liberal in the early stage and that the grip would be tightened afterwards. Most of us also think that any form of coalition government with the communists would be short-lived. I have talked to my authorities about my plan. They are in favor of the idea of sending some of us to places of safety, whenever possible.

With my best regards to you and my other friends at the Institute.

Very sincerely yours

S. S. Chern

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(524) Chern

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DAY LETTER	NIGHT LETTER

# WESTERN UNION

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INTERNATIONAL SERVICE	
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FULL RATE	DEFERRED
CODE	NIGHT LETTER

JOSEPH L. EGAN, PRESIDENT

NO. WDS.-CL. OF SVC.	PD. OR COLL.	CASH NO.	CHARGE TO THE ACCOUNT OF	TIME FILED

Send the following message, subject to the terms on back hereof, which are hereby agreed to

Prof Einstein ✓  
 Morse ✓  
 v. Krummholz ✓  
 Singel ✓  
 Veblen ✓  
 Weyl ✓

COPY

Delayed Cable  
November 19, 1948

G. B. B. B.

**Professor S.S. Chern**  
**Institute of Mathematics**  
**Academia Sinica**  
**Nanking 5, China**

If there should be any steps that you would like to have us take in the next months to facilitate your coming to this country please let us know.

Charge, Sch. of Math.

**Robert Oppenheimer**  
**Inst. for Advanced Study**

<b>A NEW CURTIS SERVICE</b>	Telegraph your order for America's favorite magazines—HOLIDAY, 1 yr., \$5 • the Post, 1 yr., \$5 • LADIES' HOME JOURNAL, 1 yr., \$3. All prices U. S. only. No charge for wire. Pay Western Union clerk for subscription or when billed by publisher.	Publisher will, on subscriber's request, refund full amount paid for copies not previously mailed. Prices subject to change without notice.
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(32) Chern

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY  
November 18, 1948

To: Professor Veblen  
From: Mrs. Leary

The Visa Division in the State Department can only suggest that either Prof. Chern is advised to travel immediately to a safer place than Nanking - taking papers and credentials - and apply at Consul there; or that we send a certified cable (including terms of our contract) to the American Consul at Nanking at same time as we cable or write Prof. Chern. The certified cable can be notarized and apparently can be expected to expedite Consul's action.

Passage is sure to be difficult. At present everyone has to go to Hong Kong or Shanghai and fly to Manila (about \$75.): wait there for ships to start again for U.S. Flight all the way would be over \$700. per person. There are family rates, etc. but it would be very high.

EWL

(Sm) S.S. Chern  
*Memorandum*

To Mrs. Leary

Date Nov. 11/48

From G. Blake

Re Professor Shiing-Shen Chern

Professor Veblen suggests that I send you the following information about Professor Shiing-Shen Chern, who was a member of the Institute's School of Mathematics 1943-45. He was born 1911 in China, is married, had one child when he was here and probably more now. After obtaining his B.S. and M.S. in China, he got his D.Sc. 1936 Hamburg, Germany. Beginning in 1937 he was Professor of Mathematics at the National Tsing Hua University of Peiping, and since his return to China has been Director (at least Acting) of the Institute of Mathematics of the Academia Sinica, now in Nanking. The most recent letter from him, this autumn, is from the address

Institute of Mathematics  
Academia Sinica  
1 Jeou Hwa Shan  
Nanking 5, China



COPY

APPENDICES  
Prof. Vahlen ✓  
Weyl ✓  
Levy ✓

INSTITUTE OF MATHEMATICS  
Academic Sinica  
1 Jeou Hwa Shan, Nanking 5

November 9, 1948

Dear Professor Lefschetz:

You perhaps have followed the recent developments in China. The general situation is very unstable and the present hardship is much more than the war years. No matter what the outcome is, I believe that it will be quite a few years before mathematical research in China can have any hope of real progress. You know I have always hoped for the best for my country and have so far not wavered about hardship. However, the conditions are now such that I may not be able to continue my work. Daily necessities such as rice, fuel, etc., are now taking a great deal of my energy.

Under such circumstances I think that you must be sympathetic to my intention to pull out of this place, although I made this decision not without pain. I am therefore asking you whether you know there is at present any possibility for me to get a position in the US, either on a temporary or permanent basis. I shall be ready to leave at any time. However, as I shall naturally bring my family (wife and two kids) along, it will probably take some time in the preparation. Looking forward for your help and advice, I am,

Yours very sincerely,

(SIGNED) S. S. Chern

(IAS) S.S.Chern

~~L.K.Hua~~

Leopoldo Nachbin

C O P Y

Consular Section - U.S. Embassy - Rio de Janeiro - July 24, 1947

Dear Professor Veblen:

There is a possibility, still in the nebulous stage, that we may be able to invite someone to Chicago on an arrangement similar to that we made (very successfully, I believe) with Marcel Riesz. My thoughts turn at once to L.K.Hua and Chern, either of whom could contribute a great deal to our graduate program. Can you throw light on the immediate plans of either Hua or Chern? At the moment, I obviously cannot stimulate anticipations by any kind of direct inquiry. I know that both would welcome further stays in the U.S.; but I have heard that Chern may receive some kind of invitation from Columbia, an arrangement with which I ought not to interfere -- and I am not informed of Hua's plans at all.

I am having an interesting time here though lecturing and sightseeing do not give me much time to work. There is one really excellent young mathematician (25) here named Leopoldo Nachbin -- born and raised in Brazil but of Austrian parentage. He is turning out real mathematics, mostly on topological lattices and related topics. As soon as his position in the Faculty is secure, he should come to the U.S. for a year or two. I think Chicago may be able to help -- I would be glad to make him a visiting lecturer (he is an excellent expositor, even in English) if I can make the necessary arrangements with the University, but then travel expenses would have to be picked up somewhere.

\* \* \* \* \*

(Signed) MARSHALL STONE

*Air mail*

*(3) 33 Chern*

April 30, 1947

Dear Professor Chern:

Professor Chiang gave me your message that you wished your reprints sent out now, and the remainder sent to you. It happens to be a lucky time, when the professors are away for the National Academy meeting. But imagine my chagrin when I came actually to put the reprints into the envelopes, to find in one package two sets of reprints, the under and hidden one being "On the curvatura integra in a Riemannian manifold", - the cause of all the delay! Since I find that Professors Veblen and Weyl already had that and the Bulletin reprint "On Riemannian manifolds of four dimensions", I assume that Professor Tuan had distributed these locally and sent you at least 20 copies. Except as this affects local distribution, I have now sent these four reprints to all the addresses you gave me + Prof. Arthur N. Milgram now here, but of the University of Notre Dame; however, there were nearly 30 copies too few of the Bulletin reprint to go around. Of that I am sending you one copy, by itself, to make sure that you are not without it. Many copies of the other three reprints will go to you at the Academia Sinica in Shanghai tomorrow, in 10 packages.

Mrs. Weyl, you may have heard, had to undergo a serious operation which involved a long stay in the hospital. But she is now recovered and looks well and beautiful again, and plans to fly in three weeks with Professor Weyl to Zürich where he will give a summer course. Professor Siegel was still in Göttingen when our latest letters were written. Professor Dirac will be a Visiting Professor here for the whole of next year. But, especially important, the new Director to follow Dr. Aydelotte on his retirement next fall has now been appointed, - Professor J. Robert Oppenheimer of California and Los Alamos, who will therefore also add another theoretical physicist to our School. Dr. Aydelotte has bought the brick house next the Tenbroecks on Battle Road, and will continue his scholarship and fellowship activities in Fuld Hall. Professor Chow and his family are now established in the new Institute apartments between Fuld Hall and the golf course; he brought over here today his lovely daughter Marian. And Professor Hua is recovering at the Johns Hopkins Hospital from an operation to cure his lameness! Miss Miller saw him there a day or so ago and found him in excellent spirits, and looking forward to learning to walk again before long.

Sincerely yours,

*R.B.*

Professor Shiing-Shen Chern  
Mathematics Department  
Tsing Hua University  
Peiping, China

December 3, 1946

Dear Professor Chern:

Yesterday Professor Chiang brought me your letter of early November and the charming filigree pin. It is most dainty and I thank you, but you should not have sent me anything. However this will be a pleasure and a happy reminder of your stay, - which we all regret could not still be continuing.

Since this letter must be both social and business, I hope you will excuse typewriting.

You thought we had 4 sets of your reprints. Three which you list we have, but not "On the curvatura integra in a Riemannian manifold," *Annals of Mathematics* 46 (1945), 474-484, which was published before the other three. I wonder whether that set could have been among the things which Professor Tuan sent before sailing to a friend in California? Mrs. Weber in Fine Hall has no record concerning orders for reprints, and doubts whether the Waverly Press keeps such records but will inquire. I am puzzled too by the large number of reprints of your *Annals* paper "Characteristic classes of Hermitian manifolds", - 300 I should estimate on unpacking them now. We shall be glad to send out these reprints; but possibly shall wait for further word about the missing set. All this year we have had a second secretary for the School of Mathematics, Miss Ray, who will go ahead now with preparing the envelopes. Of course sending the remaining reprints to you will wait until the others have been sent out.

Professor Tuan has several journals and two books, waiting for the instructions he promised me as to where to send them. One of the books is a copy of the new edition of Professor Weyl's Classical Groups, and the other something from the Chelsea Publishing Company. Belated notices from the A.M.S. etc., I hardly think he will wish to have forwarded. Perhaps you would remind him to send me word?

Professor Siegel is now in Göttingen for a temporary visiting professorship in his old chair, and Professor Morse in Paris to receive an honorary degree. Professors Borsuk and Hlavaty and Whitehead are all here at present, Professor Heinz Hopf at the University to stay, giving a course, until April 1; and Professor Marcel Riesz has already arrived for the mathematics conference (December 17-19 inclusive) of the University Bicentennial, and will talk at tomorrow's Math. Club. We have given up hope of Professors Alexandroff and Kolmogoroff for this year, but are expecting next week Dr. Minakshisundaram and Mr. Chandrasekharan from India. Travel

Professor Shiing-Shen Chern - 2


December 3, 1946

arrangements have been difficult for several people.

I am ever so glad you find conditions at least more comfortable in China. Mrs. von Neumann, just back from a visit with her family in England, thinks we do not know what shortages, and hardships and patience are,- and I suppose we don't. Personally I am most happily, though less accessibly, situated in my new room at 12 Princeton Avenue. Alf Hunt, established in Tulsa, Oklahoma, where I understand Mrs. Boyd has bought a house for them, has been free there from asthma and hay fever.

Thank you once more, and with best wishes to you and your family for Christmas and the New Year,

Sincerely yours,



Professor Shiing-Shen Chern  
Institute of Mathematics  
Academia Sinica  
320 Yoyang Road  
Shanghai, China

國立中央研究院數學研究所  
INSTITUTE OF MATHEMATICS  
ACADEMIA SINICA  
320 YOYANG ROAD  
SHANGHAI, CHINA

November 5, 1946

*Recd. Dec. 2*

Dear Miss Blake:

I have received your letter of September 6 and I am glad to learn that you are having a busy term. I only regret that I am not in Princeton.

I understand that you have now four sets of reprints of mine: 1. On the curvatura integra in a Riemannian manifold, 2. Characteristic classes of Hermitian manifolds, 3. Some new viewpoints in differential geometry in the large, 4. On Riemannian manifolds of four dimensions. As there is no more to come in prospect, I think it is time to send them out. Meanwhile I don't think it to be fair for you to take all these trouble. I therefore suggest that you send the remaining reprints to me, so that I can take care of them myself. In order to avoid possible loss in mailing, would you mind to divide the packages so that the same set of reprints will be put into different packages.

Please also add <sup>Dr.</sup> E. Stiefel, Technische Hochschule, Zurich, Switzerland on my list.

I hope you have moved into a satisfactory place to live. I imagine Princeton must have changed a great deal since I left.

I shall be in Shanghai still for some time, as I have to take care of this newly founded institute. But our University in Peiping also needs me to teach there. The probable solution is that I shall spend the spring term there. Life in China is far from being normal, but conditions are much better than during the war years and I am glad that the war is over.

With best greetings,  
Sincerely

*S. S. Chern*

*1946-11-5*

*Weyl*

國立中央研究院數學研究所  
INSTITUTE OF MATHEMATICS  
ACADEMIA SINICA  
320 YOYANG ROAD  
SHANGHAI, CHINA

September 21, 1946.

Dear Miss Blake:

I have asked my friend Mr. Chiang to bring you a small souvenir. Mr. Chiang sailed from Shanghai on September 2 and will be working in NYU. I hope it will reach you in due time.

Recently two mathematicians wrote me, asking for my reprints. Please send for me to the following addresses:

- Dr. E. F. Trombly, 10382 Roxbury, Detroit 24, Michigan *University of Chicago*  
Dr. D. B. Scott, Queen Mary College, University of London, Mile End Road, London E.1., England.

My work here is pretty busy, but rather interesting. Greetings and thanks.

Sincerely yours

*S. S. Chern*  
(Shiing-shen Chern)

(5m) Chern

September 6, 1946

Dear Professor Chern:

At last Professor Ambrose has sent me your reprints which went to Ann Arbor. I have also your Annals reprints on "Characteristic classes of Hermitian manifolds" and your Bulletin reprints "On Riemannian manifolds of four dimensions". But as you say more are coming, unless I hear from you to the contrary I will let your reprints accumulate for a few months more before sending them out again.

The League of Nations have gone, mostly to the United Nations, and most newcomers and most of our own professors have not arrived yet. But we are having extremely busy times nevertheless. Professor and Mrs. Weyl are at the seashore for a few days. Professor and Mrs. Veblen are still in Maine. Professor von Neumann's new "Computing Laboratory" has risen to its first-floor

Over



window-frames,- down Olden Lane just beyond Wes's house. To our sorrow Professor Pauli has declined the professorship here and decided to stay in Zürich. We have no further word from Professors Alexandroff and Kolmogoroff, but are expecting a long list of mathematicians including Drs. Chandrasekharan, Borsuk, Dirac, Eckmann, Minakshisundaram, F.J.Murray, Max Newman of England, Ramanathan, and J.H.C.Whitehead.

The town is crowded — the Institute has bought permanent prefabricated housing for 38 families to be installed near the Institute — Mrs. Boyd's house has been bought by the University and we all have to move in about a week, I after 10 years. Alf Hunt is suffering considerably from asthma and hay fever, and on that account may be transferred to Oklahoma. The Hunts and all of us remember your stay here with pleasure. Please remember me most kindly to Professor Tuan if you see him.

Sincerely,

Please pardon the paper — you see I couldn't stop writing!

Prof. Zhang ✓  
" " ✓  
93

Tientsin, June 5, 1946.

Dear Miss Blake:

I have been in Tientsin and Peiping for two weeks. Peiping is still the most beautiful city in China. Our university is situated in a suburb in the northwest of the city. The buildings are damaged and all the equipments lost. Fortunately we have claimed back a part of our library. The university has started moving from Kunming to Peiping and a few of <sup>the faculty members</sup> ~~them~~ have arrived. The next term is expected to begin in the end of October.

As for my own plans I shall be going back to Shanghai in a few days, in order to take care of the Mathematical Institute of the Chinese National Academy. But I shall be in Peiping when the university begins. This arrangement is of course due to the shortage of scientists in China.

I thank you very much for the trouble you have taken in distributing my reprints and in other matters. I understand that the reprints of my long paper in the Bulletin were sent to Professor Ambrose, and I have asked him to send them back to you. The mail conditions between China and the States are now quite satisfactory. So please forward to me the reprints and other printed matters to the following more permanent address:

S. S. Chern, Institute of Mathematics, Academia Sinica, 320 Yoyang Road, Shanghai

Both Professor Liqun Chiang and Professor Hua will be in the States next academic year, and Professor Chiang has started.

I imagine that you will be (or are) in vacation soon. When will you spend this summer? Hoping that you will have a pleasant vacation,

Sincerely yours  
S. S. Chern

Reply dated July 3/46

for ...  
5000 ...

Rec'd  
93

(Sh) Chern

April 15, 1946

Dear Miss Hull:

Replying to your inquiry of April 11, Professor Shiing-Shen Chern's permanent address will be

Department of Mathematics  
Tsing Hua University  
Peiping, China

But as Tsing Hua University has not yet re-established itself in Peiping since its post-war return from the South, it would be safer until you have further information from Professor Chern to use the address of his father-in-law, from which mail would always be forwarded:

1 F Passage 190  
Route Destelan  
Shanghai, China

*Through*  
*1946*

Yours sincerely,

Miss Evelyn M. Hull  
American Mathematical Society  
531 West 116th Street  
New York 27, N.Y.

Secretary, School of Mathematics

Copy sent to Prof. Chern

AMERICAN MATHEMATICAL SOCIETY

531 WEST 116TH STREET, NEW YORK 27, N. Y.

April 11, 1946

Miss Gwen Blake  
Secretary, School of Mathematics  
Institute for Advanced Study  
Princeton, New Jersey

Dear Miss Blake:

At the request of Professor Chern, the reprints of his paper, which appeared in the January issue of the Bulletin, were sent to Warren Ambrose at the University of Michigan.

Will you give us Professor Chern's present address?

Sincerely,

*Evelyn M. Hull*

Evelyn M. Hull  
Office Manager

EMH:S

*GB will answer*

(3m) S.S. Chern

San Francisco, February 24<sup>th</sup>, 1946.

Dear Miss Blake:

I thank you for your letter and the troubles you have taken in forwarding my mail to me.

At last I got news that I could sail on February 27<sup>th</sup>, as you probably already know. I am glad that I am going to join my family soon, although on the other hand I like Princeton and its people even more after I have left. The reason is probably the kind of student life I had in Princeton, and student days are always full of memories.

I enclose herewith a list of mathematicians whom I would like to have my reprints sent. There are two sets of reprints at Dr. Tuan's place, and I wonder whether you would mind to send them for me, as you have done before even when I was in China. I hope you will keep the list, as more reprints will be coming, and I am afraid that I shall bother you later too. It goes without saying that people not on the list who care for their reprints can be given ~~and that~~ I presume that you know the addresses of these mathematicians. If some of the addresses you don't know, just discard them from the list.

I saw Dr. Erdős constantly here. He showed me a letter from Kakutani, and I am glad that he is safe. Incidentally, do you have any news of a Japanese mathematician by the name of Oka, who was teaching in Hiroshima? Oka is a very good mathematician.

Best thanks and regards,

S. S. Chern

Prof. Chem's Reprint List Feb. 24, 1946

1) Princeton: 20 copies to go to Chem  
 Alexander, Muse, von Neumann, Vahlen, Weyl, Einstein, Siegel, Mayer, Segal, Kelley,  
 Zippin, Ando, <sup>Beardman</sup> Hewitt, Fan, Montgomery, Strauss, <sup>Harvard</sup> Ivan, <sup>Harvard</sup> Tschu, <sup>Harvard</sup> Chung,  
 Lipschutz, Chevalley, <sup>Harvard</sup> Hochschild, Whitehead, Fox, Bohner, Tucker, Tukay,  
 Schatten

2) Michigan: <sup>Indiana Univ</sup>  
 Hildebrandt, Myers, Rainich, <sup>Sam</sup> Eilenberg, <sup>NE</sup> Steenrod, <sup>W</sup> Ambrose

3) Harvard: <sup>Harvard</sup>  
<sup>MS</sup> Stud, Whitney, Birkhoff, MacLam, Mackey, Cohen, <sup>L.H.</sup> Lomis, <sup>J.W.</sup> Widder, <sup>J.L.</sup> Walsh

4) MIT: <sup>D.J.</sup>  
 N. Wiener, <sup>W</sup> Strick, <sup>R</sup> Hurwicz, <sup>Salem</sup>

5) Columbia: <sup>W.C.</sup>  
 J.F. Ritt, Kasner, P. Smith, <sup>Harvard</sup> Stradt

6) Chicago: <sup>EP</sup> <sup>AA</sup>  
 M.R. Hestenes, Lane, Albert, <sup>Harvard</sup> Kaplansky, <sup>Harvard</sup> Kalisch

7) Yale: <sup>W</sup>  
 Hille, Bask

8) Indiana: <sup>Harvard</sup>  
<sup>Here</sup> Artin, <sup>Harvard</sup> Bohnenbust, <sup>Harvard</sup> Thomas

9) Los Angeles: <sup>Harvard</sup>  
<sup>Harvard</sup> May Ztra, Snapper (Univ. Southern Cal), <sup>Harvard</sup> Ullman, <sup>EF</sup> Beckenbach, <sup>JW</sup> Green

10) Pasadena: <sup>AD</sup> <sup>RP</sup>  
 Z.T. Bell, <sup>W</sup> Michel, <sup>W</sup> Ward, <sup>W</sup> Dilworth

11) Other Names:  
 A. Dresden, <sup>HA</sup> Rademacher, <sup>W</sup> Zariski, <sup>W</sup> H. Cartan, <sup>C</sup> E. Cartan, <sup>W</sup> Ehmman, <sup>W</sup> Andre Weil,  
<sup>W</sup> Henry Whitehead, <sup>W</sup> Hodge, <sup>W</sup> Newman, <sup>W</sup> Mordell, <sup>W</sup> Cech, <sup>W</sup> Prizajin, <sup>W</sup> Alexandroff,  
 Kolmogoroff, Radó, F. Levi (Calcutta, India), Kosambi (Bombay, India),  
 J. Dieudonné, <sup>W</sup> Cartan, <sup>W</sup> Shifman, <sup>W</sup> Friedrichs, <sup>W</sup> Stokes, <sup>W</sup> H. Hopf, <sup>W</sup> G. de Rham.  
 Hans Sussman, <sup>W</sup> Milgram  
<sup>W</sup> Ullman  
 See additions  
 Sep 21 to Nov 5/46

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chem, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(A23) S.S. Chern

February 7, 1946

Officer in Charge  
Immigration and Naturalization Service  
U. S. Department of Justice  
Miami, Florida

Dear Sir:

I wish to report to you the departure of Professor Shing-Shen Chern on December 18, 1945. Professor Chern has left for San Francisco where he expects to obtain passage to China. He is returning to his post of Professor of Mathematics at the National Tsing Hua University in China.

Yours very truly,

Jane S. Richardson  
Assistant Secretary

Copy to Miss Blake

(923) Chern

January 24, 1946

Dear Professor Struik:

Your note of yesterday is received and I am at once forwarding your note to Professor Chern by Air Mail in Pasadena, which he expected to reach on January 18 and leave at an unknown date to sail to China. He was going first to

1 F Passage 190  
Route Destelan  
Shanghai, China

which he hoped to reach sometime in February. But his permanent address is

Mathematics Department  
Tsing Hua University  
Peiping, China

Sincerely yours,

Professor Dirk J. Struik  
Mathematics Department  
M.I.T.  
Cambridge 39, Mass.

Secretary, School of Mathematics

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA



JOURNAL OF MATHEMATICS AND PHYSICS

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE 39. MASS.

Jan 23, '46

Dear Miss Blake -

Will you kindly forward this letter to B. Chen,  
if he is still at Princeton or in the USA. If he has  
left the country, will you give me his present address?

Thanks in advance

Sincerely

Dirk J. Struik

Prof. Alexander ✓  
Einstein ✓  
Morse ✓  
Siegel ✓  
Vahlen ✓  
Weyl ✓  
L. J. Mordell ✓  
G. D. Birkhoff ✓

(5m) S. S. Chern

S. S. Chern

1F Passage 190

Route Detelan

Shanghai, China

April 2, 1946.

Dear Professors Vahlen and Weyl:

It has been ten days since I arrived at Shanghai. The trip was a very pleasant one, and I am glad to say that I found my family healthy and well, in spite of the difficult years. The difficulties are however not yet over, prices being so high that one has to spend a lot of time in worrying about family economy. But I believe that we can muddle through.

What I told you about the Institute of Mathematics of the Academia Sinica seems to materialize. To suit the present conditions its temporary site will be in Shanghai. I was asked to take care of the Institute for a certain period and shall probably be in Shanghai till the end of this year, if not longer. As in industry the most important thing at the present moment is to put everybody into work. If possible, I am also thinking of inviting some foreign mathematicians to China to stir up the situation, the difficulty being the present living conditions. Any advice from you will be deeply appreciated.

Dr. Li-Fu Chiang, who was invited to be a member of the Institute for the academic year 1945-46, is not in Shanghai. Because of his personal duties after the Japanese surrender and in part also of his illness he was unable to go to Princeton. He is planning to go to the United States next summer. As an

invitation from the Institute will be necessary for him to get the visa, I hope that the invitation could be granted to him for the academic year 1946-47 and that a letter to this effect would be sent to him when granted. In the United States he will be supported by the Chinese Government, so that no financial help is necessary. Mr. Chiang is one of the most senior mathematicians in China and has done a great deal in promoting mathematical education in China.

Hua is now in Soviet Russia and will visit the U. S. when he comes back.

I believe I have passed the period of recuperation and hope to resume work very soon.

Please send my regards to Professors Alexander, Mazur, von Neumann, Einstein, Mayer, Pauli, Siegel.

Sincerely yours

S. S. Chern

(925) Chern

March 17, 1945

Professor Shing-shen Chern  
47 Wiggins Street  
Princeton, New Jersey

Dear Professor Chern:

On the recommendation of the Faculty of the School of Mathematics, it gives me great pleasure to offer you a renewal of your stipend of \$1,800 for the academic year 1945-1946. It is a great pleasure to have you here at the Institute and I hear with satisfaction fine reports of the work you are doing. I think it is a very intelligent policy on the part of the Chinese Government to offer scholars such opportunities as you have, during the war period so as to be prepared to revive the universities in the most effective way as soon as the war is over.

With warmest good wishes, I am

Yours sincerely,

Frank Aydelotte

FA:jsr

Copies to ✓ Professor Weyl  
Miss Miller

(903) Chern

C O P Y

U. S. DEPARTMENT OF JUSTICE  
IMMIGRATION AND NATURALIZATION SERVICE  
MIAMI, FLORIDA

In replying please  
refer to this file  
number:  
0606-1197

August 29, 1944

Miss Marie C. Eichelser  
Assistant Secretary  
The Institute for Advanced Study  
Princeton, New Jersey

Dear Madam:

In re: Prof.. SHIING-SHEN CHERN

We thank you for your courteous letter of June 30th.

Our records show that Prof. Shiing-Shen Chern was admitted as a Government Official, under Section 3(1) of the Immigration Act of 1924 and is at liberty to remain while he maintains that status.

For the closing of our files, it would be appreciated if he would inform this office as to date, place and means of departure shortly before he does finally leave the United States.

Yours truly  
Francis J. H. Dever  
Officer in Charge

by  
H. B. Read  
Chief, Nationality and  
Status Section

Copy to Miss Blake  
Miss Miller

(gas) Chern

April 4, 1944

Dear Professor Chern:

On the recommendation of the School of Mathematics it gives me great pleasure to invite you to continue your membership in the Institute for Advanced Study for the academic year 1944-45, with a stipend of \$1500.

Looking forward to having you here for another year, I am

Yours sincerely,

FRANK AYDELOTTE, Director

Professor Shiing-shen Chern  
Institute for Advanced Study  
Princeton, New Jersey

FA/MCE

(705) Chern

April 1, 1944

Dear Doctor Aydelotte:

The School of Mathematics wishes to recommend the reappointment of Professor Shiing-shen Chern for the next academic year, 1944-45, with the same stipend as before, \$1500.

Very sincerely yours,

Hermann Weyl

Dr. Frank Aydelotte  
HW:GB

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

Date August 18, 1943

Full name Shiing-shen Chern

College or university, degrees, year conferred

Nankai University, B.S., 1930.Tsing Hua University, Peiping, China, M.S. 1934.Hamburg University, Germany, D.Sc., 1936.

Date and place of birth

October 26, 1911, Kashing, Chekiang Province, China

Citizenship

Chinese

Married or single

MarriedIf foreign, under what kind of visa did you enter the United States? OfficialVisa No. 332

Place and date of issue

Chungking, May 25, 1943.

When does it expire?

August 10, 1944unlimited while Chinese official

When and where did you enter the United States?

August 11, 1943, Miami

Entry permit expires when?

August 10, 1944unlimited while Chinese officialPrinceton home address 47 Wiggins Street

Telephone

555 M

Permanent address

Tsing Hua University, Kunming, China

Person to notify in case of emergency, with address

outside of PrincetonChih Meng, 119 West 57th Street, New York City, N.Y.

Honors and societies

Ting Wen-kiang Prize, Academia Sinica, 1943.Member, German Mathematical Society

Positions you have held, giving dates, or are holding (including any graduate scholarships and fellowships)

Professor of Mathematics, National Tsing Hua University, 1937—Research Fellow, National Tsing Hua University, 1934—36.Research Fellow, China Foundation, 1936—37.

Publications (Please give title and reference in full, with Vol. No., year, and page numbers.)

1. Abzählungen für Gewebe, Abhandlungen der Hamburgischen Universität, Bd. 11 (1935), 163—170.
2. Eine Invariantentheorie der Dreigewebe aus  $r$ -dimensionalen Mannigfaltigkeiten im  $R_{2r}$ , *ibid*, Bd. 11 (1936), 333—358.
3. Sur la géométrie d'une équation différentielle du troisième ordre, Comptes Rendus Acad. Paris, Tome 204 (1937), 1227—1229.
4. Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif, Bull. Sc. Math., 2 série, tome 61 (1937), 234—243

(Use other side of sheet, if necessary.) (See other side)



5. On projective normal coordinates, *Annals of Mathematics*, vol. 39 (1938), 165-171.
6. The geometry of higher path-spaces, *Jour. Chin. Math. Soc.*, vol. 2 (1940), 247-276.
7. On integral geometry in Klein spaces, *Annals of Mathematics*, vol. 43 (1942), 178-189.
8. The geometry of isotropic surfaces, *Annals of Mathematics*, vol. 43 (1942), 545-559.
9. On a Weyl geometry defined from an  $(n-1)$ -parameter family of hypersurfaces in a space of  $n$  dimensions, *Science Record*, vol. 1 (1941), 5-11.
10. On the invariants of contact of curves in a projective space of  $n$  dimensions and their geometrical interpretation, *Science Record*, vol. 1 (1941), 13-17.
11. On the euclidean connections in a Finsler space, *Proc. Nat. Acad. Sci. U.S.A.*, vol. 20 (1943), 33-37.
12. A generalization of the projective geometry of linear spaces, *Proc. Nat. Acad. Sci. U.S.A.*, vol. 20 (1943), 38-43.

O. v. has seen this. ~~How~~

(709) Chern

**CHINESE EMBASSY**

WASHINGTON, D. C.

August 2, 1943

~~Wang~~  
Mr. Frank Aydelotte, Director  
The Institute for Advanced Study  
Princeton, New Jersey

Dear Mr. Aydelotte:

I am directed by the Ambassador to acknowledge the receipt of your letter of July 19, concerning the air passage for Professor S. S. Chern.

Upon the request of the Embassy, the State Department has already wired, through the appropriate authorities, to the Air Transport commanding officer at Karachi, India, its recommendation for an early priority of air passage for Professor Chern.

Yours very truly,



K. S. Wang  
Third Secretary

KSW/da

(923) Chern

Copy for Miss Blake

July 19, 1943

His Excellency The Chinese Ambassador  
Chinese Embassy  
Washington, D. C.

Dear Mr. Ambassador:

As you doubtless know, Professor Shiing-shen Chern has been selected by the Chinese government to spend a year in mathematical research in the United States and has been elected a member of the School of Mathematics of the Institute for Advanced Study.

Professor Chern wrote to Professor Oswald Veblen of this Institute to say that he hopes to make the trip from China to the United States by air, that commercial air lines are available for most of the journey, but that from Cairo across Africa the only air transportation available is by United States Army transport planes. Professor Chern has asked us to do anything in our power to assist him in getting permission to travel by one of these planes.

I have taken this matter up with the War Department and have been informed that a request for such a priority should be made by Chinese officials through the usual channels. I should be grateful on Professor Chern's account if you would be so kind as to take this matter up with the proper authorities with a view to obtaining for Professor Chern the permission he is seeking.

Professor Chern's address, should you wish to communicate with him, is P. O. Box 96, Kunming, China.

Believe me,

Yours very truly,

FA/MCE

FRANK AYDELOTTE, Director

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

July 13, 1943

Dear Harvey:

I am writing to ask whether you could do anything to assist Professor Shiing-shen Chern of Kunming, China, perhaps the most eminent living Chinese mathematician, to obtain passage by Army transport plane from Cairo to some point on the east coast of Africa or the west coast of Brazil, where he could continue his trip to the United States by commercial air line.

Professor Chern has been given an appointment by the Chinese government to spend a year in advanced research in mathematics at the Institute for Advanced Study and we have awarded him a stipend to assist in paying his expenses. He plans to make the journey through China to the United States by air and can do most of it by commercial air lines. I understand, however, that from Cairo across to the other side of Africa or possibly to Brazil there are no commercial air lines operating and that the only way of getting across is by Army transport plane.

Professor Chern asked me for a letter to the military authorities in Cairo, vouching for him and explaining his purpose. I have sent him such a letter by air mail to China, but I am afraid this may not be sufficient and I have been wondering whether it would be possible for you to say a word in the right quarter so as to facilitate Professor Chern's passage. I should appreciate deeply anything that you feel able to do, and I am sure that any assistance given to Professor Chern would be a very friendly act to Chinese scholarship.

With kindest regards, I am

Yours sincerely,

Frank Aydelotte

Harvey Bundy, Esq.  
Office of the Secretary of War  
Washington, D. C.

(923) Chern

July 8, 1943

Dear Professor Chern:

Professor Veblen has suggested that it might be better if I furnished you with two certificates of your appointment to the Institute and a letter to the military authorities at Cairo for permission to travel on an Army transport plane. I accordingly enclose these two documents, each in duplicate, and have notarized the one concerning your appointment in case it might be more useful for you to have it in that form.

I enclose also a copy of the supplement to our Bulletin No.10 of the Institute, which you may find it convenient to have with you in order to show the military authorities and others the list of names of the officers, trustees, and members of the faculty of the Institute for the purpose of indicating to them the nature and importance of the institution. I hope this material will answer your purposes.

Looking forward with keenest pleasure to having you with us next year, I am

Yours sincerely,

FRANK AYDELOTTE, Director

Professor S. S. Chern  
P. O. Box 96  
Kumming, China

July 8, 1943

Commanding Officer  
U. S. Military Transport Service  
Cairo, Egypt

Dear Sir:

The bearer of this letter, Professor Shiing-shen Chern of Kunming, China, is an eminent Chinese mathematician, who has been designated by his government to pursue advanced research for a year in the United States. Professor Chern has been elected a member of the Institute for Advanced Study for the academic year 1943-1944 and has been awarded a research stipend. He plans to come from China to the United States by air.

I am informed that for part of the journey across Africa there is no commercial air line available and that this part of the journey must be made, if at all, in an Army transport plane. Because of Professor Chern's eminence in his field and the importance of the work which he is planning to undertake here, I venture to request from you an authorization for Professor Chern to be allowed to travel on an Army transport plane over this portion of his journey.

I should greatly appreciate any assistance you may be able to render to Professor Chern.

Yours very sincerely,

FA/MCE

FRANK AYDELOTTE, Director

July 8, 1943

TO WHOM IT MAY CONCERN:

This is to testify that by formal vote of the School of Mathematics of the Institute for Advanced Study, Professor Shing-shen Chern of Kunming, China, has been elected to membership in the Institute for Advanced Study for the academic year 1943-1944 and has been awarded a research stipend of \$1500.

FRANK AYDELOTTE, Director

Subscribed and sworn to before me  
this 8th day of July, 1943.

Notary Public, State of New Jersey

Ballistic Research Laboratory,  
Aberdeen Proving Ground, Md.,  
7 July 1943.

Dr. Frank Aydelotte,  
Institute for Advanced Study,  
Princeton, New Jersey.

Dear Frank:

*Jan 27/43*

I inclose herewith a letter from Chern which I failed to hand you last Monday. I believe it would be more effective if you would give him the letters which he asks for. I also suppose it would be desirable to inquire from sufficiently high authority about the possibility of using the Army Transport Plane. May I speak with you about this matter next Monday?

Sincerely yours,

*Oswald Veblen*

O. Veblen.

1 Incl.



(925) Chern

April 30, 1943

Board of Economic Warfare  
Technical Data License Division  
252 Seventh Avenue  
New York, New York

Gentlemen:

Please examine the attached technical data for exportation.

LICENSE NO.: None

CONSIGNEE: Professor Shing-shan Chern  
P.O. Box 96  
Kunming, China

ULTIMATE CONSIGNEE: Same as above

NATIONALITY OF CONSIGNEE: Chinese

BUSINESS OF CONSIGNEE: University professor, National Tsing Hua University,  
now in Kunming, China

BUSINESS RELATIONSHIP WITH CONSIGNEE: None. Fellow research-workers in  
mathematics.

NUMBER OF PACKAGES: 1

SUBJECT: 2 mathematical reprints by Professor Shing-shan Chern:  
"On the Euclidean connections in a Finsler space" (pp.33-37)  
"A generalization of the projective geometry of linear  
spaces (pp.38-43)  
both published in the Proceedings of the National Academy of  
Sciences, Vol. 29 (1943) at the pages noted

NEED: Author needs his own papers for further reference

SECURITY: The data has not been declared secret, confidential or restricted  
by any officer or agency of the United States.

Very truly yours,

HW:GB

(Professor Herman Weyl)

Records of the School of Mathematics : Members : Box 5 : Chern, S.S.  
Records of the School of Mathematics : Members : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

May 3, 1943

Dear Professor Chern:

I received your letter of March 4 a few days ago, and have talked over the question of your stipend more fully with Dr. Aydelotte, who has also been in communication with Dr. Hu Shih. These gentlemen are convinced that the supplement of this Institute to your Chinese stipend is hardly sufficient for the purpose intended. I am therefore happy to state that I can now officially offer you a stipend of \$1500 from our Institute. This we feel confident will be sufficient for your expenses while you are here, but it will be necessary for you to obtain from other sources the funds for coming and for your return to China.

With best greetings,

Yours sincerely,

Oswald Veblen

Professor S. S. Chern  
P.O. Box 96  
Kunming, China  
OV:GB

*Copy to Dr. Aydelotte*

April 30, 1943

Dear Professor Chern:

I have been glad to distribute the reprints of your two papers in the Proceedings of the National Academy of Sciences as requested by your letter of March 4 (with one exception) and enclose herewith a copy of each for yourself. The exception was that I could find no "Professor E. Kneser" at Columbia University, and instead sent reprints to Professor Edward Kasner of the Mathematics Department at Columbia, whom you may perhaps have meant.

Sincerely yours,

Secretary, School of Mathematics

Professor Shiing-shen Chern  
P.O. Box 96  
Kunming, China

Shiang-shien Chern  
P.O. Box 96  
Kunming, China  
April 27, 1943.

Dear Professor Veblen,

I have informed you about my decision of going to America in this summer and of accepting the stipend of your Institute for the academic year 1943-1944. I am applying for my passport and, before getting it, I have to pass a special training and service course. In order to save time of my trip, I am thinking of going to your country by air. This idea has the additional advantage that the schedules of the airways are more regular, the difference of the travelling expenses being small. A great part of the airway from China to America is run by commercial companies, about which I can arrange with them myself. But I was informed <sup>(by the U.S. Embassy in China)</sup> that the airway from Cairo (Egypt) to Monrovia or Fisherman Lake is now controlled by the U.S. Army and that that distance has to be travelled by the Army transport planes. Will you be kind enough to arrange for me a letter of recommendation or some kind of paper which I can present to the U.S. military authorities in Cairo in order to be awarded the privilege of taking the plane?

To make my statement short I beg you to send me two letters

1. A formal letter testifying that I am going to work in the Institute for Advanced Study and that I have been awarded a stipend.
2. A letter to the U.S. military authorities asking them to give me the privilege of taking the Army transport planes.

I am very sorry to bother you, in a time that you are so busily engaged in the Army, with such trivial requests. I thank you heartily in advance.

I am  
Yours sincerely  
S. S. Chern

(795) Chern

April 12, 1943

Dear Professor Chern:

Two or three weeks ago I received from you a letter which I appear to have lost. But the gist of it was that you have been chosen by your University as one of a number of professors who will receive stipends with which to come to the United States. This would on the face of it appear to make it unnecessary to seek a stipend for you from our Institute as I suggested in my letter of February 1. It is, however, clear from your letter, and I think obvious from general considerations, that it would be helpful if you could receive from our Institute an increment to your stipend.

Therefore I am happy to say that I can now officially offer you a stipend of \$1000 from this Institute for the academic year 1943-44, for the purpose of supplementing what you receive from Chinese sources. I hope that this will help to make your stay in this country more comfortable in a material way, and more profitable scientifically.

I look forward with great pleasure to welcoming you here as a colleague.

Yours sincerely,

Professor S. S. Chern  
P.O. Box 96  
Kunming, China  
OV:GB

Oswald Veblen

*Copy to D. Appleton*

Records of the School of Mathematics : Members, Visitors, Assistants : Box 5 : Chern, S.S.  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(929) Chen

S. S. Chen  
P. O. Box 96  
Kunming, China  
March 4, 1943.

Dear Professor Veblen,

I thank you very much for your kindness in arranging for me the stipend for the academic year 1943-1944.

To provide my travelling expenses to America, I am asking ~~our~~ Ministry of Education and our University for a grant. But even if this should fail, I can still depend on the subsidy of our University to cover these expenses, which will not be high if I go by boat. I am planning to leave China in July, or August this year and hope to arrive at Princeton before the first term begins.

Your State Department has invited some Chinese professors to come to the United States. Our University has chosen as its representative Dr. Y. L. Ching, professor of philosophy and a senior member of our faculty. Although most of the professors in Chinese universities have studied in the United States, much can still be done in promoting cultural relations between the two countries, particularly, as you have said, in helping Chinese scientists to do serious research works, either in the United States or in China. We hope that someday we could welcome prominent American scientists to lecture in China.

With best wishes,

Yours sincerely,

S. S. Chen

To be answered

Shiing-shen Chern  
P. O. Box 96  
Kunming, China  
March 27, 1943.

Dear Professor Veblen:

I have sent you, in September 1942, a paper by my co-worker, Mr. Hsien-chung Wang, entitled: "On a projective invariant of a non-holonomic surface". In your letters to me you have not mentioned this paper. I think that it might probably be lost, so I send you herewith a second copy.

This paper is his thesis for the Master's degree. If you think fit, please help to get it published in an American mathematical journal. Otherwise the faculty of our University will be much pleased, if you or some other mathematician would like to give some comments on this paper.

I am planning my journey and I shall try my best to arrive Princeton in September. The allowance of our University will be enough to cover my travelling expenses to America by ship. The uncertainty lies in the expenses of my return trip. I should like to know the answer to your inquiry from the Division of Cultural Relations of your State Department, because I think that it is better to make an arrangement for the return trip before I start my journey.

With best thanks and wishes,

Sincerely yours

*S. S. Chern*  
(Shiing-shen Chern)

Vanderlicia corresp. filed in hall under  
= Vanderlicia =

Shiing-shen Chern

P. O. Box 96

Kunming, China

March 4, 1943.

Dear Miss Blake,

I have been informed by Professor Veblen that two mathematical papers of mine with the titles:

1. On the Euclidean Connections in a Finsler Space;
2. A Generalization of the Projective Geometry of

Linear Spaces;

will appear in the Proceedings of the National Academy of Sciences. I was advised to send you a list of persons to whom I should like to have my reprints sent. Such a list I enclose herewith. Will you be kind enough to send my reprints to the persons named, and in addition to send two copies to myself? 70¢ x 6 = \$4.20 1 copy of each copy. maybe 1 copy

I have asked my friend Mr. Hsiao-fu Tuan (a graduate student in mathematics residing now at Princeton University) to take care of these reprints. It will certainly be good if you should do me this favour. As for the remaining copies, please keep them in your Institute, as I am coming to join the Institute in the next academic year.

With my best thanks in advance,

Yours sincerely

*S. S. Chern*

(Shiing-shen Chern)



- ✓ Professor J.W.Alexander  
" ✓ E. Artin, University of ~~Notre-Dame~~  
" ✓ G. A. Bliss, University of Chicago  
" ✓ C.Chevalley  
" ✓ John de Cicco, ~~Columbia University~~  
" ✓ J. Douglas, ~~Columbia University~~  
" A. Einstein  
" L.P. Eisenhart  
" ✓ G. Fubini
- ✓ Doctor Yung-chow Huang, M.I.T.  
" Y. K. Huang, University of Chicago
- Professor ~~van Kampen~~, Johns Hopkins University  
" ✓ Knebelmann  
" E. Kneser, Columbia University *Edward Kneser*  
" ✓ W. Mayer  
" ✓ K.Menger, University of Notre-Dame  
" ✓ M. Morse  
" ✓ J. von Neumann  
" ✓ D.J. Struik, M.I.T.  
" ✓ H.P.Robertson  
" ✓ T.Y.Thomas, California University  
" ✓ Tucker
- ✓ Mr. Hsiao-fu Tuan  
Professor O.Veblen  
" ✓ H.Weyl  
" ✓ N. Wiener, M.I.T.  
" ✓ S. Lefschetz  
" ✓ G. D. Birkhoff, *Harvard University*  
" ✓ J. L. Coolidge, *Harvard University*  
" ✓ D.D. Kosambi, *Fergusson College, India, Poona.*

✓ Prof. Chern  
✓ Siegel

(7923) Chern

February 9, 1943

Dear Mr. Peck:

Re RC

Many thanks for your letter of February 5. Since the academic year of our Institute begins about September 20, it would probably be best to arrange to have Dr. Chern come sometime in the month of September.

Yours sincerely,

Oswald Veblen

Mr. Willys R. Peck, Special Assistant  
Division of Cultural Relations  
Department of State  
Washington, D.C.  
OV:GB



DEPARTMENT OF STATE  
WASHINGTON

In reply refer to  
RC

February 5. 1943

My dear Mr. Veblen:

I have received your letter of February 1, 1943, confirming the conversation we held on January 26, 1943, at which time you recommended that the American Government provide transportation for Professor S. S. Chern to come to the United States and subsequently to return to China, in order that he may be able to spend a year in the United States in advanced study. It is noted from your letter that the Institute for Advanced Study would be inclined to award Professor Chern a stipend during his year of sojourn.

Your letter does not so state, but my recollection is that you thought Professor Chern would not desire to come to this country until the latter half of the year. If that is so, the fiscal regulations would require that the proposal be carried out, if adopted, with funds made available after June 30, 1943.

In the meantime I will investigate the possibility of favorable action on the suggestion you have made.

Sincerely yours,

Willys R. Peck  
Special Assistant  
in the

Division of Cultural Relations

Mr. Oswald Veblen,  
The Institute for Advanced Study,  
School of Mathematics,  
Princeton, New Jersey.



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From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

Please return  
T. G. B. B. B.

(945) Chern

February 1, 1943

Dear Mr. Peck:

May I recall our conversation of last Tuesday about

Professor S. S. Chern  
P.O. Box 96  
Kunming, China

You will perhaps remember that I stated that Professor Chern is a mathematician of very high rank, not only in his own country where mathematics is relatively undeveloped, but also according to the standards of Europe and America. Professor Chern has studied with the leading authorities in his own field (differential geometry) in France and in Germany. Since his return to China he has been doing original work of a very high order. He would now like to make better scientific contacts with his colleagues in the United States.

I am very strongly of the opinion that for a country in the position in which China now finds itself, it is of the first importance to develop a few citizens who are recognized in all parts of the world as leaders in their respective fields of scholarship. Dr. Chern is one of the few, and perhaps the only, Chinese whom I know who has a chance of playing this role in mathematics. On the other hand, I think it will be very much to the credit of the United States if we can contribute substantially to the development of such a man.

I have discussed the question with my colleagues and with the Director of the Institute, Dr. Aydelotte. If it were possible for your organization to bring Dr. Chern to this country and to provide for his return to China, we should be glad to give him the leading place on our list of candidates for stipends in mathematics. This would insure him an income of \$1500 for the year that he would spend with us.

If you wish to pursue this matter further, I shall be very glad to come to see you at some time when I am in Washington, or to write you again.

Yours sincerely,

Oswald Veblen

Mr. Willys R. Peck, Special Assistant  
Division of Cultural Relations  
Department of State  
Washington, D.C.  
OV:GB

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From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

(985) Chern

February 1, 1943

Dear Professor Chern:

I shall be very glad to continue to assist in finding suitable places for the publication of your papers from time to time. I only regret that I do not have time to read them myself.

I have talked with my colleagues, and we should be much pleased if you could spend a year at our Institute. I think there is no doubt that we could obtain for you a stipend of \$1,500. for the academic year 1943-1944. The difficulty, however, would be in meeting the expenses of your journey to America and back. I am making inquiries from the Division of Cultural Relations of our State Department. Perhaps you could arrange it somehow as an enterprise of the Chinese Government in promoting cultural relations with the United States. I am making the argument that it is very important for China to develop leaders in the various fields of scholarship, and for the United States to cooperate in this process.

With best wishes,

Yours sincerely,

Oswald Veblen

Professor S. S. Chern  
P.O. Box 96  
Kunming, China  
OV:GB

P.S. Regarding the reprints of your note in the Proceedings of our National Academy of Sciences, should you not like to send to the Secretary of our School of Mathematics (Miss Blake), a list of addresses to which she would send your reprints directly from here, sending also additional copies to you in China? As we understand that mail goes to China only by air, and there may be other difficulties in connection with sending printed matter at present, we are asking the editor of the Proceedings to send your reprints either to you, or to our Institute, according to his best judgment.

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From: The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

*copy of above*  
*Spencer*

S. S. Chern (929) Chern  
P. O. Box 96, Kunming, China.  
December 12, 1942.

Dear Professor Vahlen:

Your letter of October 30, 1942 was received two days ago. I thank you very much for the care you have taken in getting my papers published.

I appreciate very much the criticisms of Professor Vanduistie about my papers. It was a habit during my sejour in Paris in considering the connection in the space defined, when the problem of equivalence is solved. Although there is no essential difficulty in further discussions, much has still to be done in the actual exhibition of the connection. This is illustrated particularly clearly in my second paper on the euclidean connections in a Finsler space. I shall take account of these criticisms in my further publications.

I think I shall send to your country further papers for publication. Since you are busily engaged in wartime works, I do not know whether communications with you will cause too much disturbance in your work. If so, I hope you can introduce me to some other person to whom I can send my papers.

I am still very much interested in the present conditions in Princeton. After the Burma Road was cut half a year ago, we are practically out of contact with the outside world, and are in a sense compelled to devote our time to <sup>do</sup> research works in pure science. This is why I can still write papers on pure mathematics. If a stipend could be granted to me by the Institute for Advanced Study, I might still be able to make a trip to America, which, as I think, will be beneficial both to myself and to my fellow scientists in China. I hope you can give me some advice concerning this matter.

If you meet Professor Vanduistie, please send to him my warm greetings. With best wishes,

Yours sincerely,  
S. S. Chern

Many thanks  
for letting me  
see in this.  
S.S. Chern

(24) S S Chern

THE INSTITUTE FOR ADVANCED STUDY  
Princeton, New Jersey

October 30, 1942

Dr. S. S. Chern  
Tsing Hua University  
Kunming, China

Dear Dr. Chern:

Your two notes reached me a couple of weeks ago. Since I am very busily engaged on work for the Army I was not able to study them myself and therefore asked my old friend Professor J. L. Vanderslice to read them for me. He is very much pleased with them and I have sent them on to the Editor of the Proceedings of the National Academy of Sciences.

For your benefit I am sending you a bit of criticism which Dr. Vanderslice wrote about your papers. I agree with him that it was better not to delay publishing in order to let you take account of his critique. I thought, however, it would be interesting for you to know what he said so that you could take account of it in some future publication in case you wish to do so.

"The gist of the only criticism I can make is this. Chern takes for granted the reader's complete familiarity with Cartan's 1908 method and plunges into complicated manipulations of Pfaffians. Having succeeded in bringing his problem into a form to which Cartan's equivalence theorem applies he is satisfied to close abruptly with the briefest reference to the significance of his results, geometric or otherwise. Thus one feels a lack of motivation at beginning and end.

In paper 1, page 1 could be expanded rendering the selection of the basic equations 3 (p. 2) less arbitrary. At least a reference could be given to cover the point. Analogous basic Pfaffians have been used by Cartan and Chern before (i.e. "Geometry of Higher Path Spaces" published in an oriental journal) but in nothing I have seen is the discussion as complete as one unfamiliar with the origins of the method requires. Pages 2-5 devoted to involved manipulations condensed from his book manuscript can hardly be more concise. Then, at the end (pages 5,6) an expansion of the geometrical remarks is desirable, in particular, actual exhibition of the projective connection there referred to.

The same criticism as above applies with somewhat less force to the introduction of the simpler paper 2. Otherwise it seems all right; certainly concise enough, and a more elaborate conclusion is not necessary in a geometry so well known.

In briefest form my conclusion is this. The papers are good but need a clear cut explanation of the method and conclusions or a reference to a widely available source. If Chern wants quick publication this lack is not immediately essential and could be filled in one of his future contributions to the subject."

I do not know whether you ever see any of my other friends in China. If you do I hope that you will give them my warm greetings.

Yours sincerely,

Oswald Veblen

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From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA



703  
(204) Chern

Kunming, August 15, 1942.

Dear Professor Veblen,

I have received your letter in August of 1941 and I thank you very much for your kind interest in my application of a stipend in the Institute for Advanced Study. Since the outbreak of the Pacific War, my hopes of working in your Institute seem difficult to be realized. However, I still wish to know the decision of the Institute concerning my application.

I wish to inform you of my recent research work, in which you might be interested. In the last academic year I have concentrated my effort on a study of the different geometric objects by a constant use of Cartan's method of equivalence. With the application of this method the theories of Riemann spaces, Finsler Spaces, Cartan spaces, the geometry of paths, etc. all come under a unified treatment and the analytic manipulations are in many respects simpler. Besides I have obtained some new geometries, of which those of the isotropic hypersurfaces (the case for a space of three dimensions has been communicated to you) and the r-spreads are the most important ones. I am endeavoring to write a book and I hope to report to you its progress later on.

I enclose herewith two short notes containing the results which I obtained during the last year. As at present it is very difficult to publish anything in our country, I beg you to send them to a journal of your country. In order to secure prompt publication I have written the papers in a form which is suitable to appear in the Proceedings of the National Academy of Sciences. I enclose you also a manuscript entitled "The projective geometry of r-spreads", which contains the detailed calculations of one of the papers, so that you can check my results.

Any advice from you will be heartily appreciated.  
Thanking you in advance,

I am

Yours Sincerely

S. S. Chern (Shiing-shen Chern)

My address: Dr. Shiing-shen Chern, P.O. Box 96, Kunming, China.

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## Chapter XII

### The Projective Geometry of R-spreads

#### §1. Family of r-spreads and its Pfaffian system

A natural generalization of the results of the last two chapters is that we have in a space of  $n$  dimensions  $(x^1, \dots, x^n)$  a family of  $r$ -spreads (i. e.,  $r$ -dimensional varieties) depending on  $N=(r+1)(n-r)$  parameters. For  $r=1$  and  $r=n-1$  we get the geometries already treated. Another important particular case is given by the family of  $r$ -dimensional linear spaces in a projective space of  $n$  dimensions. These linear spaces depend exactly on  $N$  parameters and it is a well-known fact that the group of one-to-one transformations which carry a linear space to a linear space is the group of projective transformations. There is therefore reason to expect that the geometry in our space is a generalized projective geometry. In the following pages we shall show that this is the case.

Suppose the family of  $r$ -spreads in question be given. We assume the  $r$ -spreads to be such that through a point and tangent to an  $r$ -dimensional element through the point there passes one and only one spread of the family. To represent the family analytically let us write the equation of the  $r$ -dimensional element as

$$(1) \quad dx^i - h_{\alpha}^i dx^{\alpha} = 0.$$

Throughout this chapter we shall assume the range of every small Greek index to be from 1 to  $r$  and that of every small Latin index to be from  $r+1$  to  $n$ . Along a spread of the family we then have

$$dh_{\alpha}^i - r_{\alpha\beta}^i dx^{\beta} = 0,$$

where  $r_{\alpha\beta}^i$  are functions of  $x^{\alpha}, x^i, x^{\beta}$ . It follows that our spreads are integral varieties of the Pfaffian system

$$(2) \quad \begin{cases} dx^i - h_{\alpha}^i dx^{\alpha} = 0, \\ dh_{\alpha}^i - r_{\alpha\beta}^i dx^{\beta} = 0, \end{cases}$$

which, under our assumption, is completely integrable.

---

1) See, for instance, Schreier-Sperner, Einführung in die Analytische Geometrie und Algebra, II, §14, Berlin 1935.

Conversely, when such a Pfaffian system is given, its general integral can be expressed in the form

$$(3) \quad \begin{cases} F^i(x^\beta, x^j, p_\beta^j) = C^i \\ F_\alpha^i(x^\beta, x^j, p_\beta^j) = C_\alpha^i \end{cases}$$

where  $F^i, F_\alpha^i$  are  $N$  independent first integrals. Since the first integrals are independent with respect to  $p_\beta^j$ , the variables  $p_\beta^j$  can be eliminated and the result is

$$(4) \quad \Phi^i(x^\beta, x^j, C^j, C_\beta^j) = 0.$$

These equations define an  $N$ -parameter family of  $r$ -spreads. The study of the geometry of the family of  $r$ -spreads can therefore be based on the Pfaffian system (2).

To express the conditions for the complete integrability of the system (2) we introduce the notation

$$(5) \quad \frac{dF}{dx^\alpha} = \frac{\partial F}{\partial x^\alpha} + \frac{\partial F}{\partial x^i} p_\alpha^i + \frac{\partial F}{\partial p_\beta^j} r_{\beta\alpha}^i,$$

where  $F$  is any function of  $x^\alpha, x^i, p_\alpha^i$ . Then we can write

$$(6) \quad dF = \frac{dF}{dx^\alpha} dx^\alpha + \frac{\partial F}{\partial x^i} (dx^i - p_\alpha^i dx^\alpha) + \frac{\partial F}{\partial p_\beta^j} (dp_\beta^j - r_{\beta\alpha}^i dx^\alpha).$$

By Frobenius's theorem a necessary and sufficient condition for the complete integrability of (2) is given by

$$[dp_\alpha^i dx^\alpha] \equiv 0, \quad [dr_{\alpha\beta}^i dx^\beta] \equiv 0, \quad \text{mod. } dx^i - p_\alpha^i dx^\alpha, \quad dp_\alpha^i - r_{\alpha\beta}^i dx^\beta.$$

By means of (6) it is easily found that these conditions are equivalent to

$$(7) \quad r_{\alpha\beta}^i = r_{\beta\alpha}^i, \quad \frac{dr_{\beta\alpha}^i}{dx^\alpha} - \frac{dr_{\alpha\beta}^i}{dx^\beta} = 0$$

## §2. Invariant Pfaffian forms of the zeroth order

In our discussions we shall deal with properties of the family of spreads which remain invariant under general point transformations

$$(8) \quad \begin{cases} x^{i*} = x^{i*}(x^j, x^\beta), & \frac{\partial(x^{1*}, \dots, x^{n*})}{\partial(x^1, \dots, x^n)} \neq 0. \\ x^{\alpha*} = x^{\alpha*}(x^j, x^\beta), & \end{cases}$$

The left-hand members of the equations (2) are not invariant Pfaffian forms. However, if we adjoin to the variables  $x^i, x^\alpha, p_\alpha^i$  the new variables  $u_j^i, u_\beta^i, u_\beta^j, u_\beta^{\alpha\beta}, u_\beta^i$ , the Pfaffian forms

$$(9) \quad \begin{cases} \omega^\alpha = u_\beta^\alpha dx^\beta + u_\beta^\alpha (dx^\beta - p_\beta^j dx^\alpha), \\ \omega^i = u_\beta^i (dx^\beta - p_\beta^j dx^\alpha), \\ \omega_\alpha^i = u_{\beta\alpha}^{i\beta} (dp_\beta^j - r_{\beta\alpha}^i dx^\alpha) + u_{\beta\alpha}^i (dx^\beta - p_\beta^j dx^\alpha), \end{cases}$$

are invariant (in the space of all the variables). They are

called the invariant Pfaffian forms of the zeroth order.

The Pfaffian forms (9) can be modified without losing the property of being invariant. In particular, we can reduce the number of the new variables by imposing some invariant conditions. As a natural set of conditions we take the following:

$$(10) \quad (\omega^i)' \equiv [\omega^\alpha \omega_\alpha^i], \text{ mod. } \omega^j.$$

These conditions can be put in a form which involves the new variables  $u$ . We introduce, for this and for later purposes, the elements of the inverse matrices of  $(u_\beta^\alpha)$  and  $(u_j^i)$ , defined by the conditions

$$(11) \quad \begin{cases} u_\beta^\alpha v_\gamma^\beta = v_\beta^\alpha u_\gamma^\beta = \delta_\gamma^\alpha, \\ u_j^i v_k^j = v_j^i u_k^j = \delta_k^i. \end{cases}$$

Then it is easily found that the conditions (10) are equivalent to

$$(12) \quad u_{\alpha j}^{i\beta} = u_j^i v_\alpha^\beta.$$

The Pfaffian forms  $\omega_\alpha^i$  in (9), with  $u_{\alpha j}^{i\beta}$  given by (12), are further invariant Pfaffian forms of the zeroth order. From the form of the expressions in (9) we see that the exterior derivatives of  $\omega^\alpha$  and  $\omega^i$  are of the forms

$$(13) \quad \begin{cases} (\omega^\alpha)' = [\phi_\beta^\alpha \omega_\beta^\alpha] + [\phi_j^\alpha \omega^j], \\ (\omega^i)' = [\phi_j^i \omega^j] + [\omega^\alpha \omega_\alpha^i], \end{cases}$$

where  $\phi_\beta^\alpha, \phi_j^\alpha, \phi_j^i$  are newly introduced Pfaffian forms, such that

$$(14) \quad \begin{cases} \phi_\beta^\alpha \equiv du_\gamma^\alpha v_\beta^\gamma, \\ \phi_j^\alpha \equiv du_k^\alpha v_j^k - du_\beta^\alpha v_j^\beta u_k^\gamma v_\gamma^k, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i. \\ \phi_j^i \equiv du_k^i v_j^k, \end{cases}$$

It follows that, so far as  $u_\beta^\alpha, u_j^i, u_j^i$  remain independent variables, the Pfaffian forms  $\omega^\alpha, \omega^i, \omega_\alpha^i, \phi_\beta^\alpha, \phi_j^\alpha, \phi_j^i$  are linearly independent.

To find the expression for the exterior derivative of we form the exterior derivatives of the equations (13). The result is

$$(15) \quad \begin{cases} [(\phi_j^i)' - \phi_k^i \phi_j^k - \phi_j^\alpha \omega_\alpha^i] \omega^j - [(\omega_\alpha^i)' + \phi_\beta^\alpha \omega_\beta^i - \phi_j^i \omega_\alpha^j] \omega^\alpha = 0, \\ [(\phi_j^\alpha)' - \phi_\beta^\alpha \phi_j^\beta - \phi_k^\alpha \phi_j^k] \omega^j + [(\phi_\beta^\alpha)' - \phi_\gamma^\alpha \phi_\beta^\gamma + \phi_j^\alpha \omega_\beta^j] \omega^\beta = 0. \end{cases}$$

The first equation of (15) shows that the expression  $(\omega_\alpha^i)' + [\phi_\beta^\alpha \omega_\beta^i] - [\phi_j^i \omega_\alpha^j]$  will contain  $\omega^j$  or  $\omega^\beta$  in each of its terms. Since the system

$$(16) \quad \omega^i = 0, \quad \omega_\alpha^i = 0$$

is equivalent to (2) and is hence completely integrable, the terms in the last expression not involving  $\omega^j$  must be of the form  $Q_{\alpha\beta}^{i\beta}[\omega_\beta^j \omega^j]$ . We can therefore put

$$(17) \quad (\omega_\alpha^i)' = -[\phi_\alpha^\beta \omega_\beta^i] + [\phi_j^i \omega_\alpha^j] + [\phi_{\alpha j}^i \omega^j] + Q_{\alpha\beta}^{i\beta}[\omega_\beta^j \omega^j],$$

where  $\phi_{ij}^i$  are new Pfaffian forms. Using this equation, we get, from the first equation of (15),

$$(18) \quad Q_{\alpha\beta}^{i\beta} = Q_{\gamma\alpha}^{i\beta}$$

and

$$(19) \quad [(\phi_j^i)' - \phi_k^i \phi_j^k - \phi_j^\alpha \omega_\alpha^i + \phi_{\alpha j}^i \omega^\alpha] \omega^j = 0$$

We shall derive from the quantities  $Q_{\alpha\beta}^{i\beta}$  the first set of invariants of the family of spreads. The Pfaffian forms  $\phi_\beta^j, \phi_j^i$  in (13) are determined up to the transformation

$$(20) \quad \begin{cases} \phi_\beta^\alpha = \phi_\beta^{\alpha*} + a_{\beta\gamma}^\alpha \omega^\gamma + a_{\beta j}^\alpha \omega^j, & a_{\beta\gamma}^\alpha = a_{\gamma\beta}^\alpha, \\ \phi_j^i = \phi_j^{i*} + a_{jk}^i \omega^k, & a_{jk}^i = a_{kj}^i \end{cases}$$

where the  $a$ 's are arbitrary. When  $\phi_\beta^\alpha, \phi_j^i$  are replaced respectively by  $\phi_\beta^{\alpha*}, \phi_j^{i*}$ , the equation (17) retains its form, while the quantities  $Q_{\alpha\beta}^{i\beta}$  are replaced by  $Q_{\alpha\beta}^{i\beta*}$ , related to  $Q_{\alpha\beta}^{i\beta}$  as follows:

$$Q_{\alpha\beta}^{i\beta*} = Q_{\alpha\beta}^{i\beta} + \delta_j^i a_{\alpha\beta}^j.$$

From this equation we get

$$Q_{\alpha\beta}^{j\beta*} = Q_{\alpha\beta}^{j\beta} + (n-r) a_{\alpha\beta}^j.$$

Because of (18) it is possible to determine  $a_{\alpha\beta}^j$  such that

$$Q_{\alpha\beta}^{j\beta*} = 0.$$

We shall drop the asterisk and suppose the conditions

$$(21) \quad Q_{\alpha\beta}^{j\beta} = 0.$$

be always satisfied. Then  $Q_{\alpha\beta}^{i\beta}$  are independent of the choice of  $\phi_\beta^\alpha, \phi_j^i$  and are invariants.

### §3. Invariant Pfaffian forms of the first order

An important step for further discussions is to find the expression for  $Q_{\alpha\beta}^{i\beta}$ . For this purpose we shall first calculate  $\phi_\beta^\alpha, \phi_j^i$ , getting

$$(22) \quad \begin{cases} \phi_\beta^\alpha \equiv du_\beta^\alpha v_\beta^\alpha - u_\beta^\alpha v_\beta^i \omega_\beta^i, \\ \phi_j^i \equiv du_\beta^i v_\beta^j + u_\beta^i v_\beta^k \omega_\beta^k - u_{\beta k}^i v_\beta^k \omega^\beta, \end{cases} \text{ mod. } \omega^k, \omega^\alpha,$$

whence

$$(\omega_\alpha^i)' + [\phi_\alpha^\beta \omega_\beta^i] - [\phi_j^i \omega_\alpha^j] \equiv -u_{\beta k}^i v_\beta^k [\omega_\beta^j \omega^\beta] - u_{\beta k}^i v_\beta^k [\omega_\alpha^j \omega^\beta] - u_{\beta k}^i v_\beta^k v_\beta^l \omega_\beta^m \frac{\partial \omega_\beta^k}{\partial \omega_\beta^l} [\omega_\beta^j \omega^\beta],$$

mod.  $\omega^k$ .

It follows that

$$(23) \quad Q_{\alpha\beta}^{i\gamma} = -S_{\alpha}^{\beta} u_{\gamma}^i v_j^k - S_{\gamma}^{\beta} u_{\alpha}^i v_j^k - u_k^i v_{\alpha}^{\sigma} v_{\gamma}^{\tau} v_j^l u_{\beta}^{\rho} \frac{\partial x_{\rho}^k}{\partial x_{\sigma}^l} \\ + \frac{S_j^i}{n-r} (S_{\alpha}^{\beta} u_{\gamma}^l v_j^k + S_{\gamma}^{\beta} u_{\alpha}^l v_j^k + v_{\alpha}^{\sigma} v_{\gamma}^{\tau} u_{\beta}^{\rho} \frac{\partial x_{\rho}^k}{\partial x_{\sigma}^l}),$$

and hence that

$$Q_{\alpha\beta}^{i\gamma} = -(r+1) u_k^i v_j^k + \frac{r+1}{n-r} S_j^i u_{\alpha}^k v_k^{\beta} - v_{\alpha}^{\sigma} \left( \frac{\partial x_{\rho}^k}{\partial x_{\sigma}^l} u_k^i v_j^l - \frac{1}{n-r} S_j^i \delta_k^l \frac{\partial x_{\rho}^k}{\partial x_{\sigma}^l} \right).$$

It is easy to see, on account of (21), that we can arrive at the conditions

$$(24) \quad Q_{\alpha\beta}^{i\gamma} = 0$$

by properly choosing  $u_{\alpha}^i$ . The conditions (24) determine  $u_{\alpha}^i$  in terms of  $x^{\alpha}, x^i, p_{\alpha}^i, u_j^i, u_{\beta}^{\alpha}$ , and the new variables  $u_{\alpha}^i = u_{\beta}^i v_{\alpha}^k$ .

The Pfaffian forms

$$(25) \quad \omega_{\alpha}^i = u_j^i v_{\alpha}^j (dx_{\beta}^i - v_{\beta}^j dx^j) + u_{ij}^i (dx^i - v_{\beta}^j dx^j),$$

with  $u_j^i$  given by (24), are invariant and are called the invariant Pfaffian forms of the first order.

It is to be noticed that when  $r=1$  or  $r=n-1$ , the conditions (21) and (24) signify that all  $Q_{\alpha\beta}^{i\gamma}$  vanish. This suggests that it is either not possible or at least not practicable to derive further invariant Pfaffian forms by imposing conditions on  $Q_{\alpha\beta}^{i\gamma}$ . We therefore concentrate our attention to the equation (17) and form the exterior derivative of this equation. Neglecting all terms involving  $\omega^k$ , we get

$$[\{S_j^i (-\phi_{\alpha}^{\beta} - \phi_{\alpha}^{\gamma} \phi_{\beta}^{\gamma}) + S_{\alpha}^{\beta} (\phi_{\beta}^i - \phi_k^i \phi_{\beta}^k) + (-dQ_{\alpha\beta}^{i\gamma} + Q_{\alpha\beta}^{k\gamma} \phi_k^i - Q_{\alpha\beta}^{i\gamma} \phi_j^k + Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} \\ - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta}) \omega^{\sigma} - \phi_{\beta}^i \omega^{\beta} + Q_{\alpha\beta}^{i\gamma} Q_{\rho\sigma}^{k\beta} \omega^{\sigma} \omega^{\rho}\} u_{\beta}^j] \equiv 0, \text{ mod. } \omega^k$$

Now, equation (19) gives

$$(\phi_j^i)' - [\phi_k^i \phi_j^k] \equiv [\phi_j^{\alpha} \omega_{\alpha}^i] - [\phi_{\alpha}^i \omega^{\alpha}], \text{ mod. } \omega^k,$$

and the second equation of (15) gives

$$(\phi_{\beta}^{\alpha})' - [\phi_{\beta}^{\alpha} \phi_{\gamma}^{\beta}] + [\phi_{\beta}^{\alpha} \omega_{\gamma}^{\beta}] \equiv [\theta_{\beta\gamma}^{\alpha} \omega^{\gamma}], \text{ mod. } \omega^k,$$

where  $\theta_{\beta\gamma}^{\alpha}$  are newly introduced Pfaffian forms. Substituting these expressions into the last equation, we get

$$(26) \quad [(-S_j^i \theta_{\alpha\beta}^{\gamma} - S_{\alpha}^{\beta} \phi_{\beta}^i - S_{\beta}^{\alpha} \phi_{\beta}^i - dQ_{\alpha\beta}^{i\gamma} + Q_{\alpha\beta}^{k\gamma} \phi_k^i - Q_{\alpha\beta}^{i\gamma} \phi_j^k + Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} \\ - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} + Q_{\alpha\beta}^{i\gamma} Q_{\rho\sigma}^{k\beta} \omega^{\sigma}) \omega^{\rho} \omega^{\beta}] \equiv 0, \text{ mod. } \omega^k$$

From this equation we derive

$$(27) \quad -S_j^i \theta_{\alpha\beta}^{\gamma} - S_{\alpha}^{\beta} \phi_{\beta}^i - S_{\beta}^{\alpha} \phi_{\beta}^i - dQ_{\alpha\beta}^{i\gamma} + Q_{\alpha\beta}^{k\gamma} \phi_k^i - Q_{\alpha\beta}^{i\gamma} \phi_j^k + Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} \\ - Q_{\alpha\beta}^{i\gamma} \phi_{\rho}^{\beta} \equiv 0, \text{ mod. } \omega^{\rho}, \omega^k, \omega_{\beta}^{\rho}$$

On account of the conditions (21) it follows that

$$(28) \quad (n-r)\theta_{\alpha\gamma}^{\beta} + \delta_{\alpha}^{\beta}\phi_{\gamma} + \delta_{\gamma}^{\beta}\phi_{\alpha} \equiv 0, \text{ mod. } \omega^{\rho}, \omega^{\kappa}, \omega^{\lambda},$$

where

$$(29) \quad \phi_{\alpha} = \phi_{\alpha\gamma}^{\gamma}.$$

Again, putting  $\beta = \gamma$  and making use of the expression for  $\theta_{\alpha\gamma}^{\beta}$  in (28), we get

$$\phi_{\alpha\gamma}^i \equiv \frac{1}{n-r} \delta_j^i \phi_{\alpha}, \text{ mod. } \omega^{\rho}, \omega^{\kappa}, \omega^{\lambda}.$$

This allows us to put

$$(30) \quad \phi_{\alpha\gamma}^i \equiv \frac{1}{n-r} \delta_j^i \phi_{\alpha} + Q_{\alpha\gamma\beta}^i \omega^{\beta} + Q_{\alpha\gamma\kappa}^i \omega^{\kappa} + Q_{\alpha\gamma\lambda}^{i\beta} \omega^{\lambda}_{\beta},$$

where the coefficients  $Q_{\alpha\gamma\beta}^i, Q_{\alpha\gamma\kappa}^i, Q_{\alpha\gamma\lambda}^{i\beta}$  satisfy naturally the relations

$$Q_{\alpha\gamma\beta}^{\beta} = 0, \quad Q_{\alpha\gamma\kappa}^{\kappa} = 0, \quad Q_{\alpha\gamma\lambda}^{i\beta} = 0.$$

Substituting the expression (30) into (17), we get

$$(31) \quad (\omega_{\alpha}^i)' = -[\phi_{\alpha}^{\beta} \omega_{\beta}^i] + [\phi_{\gamma}^i \omega_{\alpha}^{\gamma}] + \frac{1}{n-r} [\phi_{\alpha} \omega^i] + Q_{\alpha\gamma\beta}^i [\omega^{\beta} \omega^{\gamma}] + Q_{\alpha\gamma\kappa}^i [\omega^{\kappa} \omega^{\gamma}] + Q_{\alpha\gamma\lambda}^{i\beta} [\omega_{\beta}^{\lambda} \omega^{\gamma}].$$

This equation gives the exterior derivatives of the invariant Pfaffian forms of the first order. Of the coefficients we suppose the following relations to be satisfied:

$$(32) \quad Q_{\alpha\gamma\kappa}^i + Q_{\alpha\kappa\gamma}^i = 0, \quad Q_{\alpha\gamma\beta}^{\beta} = 0, \quad Q_{\alpha\gamma\lambda}^{i\beta} = 0,$$

which is obviously possible.

Although  $\omega_{\alpha}^i$  are invariant Pfaffian forms, the coefficients  $Q$  (other than  $Q_{\alpha\gamma\beta}^{\beta}$ ) need not be invariants, because they depend on the choice of the Pfaffian forms  $\omega_{\beta}^{\alpha}, \omega_{\gamma}^i, \omega_{\alpha}$ . The coefficients or their combinations which are independent of the choice of these Pfaffian forms are invariants. From the form of the equations (13) and (31) the Pfaffian forms in question are determined up to the transformation

$$\begin{cases} \phi_{\beta}^{\alpha} = \phi_{\beta}^{\alpha*} + a_{\beta\gamma}^{\alpha} \omega_{\gamma}^{\beta}, \\ \phi_{\gamma}^i = \phi_{\gamma}^{i*} + a_{\gamma\kappa}^i \omega_{\kappa}^{\gamma}, \quad a_{\gamma\kappa}^i = a_{\kappa\gamma}^i, \\ \phi_{\alpha}^{\beta} = \phi_{\alpha}^{\beta*} + a_{\alpha\gamma}^{\beta} \omega_{\gamma}^{\alpha} + a_{\alpha\kappa}^{\beta} \omega_{\kappa}^{\alpha}, \quad a_{\alpha\kappa}^{\beta} = a_{\alpha\lambda}^{\beta}, \\ \phi_{\alpha} = \phi_{\alpha}^* + b_{\alpha\beta} \omega^{\beta} + b_{\alpha\gamma} \omega^{\gamma} + b_{\alpha\lambda}^{\beta} \omega_{\beta}^{\lambda}, \end{cases}$$

where the coefficients are arbitrary. When the  $\omega_{\beta}^{\alpha}, \omega_{\gamma}^i, \omega_{\alpha}$  in (31) are replaced by  $\omega_{\beta}^{\alpha*}, \omega_{\gamma}^{i*}, \omega_{\alpha}^*$ , the quantities  $Q$  are changed as follows:

$$\begin{cases} Q_{\alpha\gamma\beta}^{i\beta} = Q_{\alpha\gamma\beta}^{i\beta} + \frac{1}{n-r} \delta_j^i b_{\alpha\beta}, \\ Q_{\alpha\gamma\kappa}^i = Q_{\alpha\gamma\kappa}^i + \frac{1}{2(n-r)} (\delta_j^i b_{\alpha\kappa} - \delta_{\kappa}^i b_{\alpha\gamma}), \\ Q_{\alpha\gamma\lambda}^{i\beta} = Q_{\alpha\gamma\lambda}^{i\beta} + \delta_{\kappa}^{\beta} a_{\alpha\gamma}^{\kappa} - \delta_{\alpha}^{\beta} a_{\gamma\kappa}^{\alpha} + \frac{1}{n-r} \delta_j^i b_{\alpha\lambda}, \\ Q_{\alpha\beta\gamma}^{i\beta} = Q_{\alpha\beta\gamma}^{i\beta} \end{cases}$$

from which we get

$$\begin{cases} Q_{\alpha\beta}^{j*} = Q_{\alpha\beta}^j + b_{\alpha\beta}, \\ Q_{\alpha\beta\gamma}^{jk*} = Q_{\alpha\beta\gamma}^{jk} + a_{\alpha\beta}^{\gamma} - \delta_{\alpha}^{\gamma} a_{\alpha\beta} + b_{\alpha\beta}^{\gamma}. \end{cases}$$

Hence in order that the conditions (32) remain unaltered, we must have

$$b_{\alpha\beta} = 0, \quad b_{\alpha\beta}^{\gamma} = -a_{\alpha\beta}^{\gamma} + \delta_{\alpha}^{\gamma} a_{\beta},$$

where  $a_{\beta}$  is an abbreviation of  $a_{\beta}^i$ . It follows that the most general change of the Pfaffian forms  $\phi_{\beta}^{\alpha}, \phi_j^i, \phi_j^{\alpha}, \phi_{\alpha}$  is given by

$$(33) \quad \begin{cases} \phi_{\beta}^{\alpha} = \phi_{\beta}^{\alpha*} + a_{\beta j}^{\alpha} \omega^j, \\ \phi_j^i = \phi_j^{i*} + a_{jk}^i \omega^k, \quad a_{jk}^i = a_{kj}^i, \\ \phi_j^{\alpha} = \phi_j^{\alpha*} + a_{\beta j}^{\alpha} \omega^{\beta} + a_{jk}^{\alpha} \omega^k, \quad a_{jk}^{\alpha} = a_{kj}^{\alpha}, \\ \phi_{\alpha} = \phi_{\alpha}^* + b_{\beta j} \omega^{\beta} + (-a_{\beta j}^{\alpha} + \delta_{\beta}^{\alpha} a_j) \omega^{\beta}. \end{cases}$$

Of the coefficients in (31)  $Q_{\alpha\beta}^{ij}$  and  $Q_{\alpha\beta\gamma}^{ijk}$  are invariants, while  $Q_{\alpha\beta}^i$  and  $Q_{\alpha\beta\gamma}^{ijk}$  are transformed under (33) as follows:

$$(34) \quad \begin{cases} Q_{\alpha\beta}^{ij*} = Q_{\alpha\beta}^{ij} + \frac{1}{2(n-r)} (\delta_j^i b_{\alpha k} - \delta_k^i b_{\alpha j}), \\ Q_{\alpha\beta\gamma}^{ijk*} = Q_{\alpha\beta\gamma}^{ijk} + \delta_k^i a_{\alpha j}^{\beta} - \frac{1}{n-r} \delta_j^i a_{\alpha k}^{\beta} + \delta_{\alpha}^{\beta} (-a_{kj}^i + \frac{1}{n-r} \delta_j^i a_k). \end{cases}$$

#### §4. Invariant Pfaffian forms of the second order

The object of this section is to introduce from  $\phi_{\beta}^{\alpha}, \phi_j^i$  further invariant Pfaffian forms. We shall first reduce the number of arbitrary coefficients in (33) by imposing some invariant conditions. From (28) we are led to put

$$(35) \quad Q_{\alpha\beta}^{\gamma} \equiv -\frac{1}{n-r} (\delta_{\alpha}^{\beta} \phi_{\gamma} + \delta_{\beta}^{\gamma} \phi_{\alpha}) + R_{\alpha\beta\gamma}^{\delta} \omega^{\delta} + R_{\alpha\beta\gamma}^{\delta\epsilon} \omega^{\delta} \omega^{\epsilon}, \text{ mod. } \omega^k$$

from which it follows that

$$(36) \quad (\phi_{\beta}^{\alpha})' - [\phi_{\beta}^{\alpha} \phi_{\gamma}^{\delta}] + [\phi_{\beta}^{\alpha} \omega_{\gamma}^{\delta}] \equiv -\frac{1}{n-r} \delta_{\beta}^{\alpha} [\phi_{\gamma} \omega^{\delta}] - \frac{1}{n-r} [\phi_{\beta} \omega^{\alpha}] + R_{\alpha\beta\gamma}^{\delta\epsilon} [\omega^{\delta} \omega^{\epsilon}] + R_{\alpha\beta\gamma}^{\delta\epsilon\zeta} [\omega^{\delta} \omega^{\epsilon} \omega^{\zeta}], \text{ mod. } \omega^k$$

The effect of the transformation (33) on the coefficients  $R_{\alpha\beta\gamma}^{\delta\epsilon}$  is easily found to be

$$R_{\alpha\beta\gamma}^{\delta\epsilon*} = R_{\alpha\beta\gamma}^{\delta\epsilon} + \delta_{\beta}^{\delta} a_{\alpha j}^{\epsilon} + \delta_{\beta}^{\epsilon} a_{\alpha j}^{\delta} + \frac{1}{n-r} (\delta_{\beta}^{\delta} a_{\alpha j}^{\epsilon} + \delta_{\beta}^{\epsilon} a_{\alpha j}^{\delta}) - \frac{1}{n-r} (\delta_{\beta}^{\delta} \delta_{\gamma}^{\epsilon} + \delta_{\beta}^{\epsilon} \delta_{\gamma}^{\delta}) a_j,$$

from which we derive

$$R_{\alpha\beta\gamma}^{\delta\epsilon*} = R_{\alpha\beta\gamma}^{\delta\epsilon} + \frac{n+1}{n-r} a_{\beta j}^{\delta} + \frac{1}{n-r} \delta_{\beta}^{\delta} a_{\alpha j}^{\epsilon} - \frac{r+1}{n-r} \delta_{\beta}^{\delta} a_j.$$

Hence by properly choosing  $a_{\beta j}^{\delta}$ , we can make

$$(37) \quad R_{\alpha\beta\gamma}^{\delta\epsilon} = 0.$$

When (37) are satisfied, the conditions that the transformation (33) leaves them unaltered are

$$(38) \quad a_{\beta j}^{\alpha} = \frac{1}{n-r+1} \delta_{\beta}^{\alpha} a_j$$

We suppose that the conditions (37) are fulfilled and that

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between the coefficients of (33) the relations (38) hold. Then we get, from (34),

$$(39) \quad Q_{\alpha\beta}^{i\gamma} = Q_{\alpha\beta\gamma}^{i\delta} - \delta_{\alpha}^{\beta} a_{\alpha}^i + \frac{1}{n-r+1} \delta_{\alpha}^{\beta} (\delta_{\alpha}^i a_{\beta} + \delta_{\beta}^i a_{\alpha}),$$

so that we have

$$(39a) \quad Q_{\alpha\beta\gamma}^{i\delta} = Q_{\beta\alpha\gamma}^{i\delta} - r a_{\alpha}^i + \frac{r}{n-r+1} (\delta_{\alpha}^i a_{\beta} + \delta_{\beta}^i a_{\alpha}).$$

To show that  $a_{\alpha}^i$  can be chosen to make  $Q_{\alpha\beta\gamma}^{i\delta}$  vanish it is necessary to show that the quantities in question are symmetric in  $j, k$ .

For this purpose we substitute the expressions (30) and (35) into (26). The result is

$$[\{-\delta_j^i (R_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} + R_{\alpha\beta\gamma}^{i\delta} \omega^{\beta}) - \delta_{\alpha}^{\beta} (Q_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} + Q_{\alpha\beta\gamma}^{i\delta} \omega^{\beta}) - \delta_{\alpha}^{\beta} (Q_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} + Q_{\alpha\beta\gamma}^{i\delta} \omega^{\beta}) - dQ_{\alpha\beta\gamma}^{i\delta} + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\alpha}^i - Q_{\alpha\beta\gamma}^{i\delta} \phi_{\beta}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\gamma}^i - Q_{\alpha\beta\gamma}^{i\delta} \phi_{\alpha}^i - Q_{\alpha\beta\gamma}^{i\delta} \phi_{\beta}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\gamma}^i + Q_{\alpha\beta\gamma}^{i\delta} Q_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} \} \omega^{\alpha} \omega^{\beta} \omega^{\gamma}] = 0, \text{ mod. } \omega^k$$

This equation shows that we can put

$$(40) \quad dQ_{\alpha\beta\gamma}^{i\delta} - Q_{\alpha\beta\gamma}^{i\delta} \phi_{\alpha}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\beta}^i - Q_{\alpha\beta\gamma}^{i\delta} \phi_{\gamma}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\alpha}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\beta}^i + Q_{\alpha\beta\gamma}^{i\delta} \phi_{\gamma}^i = Q_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} + Q_{\alpha\beta\gamma}^{i\delta} \omega^{\beta} + Q_{\alpha\beta\gamma}^{i\delta} \omega^{\gamma}$$

We then substitute the expression (40) into the last equation and equate to zero the coefficient of  $[\omega^{\alpha} \omega^{\beta} \omega^{\gamma}]$ , getting

$$-\delta_j^i R_{\alpha\beta\gamma}^{i\delta} + \delta_k^i R_{\alpha\beta\gamma}^{i\delta} - \delta_{\alpha}^{\beta} Q_{\alpha\beta\gamma}^{i\delta} + \delta_{\alpha}^{\beta} Q_{\alpha\beta\gamma}^{i\delta} - \delta_{\beta}^{\gamma} Q_{\alpha\beta\gamma}^{i\delta} + \delta_{\beta}^{\gamma} Q_{\alpha\beta\gamma}^{i\delta} + Q_{\alpha\beta\gamma}^{i\delta} \omega^{\alpha} - Q_{\alpha\beta\gamma}^{i\delta} \omega^{\beta} = 0.$$

From this equation we get, by putting  $\alpha = \beta, \gamma = \rho,$

$$-\delta_j^i R_{\alpha\rho\alpha}^{i\delta} + \delta_k^i R_{\alpha\rho\alpha}^{i\delta} + (r+1)(Q_{\alpha\rho\alpha}^{i\delta} - Q_{\alpha\rho\alpha}^{i\delta}) = 0.$$

The first term of this equation is zero, because of (37). The same is true of the second term, since, from the second equation of (15), we have

$$(41) \quad R_{\alpha\beta\gamma}^{i\delta} = R_{\alpha\beta\gamma}^{i\delta}.$$

Hence it follows that

$$(42) \quad Q_{\alpha\beta\gamma}^{i\delta} = Q_{\beta\alpha\gamma}^{i\delta}.$$

From (39a) and (42) we see that  $a_{\alpha}^i$  can be chosen to

arrive at the conditions

$$(43) \quad Q_{\alpha\beta\gamma}^{i\delta} = 0$$

In order that the transformation (33) leaves the conditions (43) unaltered, we must have

$$(44) \quad a_{\alpha}^i = \frac{1}{n-r+1} (\delta_{\alpha}^i a_{\beta} + \delta_{\beta}^i a_{\alpha}).$$

The conditions (38) and (44) show that by the transformation (33) the Pfaffian forms  $e_{\beta}^{\alpha}, e_j^i$  are changed by terms which involve only the  $n-r$  quantities  $a_{\alpha}^i$ . We take  $a_{\alpha}^i$  to be new variables and adjoin them to the variables  $x^{\alpha}, x^i, p_{\alpha}^i, u_{\beta}^{\alpha}, u_j^i, u_j^{\alpha}, u_{\alpha}^i$ . With respect to this set of variables the Pfaffian forms  $e_{\beta}^{\alpha}, e_j^i$

are invariant. They are called the invariant Pfaffian forms of the second order. When this is the case, the Pfaffian forms  $\phi_j^\alpha, \phi_\alpha$  are determined up to the transformation

$$(45) \quad \begin{cases} \phi_j^\alpha = \phi_j^{\alpha*} + a_{jk}^\alpha \omega^k, & a_{jk}^\alpha = a_{kj}^\alpha, \\ \phi_\alpha = \phi_\alpha^* + b_{\alpha j} \omega^j. \end{cases}$$

The total number of variables in our set is equal to  $n(n+2)$ . On the other hand, we have introduced the invariant Pfaffian forms of the zeroth order  $\omega^\alpha, \omega^i$ , those of the first order  $\omega_\alpha^i$ , and those of the second order  $\phi_\beta^\alpha, \phi_j^i$ , the total number of which is  $n(n-r+1)+r^2$ . We shall prove that we can introduce  $n(r+1)-r^2$  further invariant Pfaffian forms, linearly independent between them and linearly independent from  $\omega^\alpha, \omega^i, \omega_\alpha^i, \phi_\beta^\alpha, \phi_j^i$ . If this is done we shall arrive at a situation by which we have the same number of linearly independent Pfaffian forms as the number of our variables. This situation is our ultimate goal.

Before doing this let us form the exterior derivatives of the invariant Pfaffian forms of the second order  $\phi_\beta^\alpha, \phi_j^i$ . From (36) and (19) we see that we can set

$$(46) \quad \begin{cases} (\phi_\beta^\alpha)' - [\phi_j^\alpha \phi_\beta^j] + [\phi_j^\alpha \omega_\beta^j] + \frac{1}{n-r} \delta_\beta^\alpha [\phi_j \omega^j] + \frac{1}{n-r} [\phi_\beta \omega^\alpha] \\ \quad - R_{\beta\gamma}^\alpha [\omega^\gamma \omega^j] - R_{\beta\gamma}^{\alpha j} [\omega_j^\gamma \omega^j] = [\theta_{\beta j}^\alpha \omega^j], \\ (\phi_j^i)' - [\phi_k^i \phi_j^k] - [\phi_j^\alpha \omega_\alpha^i] + \frac{1}{n-r} \delta_j^i [\phi_\alpha \omega^\alpha] - Q_{\alpha\beta}^i [\omega^\alpha \omega^\beta] \\ \quad - Q_{\alpha k}^i [\omega^\alpha \omega^k] - Q_{j\alpha k}^i [\omega^\alpha \omega_\beta^k] = [\theta_{j k}^i \omega^k], \end{cases}$$

where  $\theta_{\beta j}^\alpha, \theta_{j k}^i$  are newly introduced Pfaffian forms, of which the latter ones are subjected to the conditions

$$(47) \quad [\theta_{j k}^i \omega^j \omega^k] = 0.$$

The expressions for the exterior derivatives of  $\phi_\beta^\alpha, \phi_j^i$  are therefore determined, when we know  $\theta_{\beta j}^\alpha, \theta_{j k}^i$ . To find the latter Pfaffian forms we form the exterior derivatives of the equation (31) and make use of (46). The resulting equation, when completely expanded, is as follows:

$$\begin{aligned}
 (48) \quad & \left\{ \begin{aligned}
 & \frac{1}{n-r} \delta_j^i [(\phi_\alpha^i + \phi_\beta^i \phi_\beta) \omega^j] + [(\theta_{ij}^\beta \omega_\beta^i - \theta_{kj}^k \omega_\alpha^k) \omega^j] \\
 & + [(dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^k + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r - \frac{1}{n-r} Q_{\alpha\beta}^i \phi_\gamma + Q_{\alpha\beta}^i \omega_\alpha^k) \omega^\beta \omega^j] \\
 & + [(dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + 2Q_{\alpha\beta}^i \phi_k^l + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r + \frac{1}{n-r} Q_{\alpha\beta}^i \phi_\beta) \omega^k \omega^j] \\
 & + [(dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^l + Q_{\alpha\beta}^i \phi_k^l - Q_{\alpha\beta}^i \phi_\beta^r + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r) \omega_\beta^k \omega^j] \\
 & + [Q_{\alpha\beta}^i \omega_\alpha^k - R_{\alpha\beta}^i \omega_\beta^k] + [(dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^l - Q_{\alpha\beta}^i \phi_\beta^r + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r) \\
 & - Q_{\alpha\beta}^i \omega_\beta^k - R_{\alpha\beta}^i \omega_\beta^k] \omega_\beta^j \omega^r] + Q_{\alpha\beta}^i [\omega_\beta^k \omega_\beta^j \omega^r] + Q_{\alpha\beta}^i Q_{\beta l m} [\omega^m \omega^l \omega^j] \\
 & + Q_{\alpha\beta}^i Q_{\beta l m} [\omega_\beta^m \omega^l \omega^j] - Q_{\alpha\beta}^i Q_{\beta l m} [\omega^m \omega^l \omega^j] \\
 & + (Q_{\alpha\beta}^i Q_{\beta j r} + Q_{\alpha\beta}^i Q_{\beta k j}) [\omega^i \omega^k \omega^r] \\
 & + (2\delta_j^\beta Q_{\alpha\beta}^i - Q_{\alpha\beta}^i Q_{\beta j}^\beta + Q_{\alpha\beta}^i Q_{\beta k j}) [\omega_\beta^i \omega^k \omega^r] \\
 & + (-\delta_j^\beta Q_{\alpha\beta}^i + Q_{\alpha\beta}^i Q_{\beta j}^\beta - \delta_j^\beta R_{\alpha\beta}^i - \delta_\alpha^\beta Q_{\beta j}^i) [\omega_\beta^i \omega^r \omega^j] = 0.
 \end{aligned} \right.
 \end{aligned}$$

We shall write this equation in the form

$$(49) \quad \frac{1}{n-r} \delta_j^i [(\phi_\alpha^i + \phi_\beta^i \phi_\beta) \omega^j] + [\psi_{\alpha\beta}^i \omega^\beta \omega^j] + [\psi_{\alpha\beta}^i \omega^k \omega^j] + [\psi_{\alpha\beta}^i \omega_\beta^k \omega^j] = 0,$$

where we have put

$$\begin{aligned}
 (50) \quad & \psi_{\alpha\beta}^i \equiv dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^k + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r - \frac{1}{n-r} Q_{\alpha\beta}^i \phi_\gamma, \\
 & \psi_{\alpha\beta}^i \equiv dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + 2Q_{\alpha\beta}^i \phi_k^l + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r + \frac{1}{n-r} Q_{\alpha\beta}^i \phi_\beta, \\
 & \psi_{\alpha\beta}^i \equiv dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^l + Q_{\alpha\beta}^i \phi_k^l - Q_{\alpha\beta}^i \phi_\beta^r + Q_{\alpha\beta}^i \phi_\alpha^r \\
 & \quad + Q_{\alpha\beta}^i \phi_\beta^r + \delta_k^\beta \theta_{\alpha\beta}^i - \delta_\alpha^\beta \theta_{\beta\alpha}^i, \\
 & \psi_{\alpha\beta}^i \equiv dQ_{\alpha\beta}^i - Q_{\alpha\beta}^k \phi_k^i + Q_{\alpha\beta}^i \phi_j^l - Q_{\alpha\beta}^i \phi_\beta^r + Q_{\alpha\beta}^i \phi_\alpha^r + Q_{\alpha\beta}^i \phi_\beta^r,
 \end{aligned}$$

the congruences being taken modulo  $\omega^\alpha, \omega^i, \omega_\alpha^i$ .

That the right-hand member of the fourth equation of (50) is a linear combination of  $\omega^\beta, \omega^k, \omega_\beta^k$  has been established and is expressed by (40). When (40) is substituted into (48), the terms involving  $\omega^k$  and those not involving  $\omega^k$  must vanish separately. By a proper choice of the Pfaffian forms  $\psi$  we get from (48) or (49) the equation

$$\left[ \frac{1}{n-r} \delta_j^i \phi_\alpha^i + \phi_\beta^i \phi_\beta + \psi_{\alpha\beta}^i \omega^\beta + \psi_{\alpha\beta}^i \omega^k + \psi_{\alpha\beta}^i \omega_\beta^k \right] \omega^j = 0,$$

which expresses the vanishing of the terms involving  $\omega^k$ .

This equation allows us to put

$$(51) \quad \frac{1}{n-r} \delta_j^i [(\phi_\alpha^i + \phi_\beta^i \phi_\beta)] + [\psi_{\alpha\beta}^i \omega^\beta] + [\psi_{\alpha\beta}^i \omega^k] + [\psi_{\alpha\beta}^i \omega_\beta^k] = [\pi_{\alpha\beta}^i \omega^k],$$

where  $\pi_{\alpha\beta}^i$  are Pfaffian forms satisfying the equations

$$(52) \quad [\pi_{\alpha\beta}^i \omega^k \omega^j] = 0.$$

From (51) it follows that

$$\frac{1}{n-r} \delta_j^i (\phi_\alpha^i + \phi_\beta^i \phi_\beta) + [\psi_{\alpha\beta}^i \omega_\beta^k] \equiv 0, \text{ mod. } \omega^\alpha, \omega^i,$$

and hence that

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$$\phi'_\alpha + [\phi'_\alpha \phi_\beta] + [\psi_{\alpha k}^{\beta\beta} \omega_\beta^k] \equiv 0, \text{ mod. } \omega^\alpha, \omega^i$$

Combining these two equations, we get

$$[(\psi_{\alpha k}^{\beta\beta} - \frac{1}{n-r} \delta_j^i \psi_{\alpha k}^{\beta\beta}) \omega_\beta^k] \equiv 0, \text{ mod. } \omega^\alpha, \omega^i,$$

so that

$$\psi_{\alpha k}^{\beta\beta} - \frac{1}{n-r} \delta_j^i \psi_{\alpha k}^{\beta\beta} \equiv 0, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

Because of the conditions (43) we have

$$\psi_{\beta j k}^{\alpha\beta} \equiv \delta_k^i \theta_{\beta j}^{\alpha\beta} - r \theta_{k j}^{\alpha\beta}, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

Hence we get

$$(53) \quad \delta_k^i \theta_{\beta j}^{\alpha\beta} - r \theta_{k j}^{\alpha\beta} - \frac{1}{n-r} \delta_j^i (\theta_{\beta k}^{\alpha\beta} - r \theta_{\alpha k}) \equiv 0, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i,$$

where

$$(54) \quad \theta_{k.} = \theta_{k j}.$$

Before proceeding further, we shall draw a second consequence from the equation (48). The equation (51) gives, when the indices i and j are contracted,

$$\phi'_\alpha + [\phi'_\alpha \phi_\beta] = -[\psi_{\alpha\beta}^j \omega^\beta] - [\psi_{\alpha j k}^{\beta\beta} \omega_\beta^k] + [(\pi_{\alpha k}^j - \psi_{\alpha j k}^{\beta\beta}) \omega_\beta^k].$$

But we have

$$\psi_{\alpha\beta}^j \equiv 0, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

$$\psi_{\alpha j k}^{\beta\beta} \equiv Q_{\alpha j k}^{\beta\beta} \phi_j^\beta + \theta_{\alpha k}^{\beta\beta} - \delta_\alpha^\beta \theta_{\beta k}, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

Using (48), it is not difficult to show that  $\phi'_\alpha$  is of the form

$$(55) \quad \phi'_\alpha = -[\phi'_\alpha \phi_\beta] + [\pi_{\alpha k} \omega^k] - Q_{\alpha j k}^{\beta\beta} [\phi_j^\beta \omega_\beta^k] - [\theta_{\alpha k}^{\beta\beta} \omega_\beta^k] + [\theta_{\alpha k} \omega_\alpha^k] + S_{\alpha j k}^{\beta\beta} [\omega_\beta^k \omega^j] + S_{\alpha \beta \gamma} [\omega^\beta \omega^\gamma]$$

where

$$(56) \quad S_{\alpha \beta \gamma} + S_{\alpha \gamma \beta} = 0.$$

We now form the exterior derivative of the first equation of (46). The equation contains the term  $[dR_{\beta\gamma}^{\alpha\beta} \omega_\beta^\gamma \omega^\beta]$ .

For a fixed set of values j, p, \gamma we multiply the equation by

$$[\omega^{r+1} \dots \omega^n \omega^1 \dots \omega^{j-1} \omega^{j+1} \dots \omega^r \prod_{(k, \sigma \neq j, \beta)} \omega_\sigma^k].$$

The terms which give non-zero products when multiplied by the last expression are of the form

$$[\chi_{\beta\gamma}^{\alpha\beta} \omega_\beta^\gamma \omega^\beta],$$

so that it follows that

$$\chi_{\beta\gamma}^{\alpha\beta} \equiv 0, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

The expression for  $\chi_{\beta\gamma}^{\alpha\beta}$  is found to be

$$(57) \quad \chi_{\beta\gamma}^{\alpha\beta} = dR_{\beta\gamma}^{\alpha\beta} + R_{\beta\gamma k}^{\alpha\beta} \phi_k^\beta - R_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\alpha - R_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\beta + R_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\beta + R_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\sigma + Q_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\alpha + \frac{1}{n-r} (\delta_\beta^\alpha Q_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\beta + \delta_\gamma^\alpha Q_{\beta\gamma\sigma}^{\alpha\beta} \phi_\sigma^\beta) + \delta_\beta^\alpha \theta_{\beta\sigma}^\alpha + \delta_\beta^\sigma \theta_{\beta\sigma}^\alpha + \frac{1}{n-r} (\delta_\beta^\alpha \theta_{\beta\sigma}^\alpha + \delta_\gamma^\alpha \theta_{\beta\sigma}^\alpha - \delta_\beta^\alpha \delta_\gamma^\sigma \theta_{\beta\sigma}^\alpha - \delta_\gamma^\alpha \delta_\beta^\sigma \theta_{\beta\sigma}^\alpha) \equiv 0, \text{ mod. } \omega^\alpha, \omega^i, \omega_\alpha^i.$$

Contracting  $\rho$  and  $\gamma$ , we get, on remembering (37),

$$(58) \quad (N+1)\theta_{\beta j}^{\alpha} + \delta_{\beta}^{\alpha} \theta_{\beta j}^{\gamma} - (r+1)\delta_{\beta}^{\alpha} \theta_j + Q_{\beta\sigma j}^{k\alpha} \phi_k^{\sigma} = 0, \text{ mod. } \omega^{\alpha}, \omega^i, \omega_{\alpha}^i.$$

from which it follows that

$$\theta_{\beta j}^{\alpha} \equiv \frac{1}{n-r+1} \delta_{\beta}^{\alpha} \theta_j - \frac{1}{N+1} Q_{\beta\sigma j}^{k\alpha} \phi_k^{\sigma}, \text{ mod. } \omega^{\alpha}, \omega^i, \omega_{\alpha}^i.$$

Substituting this into (53), we get

$$\theta_{kj}^i \equiv \frac{1}{n-r+1} (\delta_j^i \theta_k + \delta_k^i \theta_j), \text{ mod. } \omega^{\alpha}, \omega^i, \omega_{\alpha}^i.$$

These two consequences permit us to put

$$(59) \quad \theta_{\beta j}^{\alpha} = \frac{1}{n-r+1} \delta_{\beta}^{\alpha} \theta_j - \frac{1}{N+1} Q_{\beta\sigma j}^{k\alpha} \phi_k^{\sigma} + R_{\beta j r}^{\alpha} \omega^r + R_{\beta j k}^{\alpha} \omega^k + R_{\beta j k}^{\alpha r} \omega_r^k,$$

$$(60) \quad \theta_{kj}^i = \frac{1}{n-r+1} (\delta_j^i \theta_k + \delta_k^i \theta_j) + V_{kjr}^i \omega^r + V_{kjl}^i \omega^l + V_{kjl}^{i\alpha} \omega_{\alpha}^j,$$

with

$$(61) \quad V_{kjr}^i = 0, \quad V_{kjl}^i = 0, \quad V_{kjl}^{i\alpha} = 0,$$

while, because of (47), we have also:

$$(62) \quad V_{kjr}^i = V_{jkr}^i, \quad V_{kjl}^{i\alpha} = V_{ilk}^{i\alpha}, \quad V_{kjl}^{i\alpha} [\omega^l \omega^k \omega^{\alpha}] = 0.$$

### §5. Invariant Pfaffian forms of the third order and the fourth order

As it has been stated we shall introduce  $n(r+1)-r^2$  further invariant Pfaffian forms, which are linearly independent of each other and linearly independent from  $\omega^{\alpha}, \omega^i, \omega_{\alpha}^i, \phi_{\beta}^{\alpha}, \phi_j^i$ . These are naturally afforded by  $\varphi_j^{\alpha}, \varphi_{\alpha}, \theta_k$ . Since they are not completely determined, we shall determine them by imposing some invariant conditions. The fact that they are completely determined then implies the property of being invariant.

Let us first put

$$(63) \quad \left\{ \begin{aligned} \Omega^{\alpha} &= (\omega^{\alpha})' - [\phi_{\beta}^{\alpha} \omega^{\beta}] - [\phi_j^{\alpha} \omega^j], \\ \Omega^i &= (\omega^i)' - [\omega^{\alpha} \omega_{\alpha}^i] - [\phi_j^i \omega^j], \\ \Omega_{\alpha}^i &= (\omega_{\alpha}^i)' - n-r [\phi_{\alpha} \omega^i] + [\phi_{\beta}^i \omega_{\beta}^{\alpha}] + [\omega_{\alpha}^i \phi_j^i], \\ \Phi_{\beta}^{\alpha} &= (\phi_{\beta}^{\alpha})' + \frac{1}{n-r} [\phi_{\beta}^{\alpha} \omega^{\alpha}] + [\phi_{\beta}^{\alpha} \phi_r^{\alpha}] - [\omega_{\beta}^{\alpha} \omega_j^{\alpha}] \\ &\quad - \delta_{\beta}^{\alpha} \left( \frac{1}{n-r+1} [\theta_k \omega^k] + \frac{1}{n-r} [\omega^r \phi_r] \right), \\ \Phi_j^i &= (\phi_j^i)' - \frac{1}{n-r+1} [\theta_j \omega^i] - [\phi_j^{\alpha} \omega_{\alpha}^i] + [\phi_j^k \phi_k^i] \\ &\quad - \delta_j^i \left( \frac{1}{n-r+1} [\theta_k \omega^k] + \frac{1}{n-r} [\omega^{\alpha} \phi_{\alpha}] \right), \\ \Phi_j^{\alpha} &= (\phi_j^{\alpha})' - \frac{1}{n-r+1} [\theta_j \omega^{\alpha}] + [\phi_j^k \phi_k^{\alpha}] + [\phi_j^{\beta} \phi_{\beta}^{\alpha}]. \end{aligned} \right.$$

From (13), (31), (46), (59), (60) we have

$$(64) \quad \begin{cases} \Omega^{\alpha} = 0, \\ \Omega^i = 0, \\ \Omega^{\alpha}_i = Q_{\alpha\beta}^i [w^{\beta} w^{\delta}] + Q_{\beta\gamma}^i [w^{\gamma} w^{\delta}] + Q_{\gamma\delta}^{\alpha\beta} [w^{\beta} w^{\delta}] + Q_{\alpha\beta}^{\gamma\delta} [w^{\beta} w^{\delta}], \\ \Phi_{\beta}^{\alpha} = -\frac{1}{n+1} Q_{\beta\alpha}^{\gamma\delta} [\Phi_{\gamma}^{\delta} w^{\delta}] + R_{\beta\gamma}^{\alpha} [w^{\gamma} w^{\delta}] + R_{\beta\gamma}^{\alpha\delta} [w^{\beta} w^{\delta}] \\ \quad + R_{\beta\gamma}^{\alpha} [w^{\gamma} w^{\delta}] + R_{\beta\gamma}^{\alpha} [w^{\gamma} w^{\delta}] + R_{\beta\gamma}^{\alpha\delta} [w^{\beta} w^{\delta}], \\ \Phi_j^i = Q_{\alpha\beta}^i [w^{\alpha} w^{\beta}] + Q_{\alpha\beta}^i [w^{\alpha} w^{\beta}] + Q_{\alpha\beta}^i [w^{\alpha} w^{\beta}] \\ \quad + V_{jkr}^i [w^r w^k] + V_{jkl}^i [w^l w^k] + V_{jkl}^i [w^l w^k] \end{cases}$$

Forming the exterior derivative of the first equation of (63), we get

$$[\Phi_{\beta}^{\alpha} w^{\beta}] + [\Phi_j^i w^j] = 0.$$

Hence we have

$$\Phi_j^i = -\frac{1}{n+1} Q_{\beta\alpha}^{\gamma\delta} [\Phi_{\gamma}^{\delta} w^{\beta}] + R_{\beta\gamma}^{\alpha} [w^{\gamma} w^{\beta}] + R_{\beta\gamma}^{\alpha\delta} [w^{\beta} w^{\delta}], \text{ mod. } w^k.$$

For the moment this is the most definite result concerning the expression for  $\Phi_j^i$ , since  $\varphi_j^{\alpha}$  is determined up to the transformation in (45).

Let us see how far the Pfaffian forms  $\theta_k$  are determined. If the Pfaffian forms  $\theta_k, \varphi_j^{\alpha}, \varphi_{\alpha}$  are replaced by  $\theta_k^*, \varphi_j^{*\alpha}, \varphi_{\alpha}^*$ , the latter ones according to (45), the expression  $\Phi_{\beta}^{\alpha}$  corresponding to  $\Phi_{\beta}^{\alpha}$  is related to  $\Phi_{\beta}^{\alpha}$  as follows:

$$\Phi_{\beta}^{\alpha} - \Phi_{\beta}^{*\alpha} = \frac{1}{n-r} b_{\beta\gamma} [w^{\gamma} w^{\delta}] - a_{\gamma k}^{\alpha} [w^{\beta} w^k] - \frac{1}{n+1} \delta_{\beta}^{\alpha} [(\theta_k - \theta_k^*) w^k] - \frac{1}{n-r} \delta_{\beta}^{\alpha} b_{\beta\gamma} [w^{\gamma} w^{\delta}].$$

On the other hand, we have, from (64),

$$\begin{aligned}
 \Phi_{\beta}^{\alpha} - \Phi_{\beta}^{*\alpha} = & -\frac{1}{n+1} Q_{\beta\alpha}^{\gamma\delta} a_{\gamma k}^{\delta} [w^{\gamma} w^{\delta}] + (R_{\beta\gamma}^{\alpha} - R_{\beta\gamma}^{*\alpha}) [w^{\gamma} w^{\delta}] \\
 & + (R_{\beta\gamma}^{\alpha\delta} - R_{\beta\gamma}^{*\alpha\delta}) [w^{\beta} w^{\delta}] + (R_{\beta\gamma}^{\alpha} - R_{\beta\gamma}^{*\alpha}) [w^{\gamma} w^{\delta}] \\
 & + (R_{\beta\gamma}^{\alpha} - R_{\beta\gamma}^{*\alpha}) [w^{\gamma} w^{\delta}] + (R_{\beta\gamma}^{\alpha\delta} - R_{\beta\gamma}^{*\alpha\delta}) [w^{\beta} w^{\delta}].
 \end{aligned}$$

A comparison of these two equations shows that  $\theta_k$  is determined up to a transformation of the form

$$(65) \quad \theta_k = \theta_k^* + C_{k\beta} w^{\beta} + C_{kj} w^j + C_{kj}^{\beta} w_{\beta}^{\beta},$$

where the coefficients  $c$  are arbitrary.

If we search for the effect of the transformation (45),

(65) on the coefficients in (64), it is easy to find

$$\begin{aligned}
 V_{jkr}^i &= V_{jkr}^i + \frac{1}{2(n-r)} (\delta_k^i b_{rj} + \delta_j^i b_{rk}) + \frac{1}{n+1} (\delta_k^i c_{jr} + \delta_j^i c_{kr}), \\
 V_{jkl}^i &= V_{jkl}^i - \delta_j^i a_{kr} + \frac{1}{n+1} (\delta_j^i c_{kl}^r + \delta_k^i c_{jl}^r), \\
 R_{\beta\gamma}^{\alpha} &= R_{\beta\gamma}^{\alpha} + \frac{1}{n+1} \delta_{\beta}^{\alpha} c_{\gamma\delta} + \frac{1}{n-r} (\delta_{\beta}^{\alpha} b_{\gamma\delta} + \delta_{\gamma}^{\alpha} b_{\beta\delta}), \\
 R_{\beta\gamma}^{\alpha\delta} &= R_{\beta\gamma}^{\alpha\delta} + \delta_{\beta}^{\alpha} a_{\gamma\delta} + \frac{1}{n+1} \delta_{\beta}^{\alpha} c_{\gamma\delta}^r
 \end{aligned}$$

Of the conditions (61) we retain the following:

$$(67) \quad V_{\alpha\beta\gamma}^j = 0, \quad V_{kjl}^{\alpha\beta} = 0.$$

Then we must have

$$C_{jr} = -\frac{n-r+1}{2(n-r)} b_{rj}, \quad c_{jl}^r = a_{jl}^r$$

Under these conditions we get

$$R_{\beta jr}^{\alpha\gamma} = R_{\beta jr}^{\alpha\gamma} + \frac{r+2}{2(n-r)} b_{rj},$$

$$R_{\beta jr}^{\alpha\gamma} = R_{\beta jr}^{\alpha\gamma} + (1 + \frac{r}{n-r+1}) a_{jk}^{\alpha}.$$

We can therefore choose  $b_{rj}, a_{jk}^{\alpha}$  such that

$$R_{\alpha\beta\gamma}^{\alpha} = 0, \quad R_{\alpha\beta\gamma}^{\alpha} + R_{\alpha\beta\gamma}^{\alpha} = 0,$$

by which  $b_{rj}, a_{jk}^{\alpha}$  are completely determined. It follows that by the conditions

$$(68) \quad R_{\beta jr}^{\alpha\gamma} = 0, \quad R_{\beta jr}^{\alpha\gamma} + R_{\beta jr}^{\alpha\gamma} = 0,$$

the Pfaffian forms  $\varphi_j^{\alpha}, \varphi_{\alpha}$  are completely determined, while  $\theta_k$  is determined up to the transformation

$$(69) \quad \theta_k = \theta_k^* + C_{kj} \omega^j$$

The Pfaffian forms  $\varphi_j^{\alpha}, \varphi_{\alpha}$  thus determined are invariant and are called the invariant Pfaffian forms of the third order.

We shall proceed to determine  $\theta_k$ . For this purpose we put

$$(70) \quad \Phi_j^{\alpha} = -\frac{1}{n-r} R_{\beta\gamma j}^{\alpha} [\varphi_{\beta}^{\gamma} \omega^{\beta}] + R_{\beta jr}^{\alpha} [\omega^{\gamma} \omega^{\beta}] + R_{\beta jr}^{\alpha} [\omega^{\beta} \omega^{\gamma}]$$

$$+ R_{\beta jr}^{\alpha} [\omega^{\beta} \omega^{\gamma}] + [\theta_{jr}^{\alpha} \omega^k],$$

where  $\theta_{jr}^{\alpha}$  are Pfaffian forms satisfying the conditions

$$(71) \quad [\theta_{jr}^{\alpha} \omega^i \omega^k] = 0.$$

The expressions for  $\theta_{jr}^{\alpha}$  can be determined from the conditions

(67), (68). Before doing this, let us put further

$$(72) \quad \begin{cases} \Phi_{\alpha} = \Phi_{\alpha}' - [\Phi_{\beta} \Phi_{\alpha}^{\beta}] + \frac{n-r}{n-r+1} [\omega_{\alpha}^i \theta_i], \\ \theta_i = \theta_i' - [\theta_j \Phi_{\alpha}^j] - \frac{n-r+1}{n-r} [\Phi_{\alpha}^i \Phi_{\alpha}]. \end{cases}$$

Then the exterior differentiation of the fourth and the fifth equations of (63) will give

$$-(\Phi_{\beta}^{\alpha})' + \frac{1}{n-r} [\Phi_{\beta} \omega^{\alpha}] + [\Phi_{\beta}^{\alpha} \Phi_{\beta}^{\alpha}] - [\Phi_{\beta}^{\alpha} \Phi_{\beta}^{\alpha}] + [\Phi_{\beta}^{\alpha} \Phi_{\beta}^{\alpha}] - [\Phi_{\beta}^{\alpha} \Phi_{\beta}^{\alpha}]$$

$$- \delta_{\beta}^{\alpha} \left( \frac{1}{n-r+1} [\theta_k \omega^k] - \frac{1}{n-r} [\omega^{\gamma} \Phi_{\beta}^{\gamma}] \right) = 0,$$

$$-(\Phi_j^i)' - \frac{1}{n-r+1} [\theta_j \omega^i] - [\Phi_j^i \omega^{\alpha}] + [\Phi_j^i \omega^{\alpha}] + [\Phi_j^i \omega^{\alpha}] - [\Phi_j^i \omega^{\alpha}]$$

$$- \delta_j^i \left( \frac{1}{n-r+1} [\theta_k \omega^k] - \frac{1}{n-r} [\omega^{\alpha} \Phi_{\alpha}] \right) = 0.$$

Contracting  $\alpha$  and  $\beta$  in the first equation, we get

$$(73) \quad \frac{1}{n-r+1} \theta_j + [(dR_{\beta jr}^{\alpha} - R_{\beta jr}^{\alpha} \Phi_{\beta}^{\alpha} + R_{\beta jr}^{\alpha} \Phi_{\beta}^{\alpha} + R_{\beta jr}^{\alpha} \Phi_{\beta}^{\alpha}) \omega_j^k]$$

$$+ [(C_{\beta jr}^{\alpha} \Phi_{\beta}^{\alpha} + R_{\beta jr}^{\alpha} \Phi_{\beta}^{\alpha}) \omega_j^k] + [\theta_{kj}^{\alpha} \omega_{\beta}^k] \equiv 0, \text{ mod. } \omega^{\alpha}, \omega^i.$$

On the other hand, the second equation gives

$$\frac{1}{n-r+1} (\delta_j^i \Theta_k + \delta_k^i \Theta_j) + [dV_{jkl}^{ir} - V_{jkl}^{ir} \phi_j^r - V_{jkl}^{mr} \phi_m^i + V_{mkl}^{ir} \phi_j^m + V_{jml}^{ir} \phi_k^m + V_{jkm}^{ir} \phi_l^m] \omega_j^r - [Q_{jkl}^{ir} \phi_j^i + Q_{jyl}^{ir} \phi_k^i] \omega_j^r + [\omega_k^i \Theta_{jk}^i] \equiv 0, \text{ mod } \omega^\alpha, \omega^i.$$

Contracting the indices  $i$  and  $k$ , we have

$$(74) \quad \Theta_j^i - Q_{jkl}^{kr} [\phi_k^r \omega_j^i] - [\Theta_{jk}^i \omega_j^i] \equiv 0, \text{ mod } \omega^\alpha, \omega^i.$$

It therefore follows that

$$[dR_{ijk}^{pr} - R_{ijk}^{pr} \phi_j^r + R_{ijk}^{pr} \phi_k^l + R_{ijk}^{pr} \phi_j^l + Q_{ijk}^{lr} \phi_l^p + R_{ijk}^{pr} \phi_j^p + \frac{r}{n-r+1} Q_{ijk}^{lr} \phi_l^p + \Theta_{jk}^i + \frac{r}{n-r+1} \Theta_{jk}^i] \omega_j^p \equiv 0, \text{ mod } \omega^\alpha, \omega^i,$$

which in turn gives

$$dR_{ijk}^{pr} - R_{ijk}^{pr} \phi_j^r + R_{ijk}^{pr} \phi_k^l + R_{ijk}^{pr} \phi_j^l + Q_{ijk}^{lr} \phi_l^p + R_{ijk}^{pr} \phi_j^p + \frac{r}{n-r+1} Q_{ijk}^{lr} \phi_l^p + \Theta_{jk}^i + \frac{r}{n-r+1} \Theta_{jk}^i \equiv 0, \text{ mod } \omega^\alpha, \omega^i, \omega^\alpha.$$

Because of the conditions (68) we get

$$\Theta_{jk}^i + \Theta_{kj}^i + (Q_{ijk}^{lr} + Q_{ljk}^{ir}) \phi_l^p + \frac{n-r+1}{n+1} (R_{ijk}^{pr} \phi_j^p + R_{ijk}^{pr} \phi_k^p) \equiv 0, \text{ mod } \omega^\alpha, \omega^i, \omega^\alpha.$$

From (71) we have

$$\Theta_{jk}^i - \Theta_{kj}^i \equiv 0, \text{ mod } \omega^i.$$

Combining these two equations, we can put

$$(75) \quad \Theta_{jk}^i = -\frac{1}{2} (Q_{ijk}^{lr} + Q_{ljk}^{ir}) \phi_l^p - \frac{n-r+1}{2(n+1)} (R_{ijk}^{pr} \phi_j^p + R_{ijk}^{pr} \phi_k^p) + T_{jka}^i \omega^\alpha + T_{jkl}^i \omega^l + T_{jki}^i \omega^\alpha,$$

with

$$T_{jka}^i = T_{kja}^i, \quad T_{jki}^i = T_{kji}^i.$$

Substituting (75) into (70), we get

$$(76) \quad \Phi_j^\alpha = -\frac{1}{n+1} Q_{ijk}^{\alpha r} [\phi_k^r \omega_j^\alpha] - \frac{1}{2} (Q_{ijk}^{\alpha l} + Q_{ljk}^{\alpha i}) [\phi_l^p \omega_j^\alpha] - \frac{n-r+1}{2(n+1)} (R_{ijk}^{\alpha p} [\phi_j^p \omega_k^\alpha] + R_{ijk}^{\alpha p} [\phi_k^p \omega_j^\alpha]) + R_{ijk}^{\alpha p} [\omega_j^\alpha \omega_k^\beta] + R_{ijk}^{\alpha p} [\omega_j^\alpha \omega_k^\beta] + T_{jka}^{\alpha} [\omega_j^\alpha \omega_k^\beta] + T_{jkl}^{\alpha} [\omega_j^\alpha \omega_k^\beta] + T_{jki}^{\alpha\beta} [\omega_j^\alpha \omega_k^\beta],$$

where, owing to a change of notation, the quantities  $T$  are only subjected to the conditions

$$(77) \quad T_{jki}^{\alpha\beta} = T_{kji}^{\alpha\beta}.$$

When the transformation (69) is applied, the effect on the coefficients  $T_{jki}^{\alpha\beta}$  is given by

$$T_{jki}^{\alpha\beta*} = T_{jki}^{\alpha\beta} - \frac{1}{n-r+1} \delta_\beta^\alpha c_{jkr}.$$

It follows that by the conditions

$$(78) \quad T_{jkr}^{\alpha\beta} = 0.$$

the quantities  $c_{jkr}$  and hence the Pfaffian forms  $\Theta_k$  are completely determined. The Pfaffian forms  $\Theta_k$  thus obtained are



invariant and are called the invariant Pfaffian forms of the fourth order.

It is interesting to remark that for  $r=1$  and  $r=n-1$  it is sufficient for our purpose to introduce the invariant Pfaffian forms up to the third order inclusive. In order that the results hold for a general value of  $r$ , however, those of the fourth order are necessary.

§6. Solution of the problem of equivalence and geometrical interpretation

Concluding the results of the above five sections we see that we have arrived at a situation by which we have  $n(n+2)$  variables  $x^{\alpha}, x^i, u_{\beta}^{\alpha}, u_j^i, u_j^{\alpha}, u_{\alpha}, a_{\beta}$  and the same number of linearly independent invariant Pfaffian forms  $\omega^{\alpha}, \omega^i, \omega_{\alpha}^i, \varphi_{\beta}^{\alpha}, \varphi_j^i, \varphi_j^{\alpha}, \varphi_{\alpha}, \theta_k$ . In order that two families of  $r$ -spreads are equivalent, i.e., that there exists a point transformation which carries the spreads of the one family to those of the other family, it is necessary and sufficient that the corresponding invariant Pfaffian forms are equal under the transformation. When the exterior derivatives of the invariant Pfaffian forms are expressed as exterior quadratic forms of themselves, the coefficients which enter in these expressions are invariants. Thus the quantities  $Q, R, V, T$  which enter in  $\Omega_{\alpha}^i, \Phi_{\beta}^{\alpha}, \Phi_j^i, \Phi_j^{\alpha}$  are invariants. Further invariants are given by the coefficients in the expressions for  $\Phi_{\alpha}, \Theta_k$ . It is well-known that a complete system of invariants is given by these invariants and their covariant derivatives. In this way, the problem of equivalence is completely solved.

A particular case of our family of  $r$ -spreads is given by the family of linear spaces. In this case, we easily find

$$(79) \quad \Omega_{\alpha}^i = \Phi_{\beta}^{\alpha} = \Phi_j^i = \Phi_j^{\alpha} = \Phi_{\alpha} = \Theta_k = 0.$$

The invariant Pfaffian forms become the relative components of the group of projective transformations in a space of  $n$  dimensions. By a process which we have repeatedly applied, we can utilize the invariant Pfaffian forms in the general case to define a generalized projective geometry or a projective connection in the space. We state this result in the follow-

ing fundamental theorem:

Let there be given in a space of  $n$  dimensions a family of  $r$ -spreads depending on  $(n-r)(r+1)$  parameters such that tangent to an  $r$ -dimensional element in the space there passes one and only one  $r$ -spread of the family. Then an invariant projective connection can be defined in the space.

(IAS) CHERN

*Eng. Morse ✓*  
*v. Rummann ✓*  
*Vollen ✓*  
*Weyl ✓*

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May 29, 1942

Mr. Frank Aydelotte, Director  
The Institute for Advanced Study  
Princeton, New Jersey

Dear Sir:

In the absence of the Ambassador, I wish to acknowledge the receipt of your letter of May 23rd, and to assure you that it will be brought to his attention upon his return to Washington next week.

Very truly yours,

LOTA LOIS ING (?)

Private Secretary

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Prof. Alexander

(925) Chern

Copy for Miss Blake

Existence ✓  
Worse ✓  
v. human ✓  
Vahlen ✓  
Weyl ✓

May 23, 1942

His Excellency The Chinese Ambassador  
The Chinese Embassy  
Washington, D. C.

My dear Dr. Hu Shih:

I take the liberty of writing to you about a very eminent Chinese mathematician, who is eager to come to Princeton to work at the Institute for Advanced Study. This is Professor Shing-shen Chern, whose academic career is summarized in the enclosed statement. Professor Chern's publications, which have been read by members of our School of Mathematics, indicate that he is one of the most promising mathematicians of this generation.

It seems to us quite clear that Professor Chern himself and Chinese mathematics in general would be advanced if it could be made possible for him to come to the Institute for a year or, still better, for two. At the end of the war Professor Chern could be immensely useful in his own professorship at National Tsing Hua University and indeed throughout all the universities of China.

We should be most happy to have Professor Chern as a member of the Institute if it could be made possible for him to come. I regret to say that we have no funds out of which we could offer to defray the cost of his travel or of his expenses in Princeton. I am writing you in the hope that it may be possible for the Chinese government to make Professor Chern's visit to Princeton financially possible. His own high qualifications and the importance of the subject would seem to us to justify the Chinese government in undertaking this expense.

With kindest regards, I am

Yours sincerely,

FA/MCE

FRANK AYDELOTTE, Director

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(925) Chern

May 23/42

This reminds me of another project which I have thought of. There is a Chinese mathematician, Professor Shing-shen Chern, who seems to all of us here to be probably the best that has yet been produced in China. He would like to come to the Institute. But we don't see how to manage it either from the point of view of funds, or from the point of view of the other arrangements that would be needed. If any suggestion should occur to you, I should be very glad to have it.

Yours sincerely,

Oswald Veblen

Dean R.G.D. Richardson  
Brown University  
Providence, R.I.  
OV:GB

Copy sent Prof. Neugebauer

Prof. Heblin ✓  
Prof. Alexander ✓  
Austin ✓  
Morse ✓  
Sturges ✓  
Woyt ✓

(925) Chern

913 -

THE ROCKEFELLER FOUNDATION

49 West 49th Street

New York

April 30, 1942

Dear Mr. Aydelotte:

In my letter to you of April 10th I remarked that we would give the question of bringing Chern to this country further thought and that I would communicate with you again.

I am writing now to say that at a conference of the officers having to do with refugee scholars it was decided that the Foundation would not undertake to bring a scholar from China at this time.

Cordially yours,

(Signed) Frank Blair Hanson

Mr. Frank Aydelotte, Director  
The Institute for Advanced Study  
Princeton, New Jersey

FBH:CR

COPY

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(905) Chern

April 22, 1942

Dear Frank:

The facts concerning Professor Shiing-shen Chern are about as follows:

He wrote me on May 8, 1941, asking whether he could get a stipend in the Institute in 1942. With his letter he enclosed an account of his researches, and a paper which he offered for publication in one of the American mathematical journals. The paper struck me as being extremely good, and the referee's report pronounces it first class. His work altogether seems to establish that Chern is the most promising Chinese mathematician who has thus far come to our attention. Our mathematical group would therefore, except for the difficult situation in which we find ourselves, have recommended him for a stipend from the Institute.

In the present circumstances our recommendation is that an attempt should be made to bring him to the Institute for a couple of years. The problem of bringing him here and of returning him to China at the end of his period might be referred to the Chinese Embassy in Washington, and the funds might be sought from one of the Foundations. We feel that Chern seems to be a man of such unusual quality, and the need of China for the development of such men so immediate, that there should be a good chance of carrying out a program of this sort.

I enclose a short outline of Dr. Chern's career, which he supplied himself, and may add that he is now a professor at the National Tsing Hua University, which was moved from Peiping to Kunming. Where it is now I don't know.

Sincerely,

Oswald Veblen

Dr. Frank Aydelotte  
OV:GB

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Records of the Society for Mathematical Research, Princeton, NJ, USA  
From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

C O P Y

My Academic Career

Shing-shen Chern

I was born on October 26th, 1911, in Kashing, Chekiang Province, China. After graduating from the Fulun Middle School, Tientsin, I entered the Nankai University, Tientsin, in 1926 and obtained my B.Sc. degree in 1930. In 1930-31 I was assistant in the Department of Mathematics at National Tsing Hua University in Peiping. In 1931-34 I was studying in the Graduate School of Tsing Hua University, and completed my work with distinction in 1934. I was then granted a fellowship by the University to study abroad for three years. I studied from 1934 to 1936 in Hamburg, Germany, under Professor W. Blaschke and received my degree of "Doktor der Naturwissenschaften" in February 1936. In 1936-37 I studied in Paris under the direction of Professor Elie Cartan. In 1937 I came back to China to be Professor of Mathematics at National Tsing Hua University, which position I am still holding.



(IAS) ~~STIEGEL~~  
~~GÖDEL~~  
~~CHERN~~  
~~TARSKI~~

96  
7  
F.AYDELOTTE TO F.B.HANSON Apr.8/46 SEE (IAS) AYDELOTTE

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(925) *Quinn*

April 8, 1942

Dr. Aydelotte:

I have checked up with Alexander, Morse, von Neumann and Weyl. They are all in favor of your going ahead and making the best arrangement that you can with the Rockefeller Foundation about the stipends for Siegel and Gödel.

We also think that it would be very desirable to do something about the case of Dr. Chern, who appears to us to be the most promising Chinese mathematician who has appeared on the horizon.

It also occurs to me that it would be desirable if you could talk over the case of Tarski with Dr. Hanson on the basis that with a little supplementary help from the Foundation he would be taken care of at Berkeley for the duration of the war.

O.V.

THE INSTITUTE FOR ADVANCED STUDY

SCHOOL OF MATHEMATICS, ~~PINE HALL~~, PRINCETON, N.J.

Application for Stipend

Full name Shiing-shen Chern Date of birth Oct. 26, 1911.  
 Present address P.O. Box 96, Kunming, China. Place of birth Kashing, Chekiang, China.  
 Citizenship Chinese  
 Permanent address P.O. Box 96, Kunming, China. Married or unmarried? Married  
 Children? One child

Degrees received when and where?

B.A. 1930, Nankai University, Tientsin, China.

M.A. \_\_\_\_\_

Ph.D. 1936, University of Hamburg, Germany.

Major subject Mathematics

Minor subjects Physics and astronomy

Have you held any fellowships or scholarships before? - Which, when, and for study where?

Fellowship of National Tsing Hua University, 1934-37, for study in the University of Hamburg, Germany, 1934-36, and for study in the University of Paris, France, 1936-37.

Former and present teaching positions Professor of Mathematics, National Tsing Hua University, China, 1937-.

Recommenders (preferably professors under whom candidate has studied, whom applicant ask to send directly to the School of Mathematics confidential information about the candidate's character and scientific abilities):

I have asked Professor E. Cartan and Professor W. Blaschke to send you informations about me; but, in view of the present situation, I am not sure whether their letters of recommendation can be duly received.

(Over)

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Recd. July 14  
 Adv. By air by hand

Thesis and other published papers (give titles and exact references; and send copies if possible):

See accompanying list. Copies are sent to you under separate cover.

Outline (in not more than 200 words) previous and intended research:

Previous research:

1. Projective differential geometry (Papers 1,2,3,19,20)  
The most important contribution in this field is the generalization of the Laplace transformation of conjugate nets to that of a class of  $p$ -dimensional varieties in  $n$ -dimensional projective space (19).
2. Differential geometry in symplectic space (Paper 18)  
Theory of curves and hypersurfaces.
3. Topological differential geometry (Papers 4,5).
4. Theory of generalized spaces or Non-Riemannian Geometry (Papers 6,7,8,9,10,11,13,21)  
Possibility of the immersion of a projectively connected space in ~~that of~~ a projectively flat space of higher dimension. Geometry of an ordinary differential equation of the third order. Geometry of higher path-spaces. Geometry of isotropic surfaces.
5. Integral geometry (Papers 12,14,15,16,17)  
Kinematic principal formula in  $n$ -dimensional Euclidean space. Integral geometry in any Klein space, in particular the generalizations of Crofton's formula and Cauchy's formula. The intrinsic parameter of G. Pick of a curve enters into these formulas.

Intended research:

1. Generalization of the formula of Gauss-Bonnet. The formula of Gauss-Bonnet is simply an "integral form" of the equation  $\omega'_2 = K[\omega_1, \omega_2]$  of Cartan for the theory of surfaces. By this observation some formulas analogous to that of Gauss-Bonnet (for  $n$ -dimensional Riemannian geometry and for some other generalized geometry) may be derived.
2. Differential geometry in the large. Topology of homogeneous spaces.
3. Geometry of a space of  $n$  dimensions with an  $m$ -parameter family of  $p$ -dimensional varieties. (Generalization of the geometry of paths)

Ans. 17 Aug.  
(903) Chern

June 28, 1941.

Dear Professor Veblen:

Your letter of June 2, 1941, was duly received. I thank you very much for your kind interest and informations concerning my application for a stipend from your Institute.

Following your advice, I enclose herewith a form of application; at the same time I am sending you from Shanghai some of the reprints of my mathematical papers. I think that a letter of recommendation directly addressed to you either from Professor Cartan or from Professor Blaschke might be of some help to my case; so I am writing to them to that effect, but, under the present conditions, the correspondence might be interrupted anywhere in its course.

With best regards and thanks,

I am

Yours sincerely

*S. S. Chern*

(Shiing-shen Chern)

Please notice, my address is:

Dr. Shiing-shen Chern

P.O.Box 96

Kunming, China

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List of Mathematical Papers by Shiing-shen Chern.

1. Pairs of Plane Curves with Points in One-to-one Correspondence, Science Reports of Tsing Hua University, Series A, vol.1(1932), pp.145-153.
2. Triads of Congruences with Generators in Correspondence, Tôhoku Mathematical Journal, vol.40(1934), pp.179-188.
3. Associate Quadratic Complexes of a Rectilinear Congruence, Tôhoku Mathematical Journal, vol.40(1934), pp.293-316.
4. Abzählungen für Gewebe, Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, Bd.11(1935), S.163-170.
5. Eine Invariantentheorie der Dreigewebe aus  $r$ -dimensionalen Mannigfaltigkeiten im  $R_{2r}$ , Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, Bd.11(1936), S.333-358.  
*Inaugural Dissertation 1936 Hamburg.*
6. Sur la géométrie d'une équation différentielle du troisième ordre, Comptes Rendus de l'Académie des Sciences à Paris, Tome 204(1937), pp.1227-1229.
7. Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif, Bulletin des Sciences Mathématiques, 2.série, tome 61(1937), pp.234-243.
8. On Projective Normal Coordinates, Annals of Mathematics, vol.39(1938), pp.165-171.
9. On Two Affine Connections, Journal of the University of Yunnan, vol.1(1938), pp.1-19.
10. Sur la géométrie d'un système d'équations différentielles du second ordre, Bulletin des Sciences Mathématiques, 2.série, tome 63(1939), pp.206-212.
11. The Geometry of higher Path-spaces, Journal of the Chinese Mathematical Society, vol.2(1940), pp.247-276.

12. Sur les invariants intégraux en géométrie, Science Reports of Tsing Hua University, Series A, vol.4(1940), pp.85-95.
13. The Geometry of a Differential Equation of the Third Order, Science Reports of Tsing Hua University, Series A, vol.4(1940), pp.97-111.
14. Some Problems in Integral Geometry, Quarterly Journal of Science of Wu-han University, vol.4(1940), pp.1-15.
15. (with Chih-ta Yen), Sur la formule principale cinématique dans l'espace à  $n$  dimensions, Rendiconti R. Accademia dei Lincei.
16. Sur une généralisation d'une formule de Crofton, Comptes Rendus de l'Académie des Sciences à Paris, juin 1940.
17. On the Integral Geometry in Klein Spaces, *submitted to the Annals of Mathematics for publication*.
18. (with Hsien-chung Wang), Differential Geometry in Simplectic Space I, to appear in the Science Reports of Tsing Hua University.
19. Sur une classe remarquable de variétés dans l'espace projectif à  $n$  dimensions, to appear in the Science Reports of Tsing Hua University.
20. Sur les invariants de contact dans la géométrie projective différentielle, to appear in Rendiconti R. Accademia dei Lincei.
21. The Geometry of Isotropic Surfaces, *submitted to the Annals of Mathematics for publication*.
22. On Projectively Flat Spaces. Unpublished.

(IAS) SHIING SHEN CHERN

Reprints 7AS reprint file  
other

Prof. Veblen has Chern reprints in his file. (June 23/41)



seen by Prof. Hays ✓  
H. H. White ✓ (9AS), Chern  
Horse ✓

Air mail

June 2, 1941

Dear Professor Chern:

I have handed your paper on "The Geometry of Isotropic Surfaces" to the editors of the Annals of Mathematics.

I am also very much interested in your desire to work at our Institute. Personally I hope very much that you will be able to do this. It will not, however, be possible to say anything about your chances of obtaining a stipend from the Institute for the academic year 1942-1943 until next autumn. In the meantime I think it would be a very good idea if you would fill out the enclosed form for application. I am also mailing you under separate cover a copy of the latest bulletin of the Institute.

With best wishes, and the hope that this letter will reach you as quickly as yours did me (less than two weeks!),

Yours sincerely,

Dr. Shiing-shen Chern  
P.O. Box 96  
Kunming, China  
OV:GB

Oswald Veblen

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From The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA

925  
see app file

925  
(OV) S.S. Chern

May 8th, 1941.

sent Vandenberg  
May 23/41

Dear Professor Veblen:

I beg to send you herewith a new paper of mine, "The Geometry of Isotropic Surfaces", which, if you think fit, I hope you will send it to an American mathematical journal for publication.

Annals 43(1942) 545

I enclose also, for your attention, a short account of my academic career and my mathematical researches--- I should like to have your advice as to whether, with such preparations, I could get a fellowship to work in the Institute for Advanced Study at Princeton. I returned from France to China in 1937. By that time, the National Tsing Hua University, of which I am a professor, had moved from Peiping to Kunming in South-west China and had lost all its books and equipments. We have endeavored to build up a center for the training of students, and in this work we have fairly succeeded. But owing to the pressure of the war, we can not, at present, afford to build up a mathematical library indispensable for serious research workers.

With such meagre equipments as you could not imagine, I have been pursuing my work for the last four years. In this period I have nevertheless achieved some results which, as my enclosed account will show, may be more significant than those done by me during my years of study in Europe. To continue my advanced work further, I should like to have a chance to work for one or two years in your Institute for Advanced Study at Princeton, and so I should like to know whether the Institute can grant me a fellowship in 1942, as there is now practically no hope of getting any such fellowship from a Chinese institution. Should the fellowship be granted to me, I shall try my best to finance the passage and travelling expenses myself.

As the ordinary mails between America and China are now quite irregular, I may be pardoned if I should

As final proof in 1941... Annals 43, 178

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ask you to send me your reply by air mail (China Clipper).

I send you by ordinary mail some reprints of my papers and I hope you will receive them later.

With thanks in advance,

I remain

Yours sincerely

*S. S. Chern*

*Proq.* (Shiing-shen Chern)

My address is:

Dr. Shiing-shen Chern

P. O. Box No. 96

Kunming, China.

C O P Y

May 8th, 1941

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As the ordinary mails between America and China are now quite irregular, I may be pardoned if I should ask you to send me your reply by air mail (China Clipper).

I send you by ordinary mail some reprints of my papers and I hope you will receive them later.

With thanks in advance,

I remain

Yours sincerely,

S. S. CHERN

*Shing* (Shiing-shen Chern)

My address is:

Dr. Shiing-shen Chern

P.O.Box No. 96

Kunming, China.

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A Short Account of My Scientific Researches  
Shiing-shen Chern

I began my researches on topics of projective differential geometry. In Papers [1], [2], [3]\*, I studied some problems in this field by following Wilczynski's method. Later I made use of Cartan's method of moving frames and gave in Paper [20] a simple geometrical treatment of the problem of invariants of contact studied by C. Segre and B. Segre, and proved some new theorems. In Paper [19] I indicated a possible generalization of the theory of Laplace transforms of conjugate nets to a class of  $p$ -dimensional varieties in  $n$ -dimensional projective space. For  $p=2$  these varieties are identical with the surfaces sustaining conjugate nets, while in the general case they may be characterized geometrically as follows: Let  $A$  be a point on the variety  $V_p$ ,  $S_p$  the tangent  $p$ -flat of  $V_p$  at  $A$ ,  $S_{p+q}$  the ambient space containing all osculating 2-flats at  $A$  of curves on  $V_p$  through  $A$ . It is well-known that, if  $S_{p+q-1}$  is a  $(p+q-1)$ -flat in  $S_{p+q}$  through  $A$ , the tangents to the curves whose osculating 2-flats belong to  $S_{p+q-1}$  form a quadric cone having an equation of the form

$$\lambda_{p+1} \Phi_{p+1} + \dots + \lambda_{p+q} \Phi_{p+q} = 0,$$

$\Phi_{p+1}, \dots, \Phi_{p+q}$  being quadratic differential forms. Our varieties in question are then characterized by the property that  $\Phi_{p+1}, \dots, \Phi_{p+q}$  are perfect squares of linear differential forms. The totality of such varieties depends on  $p(p-1)$  arbitrary functions in two variables. Every such variety has  $p(p-1)$  transforms, which are varieties of

\* The titles of these papers and the journals in which they appeared are given in the list at the end of this account.

the same nature. The whole theory of Laplace transforms and Laplace sequences of surfaces may thus be extended.

In Paper [18] I also applied the method of moving frames to study the differential geometry in the so-called "symplectic" space in which the group of transformations is the symplectic group (by using a term of Weyl).

The set of Papers [6], [7], [8], [9], [10], [11], [13], [21] deals with the theory of generalized spaces (or Non-Riemannian Geometry). In Papers [6], [13] I gave the definition of a normal conformal connection in the plane in which there is given a differential equation of the third order possessing a certain important property, the definition being invariant under the group of contact transformations. The general case has also been studied. It leads to a geometry having as fundamental group a certain five-parameter group. In Papers [10], [11] I studied the geometry in an  $n$ -dimensional space in which there is given a system of differential equations of the  $r$ -th order. This problem was proposed by D.D.Kosambi and studied by many others. I showed that, if there is a set of absolute parameters on the curves, the space is affinely connected. In Paper [9] I gave the definition of two affine connections in a four-dimensional space arising from two geometrical objects of a particular type. Paper [7] gives a theorem on the possibility of the immersion of projectively connected spaces in a projective space of higher dimension, thus generalizing the theorem of Schläfli-Cartan concerning the immersion of Riemann spaces in an Euclidean space. In Paper [8] I gave an analytic characterization of the projective normal coordinates in the sense of Cartan and showed that these coordinates are different from those of Veblen-Thomas. In Paper [21] I showed that a Weyl connection can be defined in the three-dimensional space in which there is given a

two-parameter family of surfaces.

Papers [4], [5] are concerned with the topological differential geometry of Blaschke. In Paper [4] I determined the maximum rank of an  $n$ -web of hypersurfaces in a space of  $N$  dimensions, a problem which is related to algebraic geometry. In Paper [5] I gave a theory of differential invariants of a three-web of  $r$ -dimensional varieties in a  $2r$ -dimensional space and determined in particular the conditions under which certain configurations formed by varieties of the web hold.

In Papers [12], [14], [15], [16], [17] I extended some of the results of Blaschke and Santaló on integral geometry in Euclidean space to any space of Klein in which there is given an arbitrary transitive Lie group of transformations. In Paper [15] I gave the kinematic principal formula in  $n$ -dimensional Euclidean space, while in Paper [12] I studied the integral geometry in Möbius plane (plane with the six-parameter Möbius group of transformations) and in the affine plane. Papers [14], [16], [17] deal with the integral geometry in a general Klein space. Aside from some fundamental notions there are given in these papers the generalizations of the formulas of Crofton and Cauchy. These formulas are proved in their most general form, in which the intrinsic parameter of G.Pick (parameter which generalizes the affine arc, the projective arc, etc.) plays an essential rôle. By the notions so introduced many questions in this direction can be further treated.

In Paper [22] I attempted to study the topology of homogeneous spaces, in particular, the problems generalizing the Clifford-Klein Space Problem. I showed that, under certain conditions, the only two-dimensional spaces on which there can be defined a locally flat projective connection are the projective plane and the sphere.

- List of Mathematical Papers Published by Shiing-shen Chern.
1. Pairs of Plane Curves with Points in One-to-one Correspondance, Science Reports of Tsing Hua University, Series A, vol.1(1932),pp.145-153.
  2. Triads of Congruences with Generators in Correspondence, Tohoku Mathematical Journal, vol.40(1934),pp.179-188.
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  4. Abzählungen für Gewebe, Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, Bd.11(1935), S.163-170.
  5. Eine Invariantentheorie der Dreigewebe aus r-dimensionalen Mannigfaltigkeiten im  $R_{r,r}$ , Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, Bd.11(1936), S.333-358.
  6. Sur la géométrie d'une équation différentielle du troisième ordre, Comptes Rendus de l'Académie des Sciences à Paris, Tome 204(1937),pp.1227-1229.
  7. Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif, Bulletin des Sciences Mathématiques, 2.série, tome 61(1937),pp.234-243.
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  9. On Two Affine Connections, Journal of The University of Yunnan, vol.1(1938),pp.1-19.
  10. Sur la géométrie d'un système d'équations différentielles du second ordre, Bulletin des Sciences Mathématiques, 2.série, tome 63(1939),pp.206-212.
  11. The Geometry of Higher Path-spaces, Journal of the Chinese Mathematical Society, vol.2(1940),pp.247-276.
  12. Sur les invariants intégraux en géométrie, Science Reports of Tsing Hua University, Series A, vol.4(1940),pp.85-95.
  13. The Geometry of a Differential Equation of the Third Order, Science Reports of Tsing Hua University, Series A, vol.4(1940),pp.97-111.



14. Some Problems in Integral Geometry, Quarterly Journal of Science of Wu-han University.
15. (with Chih-ta Yen), Sur la formule principale cinématique dans l'espace à  $n$  dimensions, Rendiconti R. Accademia dei Lincei.
16. Sur une généralisation d'une formule de Crofton, Comptes Rendus de l'Académie des Sciences à Paris, juin 1940.
17. On the Integral Geometry in Klein Spaces, *Submitted to the Annals of Mathematics.*
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21. The Geometry of Isotropic Surfaces. Unpublished.
22. On Projectively Flat Spaces. Unpublished.

My Academic Career  
Shiing-shen Chern

I was born on October 26th, 1911, in Kashing, Chekiang Province, China. After graduating from the Fulun Middle School, Tientsin, I entered the Nankai University, Tientsin, in 1926 and obtained my B.Sc. degree in 1930. In 1930-31 I was assistant in the Department of Mathematics at National Tsing Hua University in Peiping. In 1931-34 I was studying in the Graduate School of Tsing Hua University, and completed my work with distinction in 1934. I was then granted a fellowship by the University to study abroad for three years. I studied from 1934 to 1936 in Hamburg, Germany, under Professor W. Blaschke and received my degree of "Doktor der Naturwissenschaften" in February 1936. In 1936-37 I studied in Paris under the direction of Professor Elie Cartan. In 1937 I came back to China to be Professor of Mathematics at National Tsing Hua University, which position I am still holding.