

UPPSALA UNIVERSITETS  
MATEMATISKA INSTITUTION

*Circulated to  
Mats  
5/10/52*

May 12, 1952.

Dear Dr. Oppenheimer:

It was a great pleasure to receive your letter of April 21. I appreciate very deeply indeed the generous consideration which the Institute has shown in regard to my desire for a longer stay in Princeton.

As I review my situation in Uppsala and the prospects at the Institute, the following arrangements suggests itself. If it is convenient for you, I should like to accept now an appointment to run for two years: sept. 1952- May 1954. If, at the end of the first year, there is mutual satisfaction on the Institute's and my part with my work and teaching, I will avail myself of your generous offer for a longer period within the terms of your letter. Thus we shall have a year in which to see how my work fits into the Institute's program.

I have always considered it a great favour for any scientist to work at the Institute for Advanced Study; and I sincerely hope that I shall be able to make a contribution to the mathematical work of the Institute.

Yours very sincerely

*Arne Beurling*

Copy to Mr. Fleming  
Miss Trinneer  
Miss Underwood

April 21, 1952

Dear Professor Beurling:

For some time we have known from Selberg that it might be convenient for you if your appointment at the Institute could be extended somewhat beyond next year. This matter has been discussed with the Faculty and with the Trustees of the Institute; and I write to you with their concurrence.

We would be happy to extend your appointment for some years, up to five, for as long as you can arrange to be here on leave from the University of Uppsala. We did not ourselves feel able to judge whether that could be two, or three, or five years, and must leave it to you to make the arrangements. We have set aside \$12,000 a year for each of the years that you can be with us, and needless to say, look forward with pleasure and anticipation to your being here. As soon as I hear from you as to the period for which you would like this appointment to run, I shall write you again in more definite terms.

With every cordial wish,

Robert Oppenheimer

Professor A. K. Beurling  
Department of Mathematics  
University of Uppsala  
Uppsala, Sweden

AGENDA ITEM

Meeting of the  
Board of Trustees  
Friday, April 18, 1952

At a stated meeting on April 2, 1952 of the Faculty of the Institute for Advanced Study, the Faculty, upon motion of the School of Mathematics, approved, without dissent, the following votes:

1. (a) That the Faculty recommends, as one step, among others, to strengthen the permanent staff in theoretical physics, that George Placzek and Chen Ning Yang, now holding five-year memberships, be made Permanent Members of the School of Mathematics;
  - (b) That they each receive \$10,000 a year; and, with their concurrence, a joint 5 per cent - 5 per cent contribution to a retirement policy;
  - (c) That, in the case of Yang, there be no commitment at the present time as to whether he will or will not at a later date be offered a Professorship at the Institute; that this issue be determined by the progress of his own work, by the general situation in physics at the Institute, and by the relative merits of competing available candidates for such a Professorship; but that the appointment as Permanent Member has taken into account a substantial probability that, in the light of these conditions, he will in fact be offered a Faculty appointment at a later date.
  - (d) That, in the case of Placzek, it is not at this time anticipated that he will be offered a Professorship at the Institute; that this will not occur unless circumstances now unanticipated supervene.
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2. That the Faculty recommends **an** extension of Professor Beurling's appointment for a period of up to five years, but in no event longer than the period for which Beurling could obtain leave of absence from the University of Uppsala. Professor Beurling should receive \$12,000 a year for the period of his appointment.

Copy to Professor Selberg  
Mr. Fleming  
Miss Trinterud  
Miss Underwood

November 13, 1951

Dear Professor Beurling:

It is with great pleasure that I write to you to invite you to be a Visiting Professor and Member of the Institute for Advanced Study for the academic year 1952-53. This invitation comes to you with the unanimous approval of the Faculty in the School of Mathematics, and with an expression of hope on all our parts that it will be convenient and seem fruitful to you to accept. You know that you have many friends here; and from some of them you will have heard about the work of the Institute. I shall be glad to answer any question that you may want to put to us.

This appointment carries with it a salary of \$12,000 for the year. Our semester opens on October 1st, and ends on April 11th; but of course we earnestly hope that you will be in residence for a longer period. In any case you will find many colleagues eager to learn of your work and to talk with you.

With every good wish,

Robert Oppenheimer

Professor A. K. A. Beurling  
University of Uppsala  
Uppsala, Sweden

CC: Professors Montgomery, Morse, Oppenheimer, Pais, Selberg, von Neuman

Uppsala, November 10, 1951

[Translation by Professor Selberg of letter addressed to him.]

Dear Atle:

As I earlier communicated to you in my letter of February 10 it would be possible for me to come to Princeton in Fall 1952. During the time that has elapsed since we corresponded about this, I have only become more convinced that I should accept the offer of the Institute, and I look really forward to this opportunity for undisturbed work, which one cannot get here.

What I have written above you can consider as official, but besides this I would like to put you some questions privately. The arrangement that you mentioned in your last letter (Membership with ca. \$12,000 for the year, tax-free I understand)\*. I am very satisfied with this if the inflation has not become considerably worse during this year.

Hermann Weyl looked me up this summer and asked about my plans and spoke warmly about the Institute and its many advantages. He put forward in this connection, that the Institute would like to see me stay for a longer time, if I so should wish. In case this remark of Weyl's coincides with the official meaning of the Institute, I on my side would be prepared, without any hesitation to accept an appointment on 4 - 5 years on the above conditions.

The question is of significance, among other things in the way, that I would be reluctant to create for Uppsala University the difficulties that would arise if I started by asking for a leave of absence for one year, and only when that had elapsed gave definitive information about my intentions. In that case I should not be able to help the University in choosing my successor, which is a question of great importance for the remaining mathematicians here.

If I on the other hand could have my economic situation in America secured for 4 - 5 years ahead, I should be able at once to take care of these matters, and also avoid the expense and trouble of going back and get my furniture, etc.

Well, now you know how I look upon the situation, and I should be very grateful if you could investigate the terrain and give me some guiding informations.

(Here follows some irrelevant things, about what he is presently working on, and about a vacant professorship in Norway, where Beurling is on the committee. The letter ends then with the following sentence.)

How it in any case will be with these questions, I look with pleasure forward to spending the academic year 52-53 in Princeton, and to see you again then.

Yours faithfully,

/s/ Arne Beurling

\* This Beurling may have heard from Gårding or Fleijel who both had grants at the times when they were tax-free. A. S.

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December 15, 1951

Dear Professor Beurling:

Thank you for your good letter of November 29th. I have been much away from Princeton, and have only now seen it.

We are delighted at the prospect of your coming next year; and, as you will know from Selberg's letters, are thinking a bit about the future.

As for Hirschman, we hope that he will be successful in one of his fellowship applications. In that case, of course we shall admit him to membership at the Institute. Even if he is not successful, we feel that we should give him a grant here, not least because of your interest in having him work with you.

With every warm good wish,

Robert Oppenheimer

Professor A. Beurling  
Institute of Mathematics  
Uppsala University  
Uppsala, Sweden

Copy to School of Mathematics

THE INSTITUTE FOR ADVANCED STUDY  
PRINCETON, NEW JERSEY

SCHOOL OF MATHEMATICS

December 12, 1951

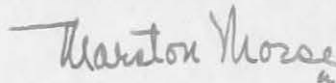
Dr. Robert Oppenheimer  
Institute for Advanced Study  
Princeton, New Jersey

Dear Robert:

I have seen the copy of the letter from Beurling to you in which he refers to I. Hirschman, Jr., whom he has strongly recommended for a Guggenheim Fellowship. Hirschman has also applied for a Fulbright Grant and I would say he stands about a forty percent chance of getting it. He will undoubtedly apply for other grants as he has in the past.

I would recommend telling Beurling that if he does not receive some one of the several fellowships or grants-in-aid for which he has applied, the Institute will take care of him. I would underwrite him only in this conditional sense, since he will probably obtain aid elsewhere.

Sincerely yours,



Marston Morse

MMcdu

CC: D. Montgomery; A. Selberg

copy to Matt  
for Circ.

December 6, 1951

Dear Professor Beurling:

Your letter of November 29th has been received in Dr. Oppenheimer's absence from Princeton. Dr. Oppenheimer will be here briefly on the 9th; and you may be sure your letter will be brought to his attention. I know how pleased he will be to see it; and I know that he will be glad to raise the question of a membership for Dr. Hirschman with the Faculty of the School of Mathematics.

Sincerely yours,

Katherine Russell,  
Secretary to the Director

Professor A. Beurling  
Institute of Mathematics  
Uppsala University  
Uppsala, Sweden



UPPSALA UNIVERSITETS  
MATEMATISKA INSTITUTION

Uppsala Nov. 29, 1951

Doctor R. Oppenheimer  
The Institute for Advanced Study  
Princeton, New Jersey.

Dear Sir:

A few days ago, I received your kind letter with the invitation from the Institute for Advanced Study for the academic year 1952-53. It is with great pleasure that I accept this favour, which will enable me to work at such an inspiring scientific centre. For my part, I will endeavour to promote the aims of the Institute, to the best of my ability.

I have recently heard that some american mathematicians have planned to spend the academic year 1952-53 with me in Uppsala. I should feel greatly obliged if the Institute for Advanced Study were able to receive one, or possibly two, such mathematicians. I refer in particular to Dr. I. Hirschman, Jr., whom I have strongly recommended for a Guggenheim fellowship. As yet, I do not know whether he will be granted it. In the event that Dr. Hirschman does not obtain a fellowship from the Guggenheim Foundation, I feel that he might be worthy of recommendation for a stipend from the Institute for Advanced Study.

Yours very sincerely,

*Arne Beurling*

UPPSALA UNIVERSITETS  
MATEMATISKA INSTITUTION

Copy to: ✓ Dr. Oppenheimer  
von Neumann  
Weyl  
Siegel  
Montgomery

February 10, 1951.

*Handwritten initials*

Professor Atle Selberg  
The Institute for Advanced Study  
Princeton, N.J.

Dear Atle,

Returning from a visit to Finland I found your very kind letter with an inquiry about my possibilities of visiting Princeton for one term or two. I should very much appreciate the favour of getting one year of research work at such an inspiring mathematical center as your Institute.

Because of the unusual great number of foreign guests who have planned to visit Uppsala for a longer stay during next academic year (51-52) I can't possibly get away from here myself.

You also mentioned the academic year 52-53 and I am happy being able to state that this period would suit me very well in all respects.

With best regards to you and your colleagues at the Institute  
Cordially yours,

/s/ Arne Beurling

Excerpt from Math Minutes 2/27/51: 1. It was agreed that Professor Selberg should write an exploratory letter to Professor Arne Beurling suggesting that the School of Mathematics would be delighted to have him become a member of the Institute for Advanced Study for the year 1952-53 with a grant of \$12,000, understanding that this would have to be confirmed by the Board of Trustees of the Institute at its next meeting.

*Dr. Oppenheimer*

Arne K. A. Beurling

Professor Arne K. A. Beurling is about 44 years of age, he studied at the University of Uppsala, Sweden, and got his doctor's degree there in 1933. From 1933 to 1937 he was docent in Uppsala, and from 1937 he has been professor of mathematics there.

Beurling's mathematical work is characterized both by originality and simplicity of the underlying ideas and by his extremely powerful technique.

His interest and work have essentially been in analysis, where he has given important contributions in several fields: in the theory of analytic functions of one variable, the theory of quasi-analytic functions, harmonic analysis or spectral theory, and in analytic number theory. He is generally recognized as one of the leading analysts of the present time.

Beurling is known as an excellent lecturer and an inspiring teacher. Under him Uppsala has become the center for training mathematicians in Sweden, and has also attracted younger mathematicians from other countries, including the United States. In 1948 he was offered the directorship of the Mittag-Leffler Institute in Stockholm but declined this. For the academic year, 1948-1949, Beurling was a visiting lecturer at Harvard, and recently was offered a full professorship there which he also declined.

At the last International Congress of Mathematicians held at Harvard in 1950, Beurling was invited to give one of the principal addresses, a distinction he has also had at the last Scandinavian Congresses, and also at a congress in harmonic analysis held in Nancy, France, in 1947.

As it would be impossible to describe Beurling's mathematical work adequately, without going to great length and into considerable technical detail, only some short remarks about a few of his papers will be made in the following.

Beurling's work on analytic functions started with his thesis in 1933. This paper ranks among the most important that appeared in that period. The problem he considers here, namely to find general bounds and estimations for the harmonic functions originating from the second boundary-value problem for simply and multiply connected domains, has been of great importance in recent function theory, because it leads to estimations for the so-called "harmonic measure." Beurling's methods lead to such estimations and inequalities which connect the harmonic measure with intuitive geometric concepts, and give a unified approach to both new and earlier known results, partly with an essential sharpening of the latter. For instance, the paper contains a purely potential theoretic proof of the famous Denjoy conjecture

- 2 -

about the number of asymptotic values of integral functions of finite order.

G. Julia<sup>1</sup> remarked about this paper: "Le travail de M. Beurling marque la voie de l'avenir pour la technique de la Méorie des fonctions."

Most of Beurling's later work on function theory has been undertaken jointly with Ahlfors. One should mention here the method now known as the "method of extremal length," which was developed by them about 1946, and has proved very fruitful for various types of problems in function theory. The basic idea underlying this method is a very simple one, namely to study conformal invariants obtained as solutions of extremal problems that are invariant under conformal mappings. The method has been taken up by a number of other mathematicians and also has been used recently by Beurling and Ahlfors to investigate the problem of what types of analytic functions there can exist on a given Riemann surface (such as bounded functions, functions with a finite Dirichlet integral, and so forth). Since this is now being used as basis for a classification of the surfaces, this question has become important during the last years, and the results obtained by Beurling and Ahlfors on this question, are the most interesting so far obtained.

In harmonic analysis and spectral theory, Beurling has done highly important work in several directions. His work on tauberian theorems, of the Wiener type, has connection with his contribution to analytic number theory, which will be mentioned later. Most of his work in spectral theory as actually part of a spectral theory of which the major part so far is unpublished and only has been to some extent expounded in lectures.

Since the problems can be described in rather non-technical terms, Beurling's work in analytic number theory will be described somewhat more fully. The central problems in analytic number theory have always been those connected with the distribution of the prime numbers. A fundamental result here, known as the "prime number theorem," states that for large  $X$  the number of primes  $\leq X$  is asymptotically equal to  $\frac{X}{\log X}$ . Earlier investigations had mainly fixed their attention on the relation between the properties of the so-called Riemann zeta-function and the prime number theorem, mainly with the point of view to prove the latter by using as few properties about the zeta-function as possible. The farthest reaching results in this direction had been obtained by Wiener.

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<sup>1</sup> In an article "Quelques applications fonctionnelles de la topologie," originally an address given at a mathematical congress in Italy.

- 3 -

Beurling replaced this point of view by a far more general and natural one, namely to study the connection between distribution properties of two sequences, one the sequence of the positive integers and the other the sequence of the primes.

A special case of the general problem he studied can be formulated as follows: Let  $\{x_n\}$  be a sequence of increasing real numbers greater than 1 and tending to infinity with  $n$ , and let  $\{y_n\}$  be the sequence one gets by forming all possible products of the  $x_n$ 's and ordering the resulting numbers in an increasing sequence. In particular, if one chooses  $\{x_n\}$  to be the sequence of the primes,  $\{y_n\}$  becomes the sequence of the positive integers. Beurling now raised the question, if one knows some asymptotic law for the distribution of the sequence of the  $y_n$ 's, what can be said about the asymptotic distribution of the  $x_n$ 's? Beurling was able to give a fairly complete answer to this question under very general assumptions, and his work throws more light over the prime number theorem than any other investigations in recent times.

One might also mention that Beurling has recently succeeded in giving the simplest characterization one knows of the Riemann zeta-function, namely as the function of minimal growth within a certain large class of Dirichlet series. Other characterizations known, due principally to Hamburger and Hecke, use much more special properties.

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