April 16, 1979

Professor John Milnor
School of Mathematics
The Institute for Advanced Study

Dear John:

Thank you for your advice concerning Hassler Whitney.

We have proposed to Hass that his material be published without the imprimatur of the Institute and have encouraged him to look for an independent publisher.

Sincerely,

John Hunt
Associate Director

THE INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY 08540

SCHOOL OF MATHEMATICS

March 29, 1979

Mr. John Hunt
Institute for Advanced Study

Dear John:

Thanks for sending me the Whitney material, which I showed to the Mathematics Faculty. Our general feeling is that it looks quite strange for the Institute to be officially associated with such a project. It may well be extremely worthwhile--we have no expertise in judging such materials. In any case, we have in recent years tried to cut down on our involvement in this kind of informal publishing which can consume a great deal of secretarial time.

Is there any way you can help Whitney to find funding and/or a publisher, without making it seem that the Institute is starting a School of Education?

Sincerely,

John

John Milnor

JMcdu
Enclosures

April 3, 1979

MEMORANDUM

To: Ms. Sabina Modzelewski

From: Aida La Brutte

Re: Publication monies for Professor H. Whitney

Just to put into writing what I mentioned to you last Friday.

Dr. Woolf would like a special budget of $1500 to be set aside from the Publication Fund for use, if needed, by Professor Whitney. This is in connection with the publication and distribution of his elementary mathematics activities. At the end of next year Dr. Woolf would like an accounting of the monies in this special budget, and all unspent monies should be returned to the regular Publication Fund.

Many thanks.

check
Princeton
Need advice
Is it approved?

THE INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY 08540

SCHOOL OF MATHEMATICS

Jan. 8, 1979

Harry Woolf, Director, IA S

Dear Harry:

I am writing you to let you know of my desire to issue some mathematical notes for schools, through the Institute, to be for sale. It goes somewhat beyond my "Mathematical Activities, Part A" (a volume I believe you know of), and I would prefer to have your OK for the project. I have talked with John Hunt about it.

Briefly, my Part A is for grades 1-2 in schools. Several hundred teachers apparently are using it in the St. Louis region, it has been made the basis for a developing program for two years work in Campinas, Brazil; to what extent elsewhere, I do not know. About 360 copies have been sold (by the IAS). The regular price is $3.50 per copy.

My proposed notes, "Meaningful Math," are for normal, and even more for remedial work, through elementary arithmetic. I should have Chapter I complete within a few weeks; hence I must know soom if possible about putting it out. I propose to write perhaps five chapters in all; the whole may take some months per chapter.

My principal purpose is to show the practical reality in getting a basically different viewpoint in teaching from what is now occuring: getting students to truly think for themselves. Only in this way can theylearn easily, and use math in applications also.

Here are some financial notes. Part A (in its second printing) cost the Institute about $1600 plus postage. If the remaining copies are sold ($3.50 per copy), about 85-90% of this will come back. (There were many complimentary copies, to help promote the methods.)

The proposed chapters (unbound, about 40-50 pp apiece) will cost about 35¢ each to produce. I propose a charge of $1 each. In terms of the usual prices of educational materials, this is low.

Part A was copyrighted by the Institute. I would like to copyright the individual chapters in my own name.

With thanks for your attention,

Hass

Hassler Whitney

June 1970

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THE INSTITUTE FOR ADVANCED STUDY

Princeton, New Jersey

SCHOOL OF MATHEMATICS

HASSLER WHITNEY

Hassler Whitney was born March 23, 1907 in New York City. His undergraduate studies were at Yale, and he received his Ph.D. degree from Harvard in 1932. He was a National Research Council Fellow at Princeton University in 1931-32 and at Harvard in 1932-33. At Harvard he was successively Instructor, 1933-35, Assistant Professor, 1935-40, Associate Professor, 1940-46, and Professor, 1946-52. He accepted the appointment as Professor of Mathematics at the Institute for Advanced Study in 1952.

Whitney's interest has been mainly in topology and its applications, especially applications to differentiable manifolds. His methods are geometric and intuitive and his work is marked by great originality and strength. He is regarded by mathematicians throughout the world as an outstanding scholar, a leader in his field , and in the front rank of mathematicians of his age. His lectures, writing and personal contacts have been of wide influence. Several mathematicians, including Chern, Eilenberg, and Steenrod have been greatly stimulated by him and the same is true of a number of younger people at Harvard and elsewhere.

A large part of his work has been on differentiable manifolds and it is on this subject that he gave the Colloquium Lectures before the American Mathematical Society at Cornell in 1946.

He helped develop the subject of cohomology and made some of its first applications. He has made important contributions to the problem of determining whether or not two given objects in a space can be continuously deformed so that they coincide.

1969

Hassler Whitney

Born March 23, 1907

Ph. B. Yale 1928

Mus. B. Yale 1929

Ph. D. Harvard 1932

Hon. Sc. D. Yale 1947

National Research Council Fellow, Princeton 1931-2, Harvard 1932-3.

Instr. Harvard 33-35, Assistant Prof. 35-40, Assoc. Prof. 40-46, Prof. 46 --

Whitney's interest has been mainly in topology and its applications especially applications to differentiable manifolds. His methods are geometric and intuitive and his work is marked by great originality and strength. He is regarded by mathematicians throughout the world as an outstanding scholar, a leader in his field, and in the front rank of mathematicians of his age. His lectures, writings, and personal contacts have been of wide influence. Several mathematicians, including Chern, Eilenberg, and Steenrod have been greatly stimulated by him and the same is true of a number of younger people at Harvard and elsewhere.

This statement attempts to describe mainly in non-technical language some of his interests which can be most easily so described and it is not intended to be an exhaustive or well rounded account.

As mentioned above, a large part of his work has been on differentiable manifolds and it is on this subject that he gave the colloquium lectures before the American Mathematical Society at Cornell in 1946. A differentiable manifold is a smooth geometric figure of any number of dimensions, some examples in low dimensions being the circle, the surface of a sphere or the surface of a ring. These manifolds arise in many ways in algebra, geometry, and analysis. To take one illustration, each point of a manifold may represent

the state of a mechanical system, and the manifold as a whole is then the totality of possible states of the system. Changes in the system cause the points to move about on the manifold. These facts were largely the motivation for the original work in the subject by Poincare. Locally a manifold is like a euclidean space (euclidean space is the ordinary space of our experience or its counterpart in higher dimensions) of the same dimension but taken in its entirety there may be great differences and the study of these differences is one of the tasks of topology.

There are two different ways to give a precise definition of differentiable manifolds, the first by abstract postulates and the second by regarding the manifold as imbedded in euclidean space by means of equations. Whitney showed that these two points of view come to the same thing so that it is permissible to adopt whichever is more convenient. He showed first that an abstract n-dimensional manifold can be imbedded in euclidean space of 2n+1 dimensions and later that 2n dimensions is sufficient. The question of how many dimensions are necessary to imbed a particular manifold is very difficult and almost all the information known has been obtained by Whitney. He has an example of a 4-dimensional manifold (the complex projective plane) which can not be imbedded in 7-dimensional space.

At each point of a manifold there is a collection of tangents which themselves form a space or a fibre, called the tangent space at the point. The totality of these tangent spaces or fibres forms a larger space called a fibre space, and an examination of this larger space has been fruitful. The idea of fibre spaces, that is spaces built up by layers of other spaces, had many origins but Whitney was one of the first, and most successful, to exploit it in this direction. He and others used these large spaces to find properties of the original manifold.

Many of his early papers were on one-dimensional complexes or graphs with some attention to the four color map problem. In another early paper he gave a characterization of a plane in the language of combinatorial topology. At about the same time he studied families of curves of the kind which are solutions of differential equations. His next papers were on properties of derivatives and were at least partly in preparation for his work on differentiable manifolds. He remarked in 1936 (in Scripta Mathematica) that he regarded his best papers up to that time as numbers 5, 10, 15, 18, 25, and 28 of the bibliography.

He helped develop the subject of cohomology and made some of its first applications. He has made important contributions to the problem of determining whether or not two given objects in a space can be continuously deformed so that they coincide.

Whitney has several hobbies including mountain climbing and music. It may be of interest that he is a descendent of Eli Whitney, inventor of the cotton gin, and of Simon Newcomb, the mathematical astronomer. His honorary degree from Yale came at the centennial celebration of the founding of the Sheffield scientific school. At about this time he was offered a Sterling professorship at Yale but did not accept. During the war he carried on mathematical research for the NDRC. He was invited to give an hour address at the International Congress of Mathematicians held at Harvard in 1950, and he was chairman of the conference on topology, held at the congress. He also addressed international meetings of topologists in Paris in 1948 and in Moscow in 1935. He has been vice president of the American Mathematical Society and is a member of the National Academy of Sciences and the Philosophical Society.

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