

THE INSTITUTE FOR ADVANCED STUDY

Princeton, New Jersey

SCHOOL OF MATHEMATICS

ATLE SELBERG

Atle Selberg was born June 14, 1917 in Langesund, Norway. He was educated at the University of Oslo where he received the Ph.D. in 1943. He was a Member at the Institute for Advanced Study in 1947-48. He went to Syracuse University as Associate Professor in 1948-49 and returned to the Institute in 1949 as a Permanent Member. He was appointed Professor of Mathematics at the Institute in 1951. In 1950 he was awarded the Fields Medal, the most distinguished international award in mathematics, for his work on the Riemann zeta function and his elementary proof of the prime number theory.

Selberg found a new and elementary proof of the prime number theorem. The prime number theorem is the following statement: The number of primes smaller than  $n$  is asymptotically equal to  $n/\log n$ , as  $n$  tends to infinity. It was proved more than fifty years before, in 1896, independently by Hadamard and de la Vallee Poussin. Their proof used the general theory of entire analytic functions and the properties of the Riemann zeta function for complex values of the variable. By the work of Landau, Hardy, Wiener and some other mathematicians, the original proof had been condensed and simplified and modified, but it still depended at an essential point upon complex function theory, namely, one had to prove that the zeta function has no zero with real part 1. Selberg's proof is entirely different and it does not need the zeta function. It has some relation to the old-fashioned method which was introduced into prime number theory by Chebyshev about 100 years ago; however, Chebyshev did not find a proof of the prime number theorem itself. The most important step in Selberg's method is the proof of a certain new formula involving prime numbers, which is done in a purely algebraic-arithmetical way.

ATLE SELBERG

November 9, 1948

Atle Selberg has written more than ten papers on function theory and number theory. Each of his papers contains at least one original and fruitful idea, and three of them are of particular interest, because they deal with three famous topics, namely the Riemann hypothesis, the existence of infinitely many primes in an arithmetical progression and the prime number theorem.

Riemann's hypothesis asserts that all non-real zeros of the zeta function have the real part  $1/2$ . Selberg has not proved the Riemann hypothesis, but he did prove in 1943 that the Riemann hypothesis is true as far as the order of magnitude of zeros is concerned. Consider the number  $N$  of zeros of the zeta function whose imaginary part lies between 0 and  $T$ , and let  $N^*$  be the number of zeros with real part  $1/2$  and an imaginary part between 0 and  $T$ . Then Riemann's hypothesis means that  $N^* \sim N$  for all  $T$ , and Selberg proved that  $N^*$  has the exact order of magnitude of  $N$ , in other words, that the ratio  $N^*/N$  has a positive lower limit as  $T$  tends to infinity. The proof uses an ingenious combination of different ideas taken from earlier papers of Hardy and Littlewood and of Bohr and Landau.

In 1947 Selberg created a new method in number theory which enabled him to give among other results the first elementary proof of Dirichlet's theorem on the existence of infinitely many prime numbers in every arithmetical progression  $ax + b$ , where  $a$  and  $b$  are given positive integers without common divisor and  $x$  runs through the values  $1, 2, \dots$ . Since Dirichlet's discovery in 1837 no other proof had been known, and

-2-

Dirichlet's proof depended upon the properties of transcendental functions; in particular, he had to prove that a certain series  $L(s)$  does not vanish at the point  $s=1$ . Selberg's proof is completely free from function theory.

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Selberg is already 31 years old. Perhaps he will never again do mathematical work comparable to his three discoveries, but he had already his place in the history of science in the 20th century.

Carl Ludwig Siegel

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Carl Ludwig Siegel

J M F  
Selberg

July 26, 1948

Memo to: Miss Trinterud

From: K. Russell

Professor Weyl has notified us that Dr. Atle Selberg has declined his membership in the School of Mathematics for the academic year 1948-49. The grant-in-aid offered Dr. Selberg is now available in the Mathematics budget and has been allocated per the attached letters.



THE INSTITUTE FOR ADVANCED STUDY  
SCHOOL OF MATHEMATICS  
PRINCETON, NEW JERSEY

7 M 7  
Selberg

21 July 1948

Dear Director R. Oppenheimer,

I am sorry to inform you that it is not possible for me to accept the membership in the Institute and stipend, which you have offered me for the next academic year. I will be leaving Princeton in near future since I have accepted a position at Syracuse University.

Yours sincerely

Atle Selberg

July 6, 1948

To Whom It May Concern:

This is to certify that Mrs. Atle Selberg is the wife of Dr. Atle Selberg, a Member of the Institute for Advanced Study in good standing. Mrs. Selberg wishes to join her husband for a visit in Canada where Dr. Selberg is on a short stay at McGill University, working with mathematicians in his field of study.

It would be appreciated if every consideration is given Mrs. Selberg, and the necessary permission granted her to visit Canada and return to the United States with her husband.

(Mrs. John D. Leary)  
Aide to the Director

July 6, 1948

Mr. Martin Clausen  
Chief, Township Police  
Princeton, New Jersey

Dear Sir:

This is to certify that the bearer of this note, Mrs. Atle Selberg, is the wife of Dr. Atle Selberg, a Member in good standing of the Institute for Advanced Study. Mrs. Selberg has been living in Princeton since October, 1947, when she joined her husband at the Institute.

We would appreciate it very much if you would see that Mrs. Selberg receives a letter from you certifying to her police record here.

Yours sincerely,

(Mrs. John D. Leary)  
Aide to the Director

797

# Memorandum

7-19-48

To Mrs Russell

Date

Mrs M.B. Stephens

Dr Atle Selberg-Math.

From

Re

I understand that the Selbergs are leaving on August 1st. They have been residing at 5A Maxwell Lane.

June 30, 1948

TO WHOM IT MAY CONCERN:

This is to certify that Dr. Atle Selberg is a Member in good standing of the School of Mathematics of the Institute for Advanced Study.

Dr. Selberg wishes to make a short visit to Canada during which he expects to visit McGill University for the purpose of discussing mathematical problems on which he has been working here. Dr. Selberg's character and standing in our community are such as to well qualify him to be granted a permit to enter Canada for this visit, and to be given whatever permission is necessary to return to his work in the United States.

Yours sincerely,

(Mrs. John D. Leary)  
Aide to the Director

July 1, 1948

The American Consul  
Montreal  
Canada

Dear Sir:

This is to certify that Dr. Atle Selberg has been a Member of the School of Mathematics of the Institute for Advanced Study for the past academic term, 1947-1948.

During his stay at the Institute for Advanced Study, Dr. Selberg has made valuable contribution to the group study at the Institute by conducting seminars and by discussion of mathematical problems with other members. During the past year Dr. Selberg has also conducted seminars at Princeton University and has lectured at the Massachusetts Institute of Technology and at McGill University.

Dr. Selberg is at present on leave of absence from his teaching position at the University of Oslo, Oslo, Norway. At that University he has been a Fellow since 1942. Since 1946 he has been teaching there in a position equivalent in the United States to an associate professorship.

The faculty of our School of Mathematics values very highly Dr. Selberg's contribution to higher mathematics. It would be of much advantage to scientific students in this country to have the benefit of his guidance and instruction, and it is hoped that his case will be given every consideration by your office.

Yours sincerely,

Robert Oppenheimer  
Director



June 23, 1948

Mr. Martin Clausen  
Chief, Princeton Township Police  
Princeton, New Jersey

My dear Mr. Clausen:

This is to certify that Dr. Atle Selberg has been a member of the Institute for Advanced Study since August, 1947.

Dr. Selberg needs to have a letter from you testifying that he has no police record in Princeton. We would appreciate it very much if you would give Dr. Selberg such a credential as he must present it to the consular authorities in making application for his visa.

Yours sincerely,

(Mrs. John D. Leary)  
Aide to the Director

UNITED STATES DEPARTMENT OF JUSTICE  
IMMIGRATION AND NATURALIZATION SERVICE

PENNSYLVANIA BUILDING  
42 SOUTH FIFTEENTH STREET  
PHILADELPHIA 2, PA.

PLEASE REFER TO THIS FILE NUMBER

February 25, 1948

IDS 0400/25666

Katherine Russell  
Secretary to the Director  
The Institute for Advanced Study  
Princeton, New Jersey

Dear Madam:

Reference is made to your letter dated September 9, 1947, reporting the arrival at your Institution of Dr. A. Selberg.

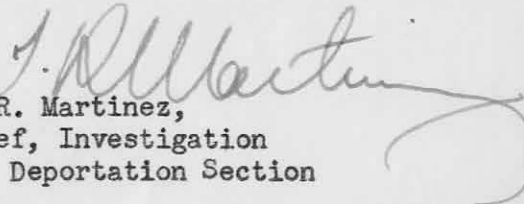
It has been determined that Dr. Selberg was admitted to the United States under Section 3(2) of the Immigration Act of 1924 for purpose of study until August 28, 1948. Under present regulations persons admitted under this Section of law are under the jurisdiction of the district director at the port of entry which, in this instance, is New York, N.Y. Dr. Selberg is being so advised today.

It is suggested that you continue to report enrollment of foreign born persons in all instances in which you are uncertain regarding the status of their admission.

Your cooperation with this Service is sincerely appreciated.

Very truly yours,

Karl I. Zimmerman,  
District Director

by:   
T. R. Martinez,  
Chief, Investigation  
and Deportation Section

December 24, 1947

TO WHOM IT MAY CONCERN:

This is to certify that Dr. Atle Selberg is a Member in good standing in the School of Mathematics of the Institute for Advanced Study. Dr. Selberg wishes to make a short visit to Canada. Dr. Selberg's character and standing in our community are such as to well qualify him to be granted a permit to enter Canada for this visit, and a re-entry permit to the United States at such time as he wishes to return to our institution to continue his researches. We should greatly appreciate your kind efforts to facilitate the grant of these permits to Dr. Selberg.

Yours sincerely,

Robert Oppenheimer  
Director

By \_\_\_\_\_  
Secretary to the Director

Sworn & Subscribed before me  
this 24th Day of December, 1947

\_\_\_\_\_  
Notary Public of New Jersey

February 10, 1948

Dear Dr. Selberg:

On the recommendation of the Faculty of the School of Mathematics, I am pleased to offer you membership in the Institute for Advanced Study for the academic year 1948-49, with a stipend of \$3,500.00.

It has been a pleasure to have you as a member of our group, and we look forward to having you associated with us during the coming academic year.

Yours sincerely,

Robert Oppenheimer,  
Director

Dr. Atle Selberg  
School of Mathematics

Copy to: Miss Blake  
Miss Trinterud

16 May 1947

Dear Dr. Selberg:

I have your letter of May 12th and hasten to send you air mail three notarized copies of a certificate of your membership in the Institute and your stipend. This is the regular certificate which I send to foreign members, and Consular officers in every country have accepted it as sufficient basis for granting a visa. I hope you will have no more difficulty in Norway.

Looking forward with great pleasure to having you here next year, I am

Yours sincerely,

FRANK AYDELOTTE

Dr. Atle Selberg  
Ullevaalsvei 82B<sup>IV</sup>  
Oslo, Norway



5/16/47

TO WHOM IT MAY CONCERN:

This is to certify that Dr. Atle Selberg of Det Matematiske Institutt, Universitetet, Blindern ved Oslo, has been elected to membership in the Institute for Advanced Study for the academic year 1947-48 with a stipend of \$3,000. This will be ample to cover his expenses for the year in Princeton. I should be most grateful for anything Consular officers of the United States can do to facilitate his arrangements for travel.

Yours sincerely,

FRANK AYDELOTTE  
Director



DET MATEMATISKE INSTITUTT  
UNIVERSITETET, BLINDERN VED OSLO

BLINDERN, DEN. May 12, 1947

Dear Director Aydelotte:

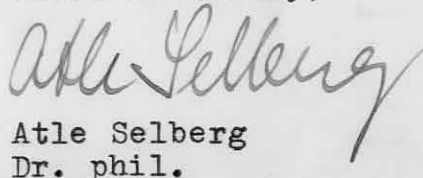
I am writing to you on occasion that I am now preparing my journey to Princeton, in accordance with the invitation to membership in the Institute for Advanced Study, which I received from you some time ago. Professor Siegel has informed me that the term would begin about the middle of September, so I am leaving Norway in the last half of August.

To obtain visa from the American Embassy in Norway I must be able to document that I have the sufficient financial resources (in American Money) for my visit in the United States. I have showed your letter with the invitation of March 11, but on the Embassy they didn't find this satisfying.

I should therefore need a certified document (in two copies) proving that I do get a stipend of \$3,000 to cover the expenses of my visit in Princeton. You would greatly oblige me if it were possible for you to send me this.

Hoping that you will excuse me that I am causing you trouble with this, I am

Yours sincerely,

  
Atle Selberg  
Dr. phil.

Address: Ullevaalsvei 82B<sup>IV</sup> Oslo, Norway

March 11, 1947

Dr. Atle Selberg  
Ullevalsvei 82B IV  
Oslo, Norway

Dear Dr. Selberg:

On the recommendation of the Faculty of the School of Mathematics, it gives me great pleasure to invite you to membership in the Institute for Advanced Study for the academic year 1947-1948 with a stipend of \$3,000.

Looking forward to having you as a member of our group next year, I am

Yours sincerely,

Frank Aydelotte  
Director

Copy to Miss Miller  
Miss Blake

at Report 1947

Arrived Aug Sept. 2/47  
Date 18 January 1947.  
Received Mar. 8/47

Full name Atle Selberg

College or university, degrees, year conferred  
Graduate in Science at the University of Oslo 1939

Phil. Doctors degree \_\_\_\_\_ 1943.  
Date and place of birth 14 June 1917 in Langesund, Norway

Citizenship Norwegian Married or single single

If foreign, under what kind of visa did you enter the United States? Non-immigrant visa

Sec. 3(2)  
Visa No. 267 Place and date of issue Oslo August 4th 1947

When does it expire? August 4th 1949

When and where did you enter the United States? August 28th 1947 at La Guardia Airport NYK.

Entry permit expires when? August 28th 1948

Princeton home address 6-D Cook Road Telephone

Permanent address Ullevålsvei 82B III, Oslo, Norway

Person to notify in case of emergency, with address  
Dr. S. Selberg, Pire Bergssvingen 3, Trondheim, Norway

Honors and societies  
Member of the Norwegian Academy of Science in Oslo, A.M.S.

Positions you have held, giving dates, or are holding (including any graduate scholarships and fellowships)  
Research fellow of the University of Oslo from June 1942. On leave & joined by Dr. Selberg

Publications (Please give title and reference in full, with Vol. No., year, and page numbers.)

List of publications is enclosed

Mr. Max Lee Appel, 2216 South 7th Street  
Philadelphia 48, Pa.

Field of work at Inst.  
Mathematics, especially analytic theory of numbers.

List of publications.

1. "Über einige arithmetische Identitäten, Avh. Norske Vid.-Akad. Oslo. I. no. 8, 23pp. (1936).
2. "Über die Mock-thetafunktionen siebenter Ordnung, Arch. Math. Naturvid. vol. 41. no. 9, 13 pp. (1938).
3. "Über die Fourierkoeffizienten elliptischer Modulformen negativer Dimension, Comptes rend. congrès math. scand. Helsingfors 1938, 3 pp.
4. Bemerkungen über eine Dirichletsche Reihe, die mit der Theorie der Modulformen nahe verbunden ist, Arch. Math. Naturvid. vol. 43. no. 4, 4 pp. (1940).
5. Beweis eines Darstellungssatzes aus der Theorie der ganzen Modulformen, Arch. Math. Naturvid. vol. 44. no. 3, 12 pp. (1941).
6. "Über ganzwertige ganze transzendente Funktionen, Arch. Math. Naturvid. vol. 44. no. 4, 8 pp. (1941).
7. "Über ganzwertige ganze transzendente Funktionen II, Arch. Math. Naturvid. vol. 44. no. 16, 11 pp. (1941).
8. "Über einen Satz von A. Gelfond, Arch. Math. Naturvid. vol. 44. no. 15, 12 pp. (1941).
9. On the zeros of Riemann's zeta-function on the critical line, Arch. Math. Naturvid. vol 45. no. 9, 14 pp. (1942).
10. On the zeros of the zeta-function of Riemann, Norske Vid. Selsk. Forh. vol. 15. no. 16, 4 pp. (1942).
11. On the zeros of Riemann's zeta-function, Skr. Norske Vid.-Akad. Oslo. I. no. 10, 59 pp. (1942).
12. On the normal density of primes in small intervals, and the difference between consecutive primes, Arch. Math. Naturvid. vol. 47. no. 6, 19 pp. (1943).
13. Bemerkninger om et multipelt integral, Norsk Mat. Tidsskr. vol. 26. 8 pp. (1944).
14. On the remainder in the formula for  $N(T)$ , the number of zeros of  $\zeta(s)$  in the strip  $0 < t < T$ , Avh. Norske Vid.-Akad. Oslo. I. no. 1, 27 pp. (1944).
15. Contributions to the theory of the Riemann zeta-function, Arch. Math. Naturvid. vol. 48. no. 5, 67 pp. (1946).
16. Contributions to the theory of Dirichlet's L-functions, Skr. Norske Vid.-Akad. Oslo. I. ( in print, ca. 60 pp. ).
17. The zeta-function and the Riemann hypothesis, Comptes rend. congrès math. scand. København 1946, 14 pp.
18. On an elementary method in the theory of primes, Norske Vid. Selsk. Forh. vol. 19. no. 18, 4 pp. (1946).



September 9, 1947

District Director  
Immigration & Naturalization Service  
Pennsylvania Building  
42 South 15th St.  
Philadelphia 2, Pa.

Dear Sir:

In accordance with the regulations, I should like to report the arrival of Dr. A. Selberg at the Institute for Advanced Study on September 2, 1947. Dr. Selberg is a member of our School of Mathematics and is engaged in research in the field of higher mathematics.

Dr. Selberg entered the United States at La Guardia Field, New York City on August 28, 1947, under Non-immigrant visa No. 267, issued at Oslo on August 4, 1947, expiring August 4, 1949.

Dr. Selberg was born on June 14, 1917 at Langesund, Norway. His Princeton address is 6-D Cook Road. Inquiries concerning him may be addressed to the above address or to Dr. S. Selberg, Øvre Bergssvingen 3, Trondheim, Norway.

Yours sincerely,

Katherine Russell,  
Secretary to the Director

Atle Selberg

11/9/48

Read by Prof. Siegel

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Selberg is already 31 years old. Perhaps he will never again do mathematical work comparable to his three discoveries, but he has already his place in the history of science in the 20th century.

Carl Ludwig Siegel

Re Selberg, my other information is a copy  
of yours.

G.Blake

Atle Selberg

~~Paul Turan~~

COPY FOR DR. OPPENHEIMER

From C.L.Siegel to O.Veblen, Copenhagen, Denmark, Nov.4/46

I should like to mention some possible European candidates, in addition to those in the Minutes of the meeting of Oct. 8, namely Atle Selberg (Oslo), Linnik (Leningrad), Eichler (Göttingen) and Turán (Budapest). All four of them have accomplished a very remarkable work in number theory. Perhaps A. Selberg (there are three brothers Selberg: Atle, Henrik, Sigmund; all three are mathematicians, but only Atle is first-rate) is the most gifted of them; he is 29 years old and now has a stipend from the university of Oslo; he has solved several entirely different problems of great difficulty, in particular a question related to the Riemann hypothesis, and he did at least two things which I was unable to do. Probably he will now obtain a chair at Trondhjem, as the successor of Viggo Brun.