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General Secretary, International Astronomical Union, 1948-52. Member Executive Committee, International Council of Scientific Unions, 1948-52.

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Abstract of lecture at the Joint Meeting of the Franklin Institute and the Rittenhouse Astronomical Society on April 1, 1959.

"The Youngest and the Oldest Stars in our Galaxy"

During the last fifteen years problems of the correlation of age, chemical composition and space distribution of the stars have been investigated by a number of astronomers.

The oldest stars in our galaxy belong to Baade's population II. It is now generally agreed that these stars differ very markedly from the young stars and those of intermediate age, with respect to chemical composition, space distribution and velocity distribution. Their relative metal content is very low. The distribution in space is such that they form a system extending into the so called galactic halo, which is much less flattened than the space occupied by the stars of intermediate age and the young stars. Finally, the space velocities are on the average much higher than for the youngest stars in the galaxy.

The investigations also tend to indicate that there is a gradual transition with respect to the properties just mentioned from the oldest to the youngest stars. Thus the ratio of the total content of heavy elements to that of hydrogen in a star is very much smaller for the oldest stars than for the youngest, however there are strong indications that the whole range of chemical compositions intermediate between the extremes is represented among the stars of the galaxy. Does this mean that the chemical composition of a star is strictly a function of its age - or what amounts to the same thing, of its epoch of formation - or do other factors enter? This and other similar questions can now be formulated and are being actively studied.

In today's lecture I would like in particular to discuss certain results concerning the ages, chemical compositions and space motions of the youngest stars. The results in questions have been obtained in recent years through the use of a new observational technique - photoelectric narrow-band photometry - and its combination with theoretical calculations. Furthermore, I wish to describe some results obtained in a newly begun survey of the unevolved part of the population of the oldest stars in our galaxy; and to discuss very briefly plans for further exploration of these stars. Finally I would like to summarize the results and their bearing on the problems of the development of our galaxy.

The very youngest stars in our galaxy are those found in stellar associations containing highly luminous stars. The luminous stars in question evolve rapidly because they exhaust their supply of nuclear energy very quickly, and it has been estimated that their ages are of the order of 5 - 10 million years, in some cases perhaps even smaller. Since it can be assumed that the stars of a stellar association were formed at nearly the same time, it follows that all the member stars, not only the most luminous among them, are very young stars.

The stellar associations are concentrated to the spiral arms of our galaxy where the density of interstellar matter is highest and conditions favorable for star formation. As a stellar association grows older it will gradually dissolve as a consequence of the space motions of its member stars, and it will also gradually

lose its most massive and luminous stars as a consequence of their evolution. The remaining stars will spread out in space beyond the spiral arm regions. The birth of stars in associations in spiral arms, and the gradual spreading out of the stars over the entire galactic disc is a process that has presumably been going on for billions of years. The association stars become field stars in a more uniformly distributed galactic population.

We can now ask the question, Is it possible through investigations of present day field stars to trace their development out of spiral arms in which they were formed hundreds of millions or even billions of years ago?

Astronomers are today very far from being in the possessions of a complete solution of the problem posed. However, important progress is being made, and certain results already obtained give considerable promise for the future. I would now like to explain how photoelectric narrow-band photometry can contribute in this field.

The properties of a star are known to be functions of two basic parameters: the mass and the age of the star. In particular, the color and the luminosity of a star are functions of its mass and age. This connection is the basis of a method for determining stellar ages: According to observations the stars are plotted in a color-luminosity diagram. In the connection between position in the color-luminosity diagram can be connected with the age, i.e. if the diagram can be calibrated in terms of age, then stellar ages can be determined. In recent years great progress has been made toward this goal through the efforts of a number of astronomers, both observational and theoretical.

For stars that occur together in associations or clusters it is now generally possible to obtain reliable color-magnitude diagrams, and from these the ages of the associations and cluster in question can be found. However, for the field stars which are in the focus of our interest, the direct approach fails. In general, the luminosities cannot be determined with sufficient accuracy here. This is where the method of photoelectric narrow-band photometry comes in.

The method is but a further development of the wellknown spectro-photographic methods of determining luminosities and spectral types, particularly Lindblad's methods based on low-dispersion spectra. The intensities in suitably selected wavelength bands varying in width from  $15\text{\AA}$  to about  $100\text{\AA}$  are measured photoelectrically with high accuracy, and the measures are then combined to certain classification indices that characterise the strength of selected spectral features, for instance the  $H\beta$  absorption line or the cyanogen band in the violet. On the basis of the measured indices the luminosity and the intrinsic color (i.e. the color unaffected by interstellar reddening) can be determined with considerable accuracy, and the mass and the age can also be found. The distance of the star can be evaluated with good accuracy, and in combination with proper motion and radial velocity give the space motion according to a standard procedure.

I would now like to explain somewhat more in detail how photoelectric narrow-band photometry leads to ages and space motions for four different categories of young stars: 1. The B8 - B9 stars, age up to 100 million years. 2. A2 - A5 stars, age up to 500 million years. 3. F0 - F5 stars, age up to 1000 million years. 4. Young G8 - K2 giants. ( In the lecture the exposition will be based on slides.)

Photoelectric narrow-band photometry has been developed beyond the framework of two-dimensional classification to yield a parameter which characterizes the ratio of heavy elements to hydrogen. (This will also be explained with the help of

slides). Thus for the field stars of the categories mentioned determinations of the age, space velocity and a quantity important in the characterization of chemical composition can be carried out.

It is immediately clear how such observational results can be utilized in our attempts to trace paths of the present field stars back to their regions of birth in the spiral arms, and to find out about the properties of the spiral arms in previous time.

We shall consider in particular two applications. 1. If we subdivide the stars in the solar neighborhood into groups according to their age, and examine the space velocity distributions for the different groups, then we note characteristic differences that are tentatively interpreted in terms of origin in different spiral regions. 2. The Fo - F5 stars prove to be particularly suited for a study of the factors that influence the chemical composition of a star at its birth. The results obtained contribute to the clarification of the question of the variation of chemical composition from star to star among a group of stars of the same age. (These two applications will be further discussed with the help of slides.)

The group that we have referred to as the youngest stars of our galaxy can conveniently be defined as containing the stars younger than about 1 billion years. For stars much older than 1 billion years it becomes very difficult to trace the path from the present location to the place of formation of the star. It is also important in this connection that big features of our galaxy - its degree of flattening, the average chemical composition, and state of motion of its interstellar matter - have presumably changed little during the last billion years. Therefore the stars of the group thus defined were formed under similar general conditions. They are part of Baade's population I.

I would now like to describe briefly some recent investigations concerning the oldest stars in our galaxy. I have already referred to the fact that these stars in comparison with the young stars have a very low heavy element content, great average distances from the galactic plane and high space velocities. The stars in globular clusters such as Messier 3 are typical representatives of this group of stars.

Stars belonging to the category of the oldest stars also occur as field stars in the solar neighborhood. This has become clear through the work of Oort, Adams and Joy, Morgan, Kuiper, Miss Roman, and that of Zwicky and Humason, and of Haro and his collaborators. The RR Lyrae stars and the blue Zwicky-Humason stars in high galactic latitude are of this kind, and represent a late stage of evolution of the category of oldest stars. On the other hand we find in this class subdwarfs of spectral types F, G and K which in spite of their great age have not changed very much by evolution. Clearly this latter kind of star is of particular importance, since systematic study of it should lead to results concerning the earliest stages of development of our galaxy. The last part of the talk will be devoted to these stars.

First we would like to have an inventory of this kind of star, at least representative samples. Although there are probably more than a billion such stars present in our galaxy, the number of field stars known at this time is less than 100. Most of the known stars were picked up among stars brighter than 9. magnitude as stars of unusually high velocity. Miss Roman contributed greatly to our knowledge of the group when she studied spectra and carried out broad-band UBV photometry for the known high-velocity stars. In particular she showed that the F- and G-type subdwarfs radiate strongly in the ultraviolet in comparison with young field stars.

- 4 -

This latter phenomenon has found its interpretation in terms of the low heavy-element content of the oldest stars. In the younger F and G stars the heavy-element absorption lines are strong and numerous in the ultraviolet, and the intensity here is considerably reduced. In the oldest star this reduction is much less pronounced.

The ultraviolet-excess of the subdwarf F and G members of the population of oldest stars has proved to be a characteristic property according to which these objects can be picked up in surveys extending to very faint stars. In collaboration with colleagues at the Bergedorf, Copenhagen and Dyer Observatories, I have recently carried out a search for F and G star subdwarfs in an area of 18 square degrees in high galactic latitude down to the 15. magnitude. First, photographic photometry in the ultraviolet and the green was obtained for all stars in the area brighter than the limiting magnitude mentioned, about 4000. Next, the stars of high ultraviolet intensity according to the photographic photometry were further examined through UBV photoelectric photometry. A number of ultraviolet-excess F and G type subdwarfs were found in this way. The work on this area is not yet completed, but the results obtained so far indicate that there are about two stars of the category in question per square degree. It is the intention to extend the work to a much larger area, and also to search a few smaller areas down to much fainter limits. The ultimate aim is to get reliable estimates of 1. the total number of these stars in our galaxy, 2. the distribution function of the ultraviolet excesses, and 3. the correlation between ultraviolet excess and average distances from the galactic plane.

Through theoretical calculations utilizing the methods of stellar atmosphere analysis it is possible to connect ultraviolet excess with chemical composition, in particular the heavy-element content. The distribution function of the latter quantity and its correlation with average distances from the galactic plane, and possibly with age, can be derived.

It was suggested by Hoyle and Lyttleton that heavy elements are formed in stars and dissipated into interstellar space, and that this is the reason why the oldest stars have a much lower heavy-element content than those formed out of interstellar matter in more recent times. This general idea has been investigated more in detail by Schwarzschild and Spitzer, and more recently by Schmidt, Salpeter and others. It is clear that observational data such as those described will be of value in the testing and further development of this picture. In particular, knowledge of the distribution function of the heavy-element content for unevolved stars will make it possible to draw conclusions regarding the distribution function of masses formed during the early phase of development of the galaxy as compared to the present phase.

Whenever we can determine the age, the space motion and the chemical composition of a star we obtain a piece of information pertaining to the development of our galaxy, because we can then state that at such time, and such location in the galaxy, a star was formed out of interstellar matter of known composition. Ultimately astronomers may in this way be able to put together rather a complete picture of the history of the galaxy. It is likely that further research during the next few years on the youngest and the oldest stars of our galaxy will contribute toward the goal. .

B. L.

## An Introduction to Astrophysics

Notes of a Lecture Series by Professor Bengt Strömberg  
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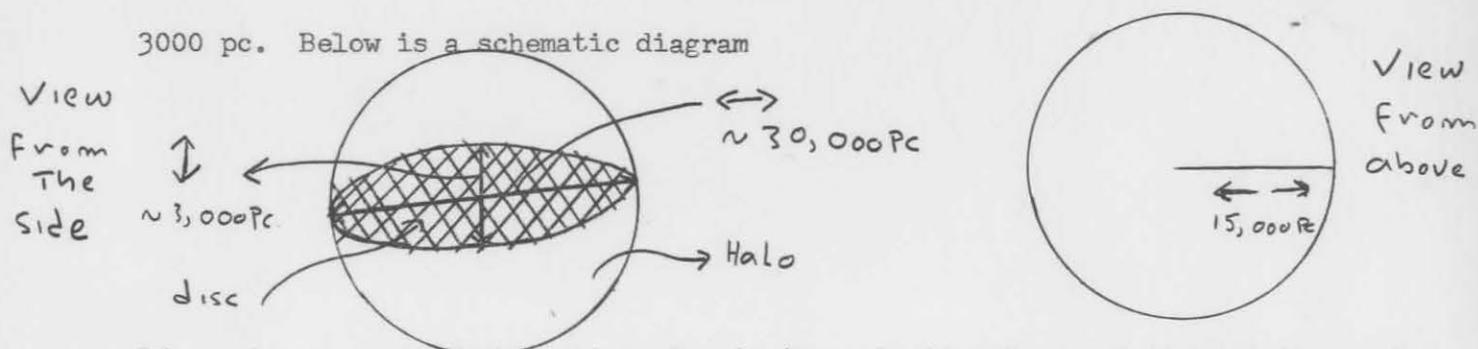
### Lecture 1.

#### Our Galaxy

We shall begin this series of lectures by making some qualitative remarks about the nature of our galaxy. These will help to present the fundamental questions and the relevant orders of magnitude. We turn first to the structure of the galaxy.

#### Paragraph 1. Structure.

The basic geometric structure of the galaxy is that of a disc. The largest diameter is of the order of 30000 pc (1.) and the central thickness is about 3000 pc. Below is a schematic diagram



If we choose an axis at right angles to the galactic plane and through the center of the galaxy then with respect to this axis there is approximate rotational symmetry. It has been found that the most luminous stars as well as interstellar matter tend toward a marked concentration on spiral arms in or near the galactic plane. The thickness of these arms is about 500 pc. Three such arms have been traced through more than a quadrant. The distance between arms is about 1500 to 2000 pc.

---

(1.) At the end of each lecture the reader will find a glossary of terms used in the lecture which are very likely unfamiliar to the non-specialist and to which we shall make frequent reference.

The spiral arms trail with respect to the galactic rotation. In the figure above we have shown the so-called Halo of the galaxy which is approximately spherical and concentric with the disc. The star density is much lower in the halo than in the disc.

#### Paragraph 2. Kinematics

The galaxy as a whole rotates. The majority of disc stars move around the center of the galaxy in nearly circular orbits of small inclination to the galactic plane. At the distance of the sun from the center (8000 pc) the speed of rotation is about 200 km/sec. It decreases with increasing distance from the center. With decreasing central distance there is first an increase in the speed of rotation, then a decrease, the maximum being at a 6000 pc distance from the center. The average random velocity relative to a point moving with circular velocity is, very roughly, 50 km/sec. The halo stars show no marked rotation. Their average velocities are very roughly 200 km/sec.

#### Paragraph 3. Stellar content.

##### Samples:

Below we give a listing of the sampling categories used in determining distributions of stellar properties:

- a.) In the first sampling group we consider the stars within 5pc. There are about 50 such known. The sample is nearly complete for stars brighter than  $L = 0.001 \times$  the sun's luminosity ( $L_{\odot}$ ) (see below). For these stars the distances are known with better than 10 per cent accuracy from the determination of trigonometric parallaxes.
- b.) In the second group we take all stars within 10 pc. There are about 200 such stars known. The sample is less complete, but is reasonable complete for stars brighter than  $\frac{L}{L_{\odot}} = 0.01$ .

c.) In the last group we consider stars within 30 pc. There are about 2,000 stars known. This is presumably less than one quarter of all stars present, but for stars brighter than  $\frac{L}{L_{\odot}} \approx 0.1$  the list is reasonably complete. For distances out to 30 pc. (but not much beyond) trigonometric parallax determines the distances fairly well (the mean error of a good trigonometric parallax is of the order of  $0''.01$  ). The luminosities which are computed from the apparent intensities and the measured distances are therefore fairly accurate for stars within 30 pc.

(It should be noted that sample b. is meant to include a., and c is meant to include them both.)

We will now discuss the luminosity distribution from samples a, b and c: There is an approximate expression for the number of stars per  $\text{pc}^3$  having a luminosity between  $L$  and  $L + dL$  . In fact  $\psi(L)dL$  is given by

$$\psi(L) \sim L^{-2} \quad \text{for } \frac{L}{L_{\odot}} > 2$$
$$\psi(L) \sim L^{-1.2} \quad \text{for } \frac{L}{L_{\odot}} < 2$$

The luminosity distribution is fairly well determined for  $100 > \frac{L}{L_{\odot}} > 0.001$  samples a.b.c. For  $\frac{L}{L_{\odot}} > 100$  even sample c contains too few stars. When  $\frac{L}{L_{\odot}} < 0.001$  the lack of completeness of sample a is significant. For very luminous stars  $\frac{L}{L_{\odot}} > 100$  the results obtained from samples a,b, and c are supplemented by the statistics of stars out to several hundred parsecs. Because the stars in question are luminous, they appear relatively bright even when they are a few hundred parsecs away. Hence it is not necessary to go much beyond the naked eye limit of apparent magnitude  $6^m$  for this supplementary investigation of very luminous stars. However, the problem is complicated in two respects:

1. The method of trigonometric parallax fails, and one must rely on average parallaxes derived from the comparison of proper motions and radial velocities, and luminosities derived from the analysis of stellar spectra.

2. The star density is no longer uniform within the volume investigated. There are appreciable variations with distance from the galactic plane, and variations from regions inside to regions outside the spiral arms. However, reasonably accurate luminosity distributions can be derived for regions in the galactic plane and separately inside and outside the spiral arms. However, there is another respect in which the two samples (stars within 30 pc. and stars brighter than apparent magnitude  $6^m$ ) are not quite adequate (population II, see below).

Next we turn to the question of stellar-classification. The spectrum emitted by a star (a continuous spectrum traversed by a large number of absorption lines) is a function of:

1. Two physical parameters which describe the influence of the star as a whole upon the directly visible atmospheric fringe; namely the net flow of energy  $F$  per unit area and unit time outward through the atmosphere, and the gravitational acceleration  $g$  toward the center of the star.

2. The chemical parameters, namely the relative abundances of the elements.

If the spectra of large numbers of stars (10,000 - 100,000, say) are carefully examined it is found that they can be divided into relatively few groups (of the order of 200 such groups) such that the spectra of the stars within one group look practically identical. The variety does not at all correspond to what one might expect if the chemical parameters varied widely. This observational fact suggests great regularity, if not complete uniformity, with regard to the chemical composition of stellar atmosphere. Quantitative analysis based on observed stellar absorption line strengths has largely confirmed this conclusion.

It has been found that the great majority of stars for which spectra have been obtained can be classified in a two-dimensional system. As the two parameters we can choose the net flux  $F$  and the gravitational acceleration  $g$ . However, it is customary to consider, instead of  $F$ , an equivalent quantity called the effective temperature  $T_e$  and defined through the black body radiation law  $F = \sigma T_e^4$  where  $\sigma = 5.67 \times 10^{-5}$  ergs x  $\text{cm}^{-2}$  x  $\text{time}^{-1}$  x  $\text{Temp}^{-4}$  (in deg. Kelvin). From the definition it follows

that, if the radius of the star is  $R$ , the total energy radiated into space per unit time,  $L$ , is given by  $L = 4\pi R^2 \sigma T_e^4$ .  $L$  is usually called the luminosity.  $L$  for the sun is denoted by  $L_\odot$  and is equal to  $4 \times 10^{33} \text{ erg sec}^{-1}$ . As above (in our discussion of stellar samples)  $L$  is usually measured in terms of  $L_\odot$ . Since  $g = GM/R^2$  where  $M$  is the mass of the star, it follows that  $g \sim MT_e^4/L$ . According to this relation if the mass were a single valued function of  $g$  and  $T_e$  (see below) the same would be true of  $L$ . This would suggest that the effective temperature  $T_e$  and the luminosity  $L$  may be taken as the two parameters in our two dimensional classification scheme. This is usually done and is found adequate.

For a relatively small number of stars (10-20) the value of  $F$ , and hence the effective temperature  $T_e$ , is known (for example for the sun, certain eclipsing binaries and some supergiant stars whose diameters have been measured interferometrically). For a much larger number of stars the luminosity is known with fair accuracy from measured apparent intensities and distances determined by the method of trigonometric parallaxes. On the basis of this material it has been possible to calibrate empirical spectral classifications in terms of  $T_e$  and  $L$ . (For the most luminous stars which are not represented in sample  $c$  distance information derived from proper motions and radial velocities has been used for the calibration.)

The empirical classification system uses the spectral classes (O,B,A,F,G,K, and M) and the luminosity classes ( I,  $I_e$ , the most luminous stars, II...  $V$  ) defined by typical stars (Subdivision of the spectral classes are used, e.g. B0, B1, ... B2, ... B9, A0, A1 ...). The calibration gives  $T_e$  and  $L$  in terms of spectral class. ~~The~~ The spectral class and  $T_e$  are closely correlated.

The changes in the spectrum with  $L$  are much less conspicuous than the changes with  $T_e$  (and correspondingly require spectra of higher dispersion for detection and evaluation). The broad features of empirical stellar classification can therefore

be described in terms of the spectral class alone. The O and B stars which correspond to the highest  $T_e$  (20,000 - 50,000) are characterized by the presence of helium absorption lines in the spectrum. The A and F stars ( $T_e \sim 10,000$ ) exhibit very strong Balmer lines that dominate the spectrum. The G and K stars ( $T_e \sim 5,000$ ) manifest a large number of metal lines, while the M stars ( $T_e \sim 3,000$ ) show strong molecular bands.

The Hertzsprung-Russell diagram - In the Hertzsprung-Russell diagram (H-R diagram) a star is represented by a point with abscissa  $T_e$  and ordinate  $L$  (usually on a logarithmic scale). Since  $L = 4\pi R^2 \sigma T_e^4$ , the radius of the star is a single valued function of its location in the H-R diagram. Because  $T_e$  and the spectral type are closely correlated, a spectral class-luminosity diagram is very similar to the  $T_e$ -luminosity diagram.

Stellar masses - For a relatively small number (20-30) of visual and spectroscopic binaries the period and the semi-major axis of the absolute orbit of each component is known with sufficient accuracy <sup>for the masses to be determined</sup>. The mass range is  $10^{-2}$  to  $10^{+2}$  solar masses. When these stars and the corresponding mass values are plotted on the H-R diagram, it is found that the observational material is compatible with the assumption that the mass is a well defined function of the location in the H-R diagram. Lines of constant mass can be drawn through most of the diagram area where stars occur, and the corresponding mass can be estimated for any relevant point in the H-R diagram. The mass increases with luminosity, but is not a simple function of it.

Stellar rotational velocities. - If a star has axial rotation, the absorption lines in the spectrum will be correspondingly broadened by the Doppler effect.

Within the framework of the theory of stellar absorption lines it is possible to determine the amount of axial rotation from the observed profiles of the absorption lines. The quantity obtained is the projection of the rotational velocity vector on the plane at right angles to the line of sight. The effect of the projection shortening can be allowed for statistically. For O and B stars rotational velocities of 100-200 km/sec. are common. For G, K, and M stars the rotational velocities are at most a few km./sec. A and F stars are intermediate.

#### Distribution of Stars in the H-R diagram:

For the stars within 30 pc. (sample c.) and for stars brighter than apparent magnitude  $6^m$ , the effective temperature  $T_e$  can be determined from the two dimensional spectral classification. The luminosity can also be found in this way or from the apparent intensity and the distance derived from the trigonometric parallax method. With proper reductions to equal sample volume, this material gives a fairly reliable determination of the stars in the H-R diagram for a local sample of our galaxy. The principal features of the distribution are the following:

1. About 90 per cent of all stars (total star density being about 0.1 stars per pc.<sup>3</sup>) belong to a so-called main sequence of stars, with points close to a curve in the H-R diagram which extends from the O stars with  $L = 10^5 L_\odot$  through a point corresponding to the sun (spectral type G,  $L = L_\odot$ ) to the lowest temperature M stars with  $L \sim 10^{-4} L_\odot$ . For the part of the main sequence which is below the point corresponding to the sun, the scatter of points is very small (when only stars with observations of high precision are used) the variations of  $L$  for equal effective temperatures being within limits  $\pm 10$  per cent. For the part above the sun the scatter is considerable, the total range in  $L$  for the

same effective temperature being given by a factor of about 3-4.

The main sequence of stars form to a good approximation a one parameter series of stars: radius, luminosity, effective temperature are unique functions of the mass. As the mass increases, the radius, the luminosity and the effective temperature all increase.

2. About 5 per cent of the stars are so called white dwarfs. They form a sequence roughly parallel to the main sequence, but located below it in the H-R diagram. For equal effective temperature, the luminosity of the average white dwarf is about  $10^4$  times smaller than for a main sequence star. The scatter of luminosity around the white dwarf sequence is considerable, the range of L for equal temperature being measured by a factor of over a hundred. However, even the most luminous white dwarfs are about 1000 times fainter than the main sequence stars of the same temperature.

It follows that the radii of the white dwarfs are small compared to the main sequence stars,  $R \sim 0.01 R_{\odot}$ , or on the average about the size of the earth. The few well-determined masses of white dwarfs are in the range  $0.1 - 1 M_{\odot}$  indicating average densities  $\sim 10^5 \text{ g cm}^{-3}$ .

3. About 0.5 per cent of the stars are K giants (effective temperature  $\sim 4,000^{\circ}$ ) with L about 40-200  $L_{\odot}$ . The radii are in the range 10-20 while the masses are 2-4 solar masses. The radii of the K giants are much larger than for main-sequence stars of the same mass, or the same effective temperature.

4. About 0.5 per cent of the stars are so-called K subgiants, with  $T_e \sim 4000^{\circ}$ , and  $L \sim 10 L_{\odot}$ . The radii are around  $5 R_{\odot}$ , The masses are  $\sim 1 - 1.5 M_{\odot}$

We shall discuss the relation of subgiants and giants later in connection with the evolution of stars in galactic clusters.

5. Giants and subgiants intermediate between the K giants and the K subgiants respectively and the main sequence exist but they are scarce. There is a pronounced gap in the H-R diagram between the main sequence and the K giants. There is also a gap between the main sequence and K subgiants but it is much less pronounced.

6. Stars of very high luminosity  $10^4 - 10^5 L_{\odot}$  exist for all spectral types everywhere above the main sequence giants and subgiants. However, they are extremely scarce. They hardly occur outside the spiral arms, and there they form the order of one millionth of all stars. The radii are 10-1000 solar radii, and the masses presumably in the range of 10-100 solar masses.

Although the supergiants are extremely scarce in samples where the stars of all luminosities are represented by equal volumes, they are rather prominent in samples selected according to apparent intensity. Thus, among the 200 hundred brightest stars in the sky, about ten percent are supergiants. The same selection effect has the consequence that the category of main sequence M stars (red-low luminosity stars), to which more than 50 per cent of all stars belong, does not have a single representative among the stars visible to the naked eye.

7. When the stars within 10 par sec (sample b.) for which luminosity and effective temperature (or an equivalent quantity) had been determined with high precision are plotted in the H-R diagram, it appears that there are a few G-M stars below the main sequence which form a separate sequence. They are about 3 times less luminous than the main sequence stars of the same temperature. All are below the part of the main sequence where it is very well defined (cf. above). Therefore, they cannot be regarded as just the largest deviations in a continuous distribution. Furthermore they stand out from normal main sequence stars by their very high space velocities. Their metal absorption lines are weaker than for a main sequence star

of the same effective temperature, and this cannot be explained as a consequence of the lower luminosity, but indicates relatively lower heavy element content.

Sample b contains only 5 stars which can be assigned to this category with certainty (there are also three or four doubtful cases). The stars are referred to as subdwarfs. The subdwarfs presumably make up about 4 percent of all stars of the local sample of our galaxy. However, because of their high space velocities they must populate a volume of space that extends much farther away from the galactic plane than the corresponding main sequence star space. Therefore the subdwarfs may well represent 20 per cent or more of all stars in our galaxy.

Presumably they predominate in the galactic halo. Samples extending to a radius of 100pc. have been searched for more luminous subdwarfs. A group of F stars (about 20 stars) has thus far been established, with the following properties: a.

On the average the stars fall below the main sequence ( by a factor of three in luminosity) but the data are not accurate enough to establish this in individual cases. b: The spectra are strikingly different from those of the main sequence stars indicating a low heavy element content (lower by a factor of 10-20) c: The space velocities are high. Clearly this is an extension to higher luminosity F Stars of the subdwarf sequence found in the sample of stars within 10pc. ( the expected number of F type subdwarfs within 10 pc. is less than 1, according to the statistics obtained from the larger sample).

Population I and population II - As we have just seen the main sequence stars and the subdwarfs differ greatly with respect to chemical composition, space distribution and space velocity. They belong to two different star categories which are usually called population I and population II.

Actually the names population I and population II were introduced in connection with investigations of galaxies. When the studies are extended beyond

our nearest surroundings in the galaxy, and to other galaxies, it is found that differences corresponding to differences between ordinary main sequence stars (population I) and subdwarfs (population II) exist among stars of high luminosity. The stars in question are very much smaller in number than the low-luminosity main-sequence stars and subdwarfs that were just considered, but they are very important since representative stars can be observed in distant parts of our galaxy and in other galaxies. The most important are the following (most of the categories of population I have already been mentioned):

Stars of high luminosity

Population I

Main sequence O-B3 stars  $L = 10^4 - 10^5 L_{\odot}$

Supergiants B-M  $L = 10^4 - 10^6 L_{\odot}$

$\delta$  - Cephei stars  $L = 10^3 - 10^4 L_{\odot}$

The brightest stars in many galactic clusters

Population II

RR Lyrae stars  $L \sim 10^2 - 10^1 L_{\odot}$

The brightest stars in most globular clusters, namely

K stars with  $L \sim 10^3 L_{\odot}$

The high luminosity population I stars have small average distances from the galactic plane and are almost exclusively found in the spiral arms. The scatter of their velocities around the velocity vector of the galactic circular rotation is very small. We shall presently discuss the fact that these stars are all very young ( $\sim 10^6 - 10^8$  years). They have formed out of interstellar matter in the spiral arms and have not yet had time to spread out into the space between the arms.

The great majority of population I stars are ordinary main sequence stars of low luminosity. They are on the average much older (average age  $\sim 3 \times 10^9$  years) and have spread out over the entire space of the galactic disc. In other respects (chemical composition) they are akin to the high-luminosity population I stars. Population II stars are all old stars (age  $\sim 6 \times 10^9 - 10^{10}$  years), even the

brightest among them. They have larger average distances from the galactic plane and thus populate both the galactic disc and the galactic halo. They do not participate much in the galactic rotation and have high random space velocities.

Stars with properties intermediate between those of population I and population II exist. There may, or may not be a continuous transition between population I and population II stars.

Galactic structure and kinematics as a function of position in the H-R diagram.

It is clear from what has been mentioned that the distribution of stars in space should be determined not only separately for population I and population II stars, but, in that it is possible, individually for stars in various regions of the H-R diagram.

The same consideration applies to studies of the space velocities where a subdivision of the stars according to location in the H-R diagram is essential.

Space velocities relative to the sun are determined by combining proper motion (angular motion per year), distance, and radial velocity (determinable from the Doppler effect in absorption lines to an accuracy of a few km/sec. or better). Complete data are generally available only for stars in the neighborhood of the sun, but the samples go somewhat beyond sample c and the naked eye stars considered in connection with the H-R diagram. The space velocities can be reduced to the local centroid of circular galactic rotation, or to the center of the galaxy; the circular galactic rotation velocity being known. Complete data are available for several thousand stars, and partial data for about 200,000 stars. Thus statistical studies of the velocity is subdivided according to location in the H-R diagram. The dispersions of space velocity in the direction of the galactic center, the direction  $90^\circ$  from the galactic center in the galactic plane, and in the direction perpendicular to the galactic plane have been determined. This has

been done separately for about a dozen subgroups of stars in various parts of the H-R diagram.

Interpretation of stellar distribution in the H-R diagram. According to quite general considerations one expects the structure of a star, its luminosity radius and emitted spectrum to be a function of the following parameters:

1. The mass,  $Z$ , the initial chemical composition, and  $t$ , the age. Further parameters that might be of importance are the total angular momentum and possibly the total magnetic energy contained in the star.

A considerable part of our discussion will be concerned with the problem of connecting the parameters mass, initial chemical composition and age on the one hand, with location in the H-R diagram on the other.

Variable stars, novae, supernovae. - Brief mention will now be made of certain special categories of stars that play an important role in discussion of stellar evolution.

Of the several classes of variable stars we shall mention only four: the RR Lyrae stars, the  $\delta$  - Cephei stars, the red long-period variables (Mira stars), and the T Tauri stars.

The RR Lyrae stars show periodic very regular light variations with periods in the range from 0.1-1 days. The maximum luminosity is about 3 times the minimum luminosity. These stars are of spectral class A and luminosity  $\sim 100 L_{\odot}$ . According to their space distributions and space velocities they are population II stars.

The  $\delta$  - Cephei stars also show periodic, very regular light variations, however the periods are in the range from somewhat over one day to about 40 days, while the amplitudes are similar to those of the RR Lyrae stars. Spectral class and luminosity are functions of the period. Short period  $\delta$  - Cephei stars are in

spectral class F and have a luminosity of  $\sim 400 L_{\odot}$ . Long period  $\delta$ -Cephei stars belong to spectral class K and have luminosity  $\sim 30,000 L_{\odot}$ . They are population I stars with small average distance from the galactic plane, and small random space velocities relative to the local galactic rotation.

The variability of RR Lyrae stars and  $\delta$ -Cephei stars can be interpreted in terms of a theory of periodic radial pulsation, the radius varying about 20 per cent during a period.

The red long period variables show periodic light variations which are much less regular than those of the RR Lyrae stars and the  $\delta$ -Cephei stars. The periods are in the range 40-600 days. The amplitudes in energy output (luminosity) are similar to what is found for RR Lyrae and  $\delta$ -Cephei stars, however the amplitudes in visual light are considerably larger. The spectral type is M and the luminosity is about  $5000 L_{\odot}$ . With respect to space distribution and space velocity the red long-period variables are intermediate between RR Lyrae stars and  $\delta$ -Cephei stars, however closer to the former so that they are usually classed as population II.

The T Tauri stars are irregular variables that show emission lines in their spectra. The spectral types are F, G, or K. The luminosities are much smaller than for giants, but larger than the main sequence luminosity corresponding to the spectral type. The T Tauri stars are much more numerous per unit volume than the other classes of variables. They are population I objects, generally concentrated in regions where the density of inter stellar matter is high. As we shall see, they are considered to be very young stars of relatively small mass which are still in the contraction stage that precedes the stage where nuclear energy is released in the interior of the star.

The novae are characterized by outbursts during which the luminosity increases by a factor of 100-10,000, the violent phase of the outburst, lasting about 10-100 days. At maximum  $L \sim 10^5 L_{\odot}$ . After 10-20 years the novae return

to a luminosity nearly equal to that before the outburst. During an outburst the mass-loss is  $\sim 10^{-6}$  solar masses and the total energy loss is  $\sim 10^{44}$  ergs. About 100 nova outbursts occur per year in our galaxy. Before and after the outburst the stars in question are presumably 0 stars with  $L \sim 1-100 L_{\odot}$  i.e. ., well below the main sequence, but above the white dwarf region. These outbursts are probably recurring phenomena, with perhaps many thousands of occurrences at intervals of  $10^4 - 10^6$  years.

The supernovae during their outbursts reach  $L \sim 10^8 - 10^9 L_{\odot}$ . The total energy radiated during a supernova outburst is  $\sim 10^{48} - 10^{49}$  ergs and the mass loss is probably  $\sim 1 M_{\odot}$ . The frequency is of the order of 1 in a hundred years in our galaxy.

Double and multiple stars. About 30,000 stars are known to be double or multiple. However, surveys of the nearest stars, for which the chances of discovery of duplicity are good, show that the number of double or multiple systems is of the same order as the number of single stars.

Globular clusters, Galactic clusters, Associations. Globular clusters consist of  $\sim 10^5$  stars, contained in a roughly spherical volume with a diameter  $\sim 30$  pc., so that the star density is  $\sim 10^2$  ~~times~~ times higher than in normal region in the solar neighborhood. About 100 globular clusters are known. They are largely found in the galactic halo, and the stars belong to population II. The space velocities of globular clusters are characteristic of population II.

Galactic clusters consist of  $20 - 10^4$  stars contained in a volume of space about 5 pc in diameter. The Pleiades and the Hyades are well known examples of galactic clusters. About 500 galactic clusters are known. The stars belong to population I ( see above).

Associations are clusterings of O and B stars. They do not stand out through

high star density when stars of all types are considered, but if O and B stars only are plotted, they stand out very clearly. In some associations A stars and T Tauri stars are known to be present. About 30 associations are known. The established membership varies from less than ten to many hundred stars.

Globular clusters, galactic clusters and associations are key objects in the study of stellar evolution. We shall have occasion to discuss their properties and in particular the distribution of the member stars in the H-R diagram.

Some numerical values and definitions. Supplementary remarks.

1 pc = 206,265 astronomical units (mean distance earth-sun) =  $3 \times 10^{18}$  cm = 3.26 light years.

1 pc /  $10^6$  years  $\sim$  1 km./sec.

Apparent magnitude  $m = \text{const.} - 2.5 \log_{10} I$  ( $I = \text{intensity}$ ). The constant is chosen in such a way that stars at the limit of naked-eye visibility are of magnitude  $6^m$ . The absolute magnitude  $M$ , is defined as the apparent magnitude that would be observed if the star were placed at a distance of 10 pc. If the actual distance is  $r$ , then  $M = m + 5 - 5 \log_{10} (r)$ , assuming that the inverse square law of intensities holds. If there is interstellar absorption in the light path to the star, then a corresponding correction has to be applied.

Mass of the sun =  $2.0 \times 10^{33}$  g.

Radius of the sun =  $7 \times 10^{10}$  cm.

Luminosity of the sun  $4 \times 10^{33}$  erg/sec.

Distance of the sun from the center of the galaxy  $\sim 8000$  pc.

Galactic rotation, circular velocity at the distance of the sun from the center  $\sim 200$  km./sec. Corresponding period of 1 revolution  $\sim 2 \times 10^8$  years, corresponding acceleration  $2 \times 10^{-8}$  cm./sec<sup>2</sup>.

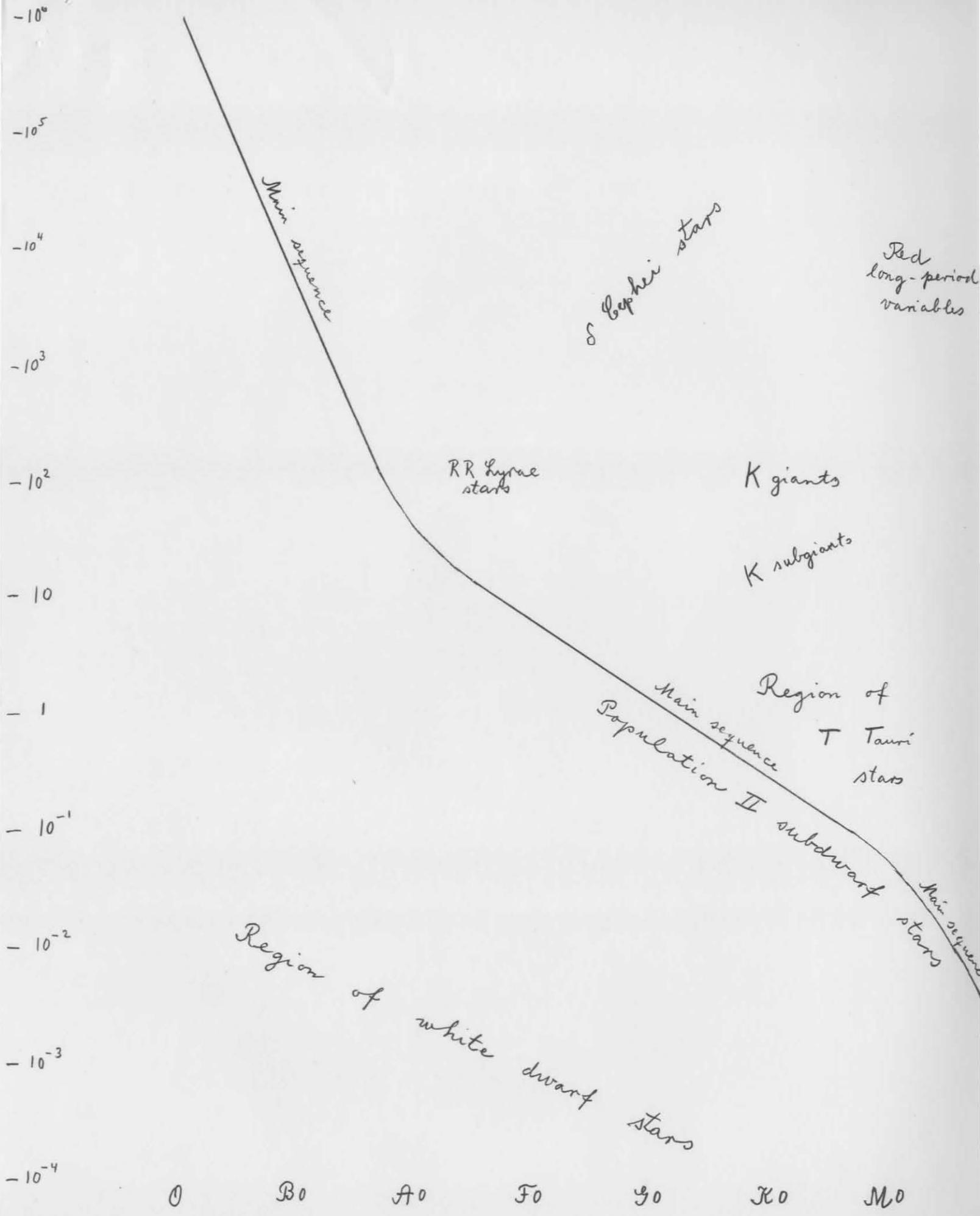
Total mass of galactic system  $\sim 2 \times 10^{10}$  solar masses.

The sample of the nearest stars contains many faint stars, for instance, there are several of apparent magnitude  $14^m$  in sample b. (stars within 10 pc.). The work involved in measuring trigonometric parallaxes for all stars down to  $14^m$  in order to find those with distances less than 10 pc. would be quite prohibitive. The procedure used to find these stars is the following: nearly all stars nearer than 10 pc. will have proper motions larger than  $0''.2$  per year (only the stars with tangential velocities less than 10 km/sec. may not). Stars with proper motions larger than  $0''.2$  per year can be picked out with relative ease by comparison of pairs of celestial photographs taken, say, 20 years apart. This has been done for stars down to magnitude  $14^m - 15^m$  for most of the sky (to fainter stars for certain areas). The stars with proper motion larger than  $0''.2$  are only a very small fraction of all stars brighter than  $14^m$ . These can be further investigated and their distances found to determine whether or not they belong to the sample in question.

This may be further illustrated by the following example. There are about 10 million stars brighter than  $13^m$ . Investigations of stars with large proper motions down to  $13^m$  yielded 5 population II subdwarfs. If the result is projected from the neighborhood to the whole galaxy, taking into account the halo character of the population II subdwarfs (see above), then it is estimated that our galaxy contains of the order of  $10^{10} - 10^{11}$  population II subdwarfs.

Properties of main sequence stars

	Te	$L/L_{\odot}$	$R/R_{\odot}$	$M/M_{\odot}$	Average rot. vel.	Average <del>sp</del> <sup>sp</sup> vel. rel. to local centroid	Average distance from gal. plane
O	40 000°	$10^5$	6	30	200 km/sec.		
B0	30 000	$10^4$	4	15	200	10 km/sec	30 pc.
A0	12 000	40	1.5	2.5	100	20	100
F0	8 000	10	1.2	1.8	50	25	200
G0	6 000	1	1	1	0	30	300
K0	4 500	0.2	0.7	0.6	0	45	500
M0	3 500	0.02	0.5	0.5	0	60	800



## The Composition of Stars and their Ages

By Bengt Strömgen

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The Halley Lecture for 1958, delivered at Oxford on May 27

Mr. Vice-Chancellor, Ladies and Gentlemen:-

Twenty-five years ago Henry Norris Russell delivered the Halley Lecture on "The Composition of the Stars". Russell described his investigations of the chemical composition of the atmospheres of the sun and the stars through quantitative spectral analysis.

The methods for determining relative abundances of the elements from the strengths of their spectral lines have been further developed and made more precise since Russell carried out his pioneer work in the field. Through the efforts of a number of astrophysicists detailed information on the chemical composition of the atmospheres of the sun and several bright stars has been obtained.

In the investigations in question the strengths of large numbers of absorption lines in stellar spectra were determined through photographic photometry of high-dispersion spectra, and the line strengths were then evaluated and converted into relative abundances on the basis of theoretical calculations of the physical properties of model stellar atmospheres.

What I wish to talk about today are investigations on the composition of stars which in one sense are much more limited in scope than those just referred to. They aim at the determination of certain broad features of chemical composition, such as the relative abundances of hydrogen, helium and the group referred to as the heavy elements (which consists of all the other elements). Through this limitation of scope it becomes possible to investigate large numbers of stars, and to extend the investigations to very faint and relatively distant stars.

More specifically, I am going to discuss methods of photoelectric narrow-band photometry and their application to problems concerning the composition of stars and their ages.

In his lecture Russell emphasized the fact that stellar spectra are remarkably similar inter se. The variety is much less than it would be if the relative abundances of many elements varied appreciably from star to star. This observed uniformity suggests that the problems of the interpretation of stellar spectra may be tackled successfully, in a first approximation, on the assumption of uniform chemical composition of the stars. As the analysis progresses deviations from this simple picture, if they exist, will become apparent.

Consider a group of stars of the same initial chemical composition (composition at formation). The stars may then differ in two respects, namely in mass and in age. A star of a given mass will go through a sequence of stages of evolution. In other words, the radius, the luminosity, and the emitted spectrum of the star will be functions of the age of the star. For a star of different mass the course of evolution will be different, so that the properties of a star, including its spectrum, will depend upon its mass and its age.

Theoretical investigations of the internal constitution of stars and of stellar evolution carried out by a number of astrophysicists during the last few decades have shown that the normal course of evolution of a star is as follows. After a relatively brief period of contraction the star reaches a stage where thermonuclear processes in the central region generate an amount of energy that balances the energy radiated by the star into space. When the star's store of nuclear energy has been exhausted it will relatively quickly go through a phase of evolution, characterized on the whole by contraction, and for the more massive stars by loss of mass into the surrounding space. Finally part of the stellar mass may form a white dwarf, or if the mass of the star is below the Chandrasekhar limit, all of it may go into a white dwarf.

We shall be concerned here mainly with the phase of evolution where the thermonuclear energy generation equals the luminosity. During this phase there is a continuous change of the properties of the star which is caused by the gradual exhaustion of the star's nuclear fuel. A detailed analysis of the process in question

has been carried out in recent years through the combination of observational work on the properties of stars in galactic clusters and associations and extensive theoretical calculations on stellar evolution. Quite briefly, a galactic cluster or association gives a picture of the properties of a number of stars that differ in mass but which are all of the same age. Some clusters and associations are very young, and from these it is learned what the properties of stars are when they have just reached the phase when thermonuclear processes supply the radiation of the star and only very little of the nuclear fuel has been used up (age-zero stars). Other clusters and associations are older and show the effect of the gradual exhaustion of hydrogen through its transmutation into helium. They can be arranged according to their ages, and when the cluster and association material is put together in this fashion a good picture of the dependence of the properties of a star upon the two parameters, mass and age, emerges (cf. e.g. recent expositions by H.L. Johnson (1) and A.R. Sandage (2)). When the discussion is limited to population I clusters and associations the observed facts can be well accounted for on the basis of the assumption that the stars all have the same initial composition. The stars in question differ widely with respect to luminosity, radius and surface temperature, and the appearance of the absorption line spectrum varies greatly from star to star, however in the main the differences can all be interpreted in terms of differences in the two basic parameters, mass and age.

We now ask the question, Is it possible to devise accurate and convenient methods for the determination of mass and age of a star from observed properties of the spectrum emitted by the star. We desire methods that can be used for stars in general where we do not have the additional information available in the case of cluster and association stars.

Let us take as the starting point of the discussion the fact that the spectra of the great majority of population I stars fit into a two-dimensional classification scheme. This corresponds to the dependence of the spectrum upon two parameters, mass and age. In the Morgan-Keenan classification scheme the two observational parameters,

which can be determined by visual inspection of the stellar absorption line spectrum, are the spectral class and the luminosity class. The former is closely correlated with the surface temperature of the star, the latter (for given spectral class) with the luminosity. Now, the information derived from observations of stars in clusters and associations makes it possible to calibrate the two-dimensional MK classification in terms of mass and age. For some purposes this method of age determination is most valuable. However, the accuracy obtained is insufficient in certain other contexts, and it would seem quite desirable to improve the accuracy of the spectroscopic method of determination of mass and age. This has in fact proved possible through the method of photoelectric narrow-band photometry of selected features of stellar spectra that I mentioned earlier.

Before describing the method I wish to refer briefly to the efforts made in recent years to improve the accuracy of two-dimensional spectral classification through photographic spectrophotometry of both continuum and absorption lines. Very important progress has been made through the work of Chalonge and his collaborators(3), Petrie (4), and others. The use of a photographic spectrum for the purpose has many obvious advantages. However, in certain applications the method of photoelectric narrow-band photometry, which as we shall see allows fast and accurate work and which reaches much fainter stars, may be preferred.

Since 1951 I have investigated possibilities of the method of two-dimensional spectral classification through photoelectric narrow-band photometry, chiefly at McDonald Observatory. I shall first describe work on B stars, then briefly mention investigations on A stars, and discuss finally population I F and population II F stars. The work in question developed from Lindblad's investigations of two-dimensional classification with the help of short objective-prism spectra. The success of Lindblad's method indicated that two-dimensional classification could be made on the basis of determinations of the intensity of certain strong absorption features in the spectrum. The features in question are in fact so strong that they can be conveniently and accurately measured through photoelectric narrow-band photometry.

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Consider now the B stars. Since the investigations of Barlier, Chalonge and Vassy, and of Ohman more than twenty years ago it has been known that the magnitude of the Balmer discontinuity in the ultraviolet spectrum is a very good indicator of surface temperature (or effective temperature) for the high-temperature stars in question. Further, the work of Lindblad just referred to had shown that the strength of the Balmer absorption lines, for instance that of the H $\beta$  line, indicates the stellar luminosity. The spectral features in question are both relatively strong, and one would therefore expect that the effective temperature and the luminosity of a star could be determined through photoelectric narrow-band photometry that measured the magnitude of the Balmer discontinuity and the strength of the H $\beta$  line. From the previous discussion it is also clear that one could hope to determine from such measures not only the effective temperature and the luminosity, but also the two basic parameters, mass and age. These expectations proved to be correct.

In other words, the suggested procedure would be to determine the mass and the age of a star by asking two questions: the magnitude of the Balmer discontinuity and the strength of the H $\beta$  absorption line. In principle, if we assume that our stars all have the same chemical composition, the two basic parameters mass and age will determine all the properties of the star, including the two quantities measured. The determination of mass and age from the observed quantities is then a problem with two unknowns and two conditions. If we have chosen the observational criteria well, the problem will have an unambiguous solution of adequate precision, at least within a certain range of masses and ages.

I shall now briefly describe the photoelectric photometers that have been employed for the measures of the Balmer discontinuity and the H $\beta$  strength. The first photometer used was a standard type instrument to which a filter box holding six narrow-band filters had been added. Five of the filters are interference filters, the sixth is an ultraviolet glass filter. The wave-lengths of maximum transmission and the halfwidths of the filters are as follows: a 5000 Å (90Å), b 4861(35), c 4700(100),

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d 4500(80), e 4030(90), f 3600(350). For each program star the intensities through the six filters were measured in succession. The measured intensities  $I(a)$ , ...  $I(f)$  were then combined to yield two indices  $c$  and  $l$ , of which the first measures the Balmer discontinuity, the second the  $H\beta$  strength:

$$c = \text{constant} + 2.5 \left[ 2 \log I(e) - \log I(d) - \log I(f) \right]$$
$$l = \text{constant} + 2.5 \left[ 1/2 \log I(a) - 1/2 \log I(c) - \log I(b) \right]$$

The wave-length regions are chosen in such a way that the indices  $c$  and  $l$  are practically insensitive to the effect of interstellar reddening. Filter  $f$  transmits largely in the wave-length region below the Balmer discontinuity at  $3647\text{\AA}$  and the region just above  $3647\text{\AA}$  which is affected by the higher members of the Balmer series. When the Balmer discontinuity is strong,  $I(f)$  is relatively low and  $c$  is correspondingly large. Filter  $b$  transmits in the region around  $H\beta$ . When the  $H\beta$  absorption is strong  $I(b)$  is low and hence  $l$  is large. The width of filter  $b$  is so chosen that even the strongest  $H\beta$  lines are almost completely within the band of high transmission, while faint  $H\beta$  lines can still be adequately measured through precision photoelectric photometry. In recent observing series an  $H\beta$  filter with a half-width of  $15\text{\AA}$  has been employed. The sensitivity of the  $l$ -index to  $H\beta$  absorption was hereby approximately doubled, and the measures improved for all stars but those with the strongest  $H\beta$  lines.

For B stars a measure of the Balmer discontinuity which is equivalent to  $c$ , and as accurate, can be derived from standard UBV photometry (cf. e.g. Morgan and Harris (5)). The only narrow-band photometry required is then the determination of the  $H\beta$ -index  $l$ .

A second photoelectric photometer for measuring the  $H\beta$ -index  $l$  was constructed a year ago and used with the 36-inch reflector of the McDonald Observatory. In this photometer a 30 per cent - 70 per cent beam splitter is used in connection with two photomultipliers (P1 and P2). First, one set of measures is made with a  $15\text{\AA}$   $H\beta$  filter in front of P 1 and a  $150\text{\AA}$   $H\beta$  filter (comparison filter) in front of P 2. The signal ratio is a measure of the  $H\beta$

strength. A second set of measures is made with a  $150 \text{ \AA}$  H $\beta$  filter in front of P 1 while the  $150 \text{ \AA}$  comparison filter remains in the light path to P2. If the first signal ratio is divided by the second an H $\beta$ -index is obtained which is independent of the relative sensitivity of P1 and P2. Furthermore, this index is quite independent of fluctuations in the transparency of the terrestrial atmosphere since it is derived from the ratio of signals obtained through simultaneous measures in two practically equal mean wave lengths. I designed the second photometer to gain this advantage, even though it meant a light loss of a few tenths of a magnitude. It is of course a great advantage to be able to obtain precision photometry even with a non-photometric sky. Experience has fully confirmed the expectations in that it proved possible to measure I-indices through thin clouds of  $1^m$ - $2^m$  extinction without any reduction in photometric accuracy.

The probable error of one observation of an H $\beta$ -index was found to be  $\pm 0^m.004$  for measures with the first photometer and a good sky. For observations with the second photometer the p.e. was also  $\pm 0^m.004$  irrespective of the quality of the sky. The p.e. for a determination of the index c(one observation) was  $\pm 0^m.008$ .

With the second H $\beta$  photometer attached to the 36-inch reflector of the McDonald Observatory the I-index can be obtained with a few minutes observing time for stars brighter than  $11^m$ . It has proved possible to observe at a rate of about 15 stars per hour. Fainter stars can be reached by extending the integration time with the current integrators employed in the photometer to 1 hour. The corresponding limiting magnitude for I-observations with the McDonald Observatory 82-inch reflector is then  $(16^m$ - $17^m$  depending on the accuracy required. Since it takes an exposure of about 1 hour with the 82-inch to obtain a slit-spectrum for classification purposes of 12-magnitude star, the gain in limiting magnitude of the photoelectric method over the photographic method is  $4^m$ - $5^m$ , as was to be expected. It should be emphasized in this connection that the photoelectric observation in question yields only one index, whereas the photographic observation of course gives all the information contained in a classification spectrum. When feasible a combination of the

two methods would in many cases yield the most satisfactory results.

When the Balmer discontinuity index  $c$  and the  $H\beta$  index  $l$  have been determined for a star, its position in a  $c-l$  diagram can be plotted. Fig. 1 summarizes results obtained for a number of B stars at Mc Donald Observatory by myself, and by D.L. Crawford (6). The measures have all been reduced to one system. It should be noted that the  $l$ -index system of Fig. 1 corresponds to measures with the  $H\beta$  filter of  $35 \text{ \AA}$  halfwidth. When the measures made with the  $15 \text{ \AA}$  filter are reduced to the  $l$ -scale of Fig.1, the probable error (one observation) becomes  $\pm 0.^m002$ .

In Fig. 1 the lower-limit line in the  $c - l$  diagram represents very young stars (the individual stars in question are not plotted in the diagram). It was obtained through measures of stars in the I Ori and II Sco associations which are known to be very young (only a few million years) because they contain stars of very high luminosity that exhaust their supply of nuclear energy very quickly. At the top of the diagram are points corresponding to supergiants. We shall not be concerned with these stars here, but notice in passing that they are well segregated from the stars of luminosity classes II - V in the  $c-l$  diagram.

Above the line which is the locus of stars of age zero we find a fairly wide distribution of points corresponding to luminosity classes II-V. On the average the height above the age-zero line increases as we go from stars of luminosity class V to luminosity class III and II stars. In particular we notice that the height above the age-zero line is generally considerable in comparison with the probable error of a determination of the location of a point in the  $c-l$  diagram, even for the stars of luminosity class V, particularly for the spectral range B5-B9.

In a recent investigation (7) I have attempted a calibration of the  $c-l$  diagram in terms of spectral class and absolute magnitude, using absolute magnitudes determined by Blaauw for stars in II Sco, and absolute magnitudes for Pleiades stars according to Eggen, Harris and H.L. Johnson. Absolute magnitude calibration was obtained for points close to the age-zero line, and also for the region above this line in the spectral range B7 - B9. The precision of absolute magnitudes obtained from the  $c-l$  classification is measured by a p.e. (one observation) of  $0.^m2-0.^m3$ , while

the p.e. of the spectral classes is approximately 0.01 of a class. We shall not discuss these problems any further, but turn to the main question of the calibration of the  $c-l$  diagram of the B stars in terms of mass and age, with a particular view to age determination.

It is possible to derive an approximate solution of the problem through the use of data on stars in clusters and associations. D.L. Crawford (6) has derived the average location curves in the  $c-l$  diagram for a number of clusters and associations and studied the shift of the location curve in the direction of smaller  $l$  with increasing age. However, I wish to discuss here a calibration of the  $c-l$  diagram in terms of mass and age which rests on theoretical calculation, and which applies to the spectral range B3 - B9. At the present moment these calculations are not quite as accurate as one might wish, but there is hope that further calculations now in progress will yield results of very good precision.

We begin by considering the sequence of stars on the age-zero line in the  $c-l$  diagram of the B stars. We know  $l$  as a function of  $c$ , and we also know the luminosity  $L$  as a function of  $c$  from the calibration just referred to. From the work of D.L. Crawford (6) the relation between equivalent width  $W$  of the  $H\beta$  line and  $l$  is known. The relation between  $c$  and the effective temperature  $T_e$  can be evaluated with fair accuracy on the basis of model atmosphere work by Osawa, Saito, Miss Underhill, Miss Mc Donald, Aller and others. Good estimates of mass  $M$  as a function of  $L$ , and hence of  $c$  are available from spectroscopic binary material. From  $L$  and  $T_e$  the stellar radius  $R$  is deduced in the customary way, and the gravity  $g$  in the stellar atmosphere is computed from  $M$  and  $R$ , again as a function of  $c$ , for the age-zero line.

As is well known the atmospheric gravity does not vary much along the age-zero line in the spectral range considered, i.e. B3-B9. By applying very small corrections we can convert the known relation of the equivalent width  $W$  and  $T_e$  valid for the age-zero line into a relation between  $W$  and  $T_e$  for a given constant  $g$ , say  $g = 1.0 \times 10^4$ .

Next we consider what happens to the emitted spectrum, or rather to  $c$  and  $l$ , as a star evolves from age zero to a given age  $t$ . We restrict ourselves to the consideration of evolution during the phase when there is still hydrogen fuel left at every point in the stellar interior, even at the center where the conversion of hydrogen into helium is the fastest. Recent work by Tayler (8) and by Kushwaha (9) provides the basis for a calculation of the rate of change of radius  $R$  and luminosity  $L$  as a function of time  $t$  for a star of given mass  $M$ .

Starting with the data corresponding to a star on the age-zero line of given  $c$ , we can compute the radius  $R$  and the luminosity  $L$ , and hence the effective temperature  $T_e$  and the atmospheric gravity  $g$  as a function of time  $t$ . Next we derive  $c$  from the established relation between  $T_e$  and  $c$  which is known to be quite insensitive to variations in  $g$  in the range considered. Then we compute the equivalent width  $W$  from  $T_e$ , first for  $g = 1.0 \times 10^4$  (cf. above). Finally we derive the change in  $W$  corresponding to the change of  $g$  from  $1.0 \times 10^4$  to the proper value of  $g$ , using the approximate relation  $W = g^{0.2}$  valid for the range of  $T$  and  $g$  in question. Model atmosphere calculations by de Jager and Neven (10) indicate that the inaccuracies due to the use of this approximation are not very great. The last step is the derivation of  $l$  from  $W$  with the help of the standard  $W(l)$  relation.

Summarizing, we see that it is possible to follow the path in the  $c-l$  diagram of a point corresponding to an evolving star, starting from the zero-age line, and to indicate from point to point the age of the star.

Results of calculations according to this procedure are represented in Fig. 1. Five paths of evolution starting at five different points of the age-zero line, corresponding to masses equal to 3.5, 4.0, 4.7, 5.5 and 6.8 solar masses are shown. Also shown are lines of equal age corresponding to the ages  $25 \times 10^6$ ,  $50 \times 10^6$ ,  $75 \times 10^6$  and  $100 \times 10^6$  years. The lines corresponding to 25 and 50 million years form only a small angle with the age-zero line, whereas the 75 and 100 million year curves bend sharply to the right for the larger masses. This corresponds to the fact that the speed of evolutionary change is high here due to the fact that the hydrogen fuel

in the central region of the star is almost exhausted.

With the aid of the computed theoretical curves of equal age it is now a simple matter to derive the age of a star from observed values of  $c$  and  $l$ . Let us summarize the whole procedure for the determination of the age: 1. Standard UBV photometry or  $c$ -photometry yields  $c$ . 2. The index  $l$  is measured with the  $H\beta$  photometer. 3. The point corresponding to the star is plotted in the  $c$ - $l$  diagram and the age is read off through interpolation between the theoretical curves of equal age.

For example, the points corresponding to Pleiades stars shown in Fig. 1 with one exception lie between the curves corresponding to the ages  $50 \times 10^6$  and  $75 \times 10^6$  years. The average age for the stars in question is found to be about  $70 \times 10^6$  years. This value agrees rather well with age determinations by Lohmann(11) and v. Hoerner (12) based on the distribution of the brighter Pleiades cluster stars in the H-R diagram (100 and 80 million years, respectively).

It should be emphasized that the procedure based on the use of the  $c$ - $l$  diagram is applicable to any star in the spectral and luminosity range in question. There is no limitation to cluster and association stars.

We can obtain an estimate of the precision of the determined ages as follows. Investigations of member stars of the associations I Ori and II Sco show that for these stars the scatter around a mean curve in the  $c$ - $l$  diagram is measured by a root mean square deviation in  $l$  equal to  $\pm 0^m.007$ . This scatter includes the effect of errors of observation, but the latter contribution is small as the p.e. of the  $l$ -value in question is  $\pm 0^m.002$  or less. Since the stars in question are practically of the same age, the scatter indicates the uncertainty which must be expected in the age determinations. At spectral class B 8 the separation between the curves of zero age and age  $25 \times 10^6$  years is about  $0^m.007$ , and we therefore estimate the p.e. of an age determination to be about 15-20 million years here. At B5 the corresponding separation is  $0^m.016$  and the estimated p.e. of an age determination is about 7 million years. In a general way the precision of the age determination increases as we go upward or to the right in the  $c$ - $l$  diagram, because the speed of evolutionary change is

higher here. The estimated uncertainties do not include the effect of errors in the theoretically computed calibration curves.

The fact that ages of individual B stars can be determined with accuracies as indicated might be of some importance in galactic studies. I would like to refer to one possibility, namely investigation of the distribution in space of the place of formation of B5-B9 stars in our galactic neighborhood.

Let us return now to the question of chemical composition. If the B stars considered differ among themselves only with respect to mass and age, then we would expect all their properties to be uniquely determined by the location of the star in the c-l diagram. As far as the available material on spectra and absolute magnitude goes it is in general agreement with this prediction. However there is some evidence of a small scatter, of the order of  $\pm 0.2^m$ , in the absolute magnitudes of stars with the same c and l.

Also, if mass and age are the only variable parameters, we would expect the stars of a cluster or association to show negligible scatter around a line in the c-l diagram, since the ages of such stars are very nearly the same. As already mentioned some scatter is actually observed in this case, but it is small.

There may be more than one reason for this scatter: 1. The scatter in age might not be completely negligible. 2. There might be effects of differences in rotational velocity and in the magnetic properties of the stars. 3. The stars might differ in chemical composition.

The general conclusion in that differences in chemical composition among the B stars investigated so far are not very pronounced. I would like to discuss quite briefly a test for difference in chemical composition that was applied to the I Ori and II Sco associations. As was already mentioned these associations are so young that their curve in the c-l diagram corresponds practically to the zero-age curve, at least in the B5 - B8 range. The mean curve in the c-l diagram is very well determined from the observations for these two associations. It is found that the mean curves for I Ori and II Sco agree within about  $0.004^m$  in l for equal c. This indicates that the chemical composition of the two associations is nearly the same.

We can ask the question, How closely the same. In order to answer this, let us consider the dependence of the location of the zero-age line in the  $c-l$  diagram upon chemical composition. The location is a function of the relative abundances of hydrogen, helium and the group of all heavy elements. If the ratio of helium to hydrogen changes while the abundance of the heavy elements group remains constant, the radius  $R$  and the luminosity  $L$  for a given mass will change. A change of atmospheric gravity  $g$  for given  $T_e$  results, but it so happens that it is very small. However, the change of the helium-hydrogen ratio will affect atmospheric conditions in the following way (in the range of effective temperature in question). Helium does not add appreciably to the atmospheric opacity which is contributed mainly by hydrogen, but it contributes weight. If the relative abundance of helium increases the effect is equivalent to an increase in gravity as far as the electron pressure in the atmosphere is concerned. Since the Stark effect broadening of the hydrogen lines is proportional to electron pressure, the strength of the hydrogen lines will increase with increasing helium content, and the zero-age line will be shifted downward in the  $c-l$  diagram. If we combine the result of these considerations with the fact referred to above namely that the location of the zero-age line in the  $c-l$  diagram for an association can be determined with an accuracy of a few thousands of a magnitude in  $l$ , we can conclude that differences in hydrogen content  $X$  should be detectable if they amount to more than 10 per cent of  $X$ .

A similar analysis can be carried out with regard to changes in the heavy element content. For the B stars considered there is here a direct effect on  $g$  (for given  $T_e$ ) while the atmospheric opacity effect is very small.

We are thus lead to the conclusion that the chemical compositions of I Ori and II Sco are not very different, unless of course the effect of differences in helium content and heavy element content were equal and of opposite sign. Additional information is required to give a definite answer to the question raised hereby.

In this context it is of importance that I Ori and II Sco belong to the same

spiral arm. Comparison of associations in different spiral arms with regard to the exact location of the age-zero line in the  $l$ - $c$  diagram might yield interesting results. We note that an investigation of this effect for B stars in the Magellanic clouds is not beyond the limits of the type of equipment described above.

I shall refer to investigations of A stars through photoelectric narrow-band photometry only very briefly. A discussion (12) of two-dimensional classification of A stars by this method has shown that the accuracy is very satisfactory for A3-A9 stars, the p.e. (one observation) being  $\pm 0.02$  of a spectral class and  $\pm 0.12$  in absolute magnitude.

Age determinations can be made for A stars by plotting the H $\beta$  index  $l$  against  $(B-V)_0$ , i.e. the standard color index corrected for interstellar absorption. Only preliminary results are available at present. It may be mentioned that member stars of the Hyades, Praesepe, and the Ursa major stream are located considerably above the zero-age line in the  $(B-V)_0$  -  $l$  diagram. As the age of these cluster stars is probably  $5 \times 10^8$  -  $10^9$  years it appears that ages for A stars of luminosity classes III - V can be determined with an accuracy of perhaps 100-200 million years. The precision is of course inferior to that obtained for the spectral range B5 - B9 because evolution is much slower.

Turning finally to the F stars, I shall summarize very briefly results concerning two-dimensional classification (cf. (13)) and age determination, and then discuss a question regarding the chemical composition.

For population I F stars the  $c$  -  $l$  diagram has been calibrated in terms of color index  $(B - V)_0$ , or spectral class, and visual absolute magnitude  $M_V$ . Nearby stars, which could be assumed to be practically unaffected by interstellar reddening, and for which relatively accurate trigonometric or cluster parallaxes are available, were measured for this calibration. It was found that  $(B - V)_0$  can be predicted from  $c$  and  $l$  with a p.e. (one observation) of  $\pm 0.008$ , while the corresponding p.e. for  $M_V$  is  $\pm 0.2$ .

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As in the case of the B stars there is for the population I F stars a very well defined lower-limit line of the distribution in the  $c - l$  diagram. This is interpreted as the zero-age line. As the stars evolve they move upward relative to this line. The rate of evolution is of course much lower again than for the A stars, and the accuracy of age determination correspondingly lower. It is estimated that the age of an F star can be determined from  $c$  and  $l$  with a p.e. roughly equal to  $1 \times 10^9$  years. The life-time of these stars being about  $3 \times 10^9 - 1 \times 10^{10}$  years, such age determinations may prove to be of value. No theoretical mass-age calibration of the  $c - l$  diagram for population I F stars has as yet been carried out, but it is quite feasible to do so.

We now ask the question, whether or not the available photometric data for the population I F stars are compatible with the assumption that these stars all have the same initial chemical composition. If the assumption is correct we expect that all stars with the same location in the  $c - l$  diagram are identical. The first test was carried out with the  $(B - V)_0$  material (for practically unreddened stars). It was found that the scatter of  $(B - V)_0$  for stars with the same  $c$  and  $l$  was indeed small, perhaps vanishingly small. The next test made use of the  $(U - B)_0$  material. Here the situation proved to be different. The scatter was easily recognizable, with a total range of  $0^m.15$ , which is many times the range of scatter to be expected from errors of the photometric observations. There is thus in this material clear evidence of the importance of a third variable parameter, in addition to the parameters mass and age.

Already five years ago H.L. Johnson and Morgan (14) in discussing the  $(U - B)$ ,  $(B - V)$  relation for bright nearby population I stars had found evidence for a third classification parameter. Confirmation was obtained by H.L. Johnson (15) through UBV photometry of galactic clusters. Miss Roman's work on weak-line and strong-line F stars pointed in the same direction. Conclusive evidence demonstrating the importance of the third classification parameter has been obtained by Chalonge and

his collaborators through extensive spectrophotometric investigations (cf.(15)).

The cause of the additional scatter in question is undoubtedly a variation from star to star of the abundance ratio heavy elements-hydrogen. Variation in this ratio affects the structure of the star and the emitted spectrum in various ways, but for the F stars the most important effect is that of the metal absorption lines on the intensity distribution in the spectrum. The lines are particularly crowded in the ultraviolet, and a deficiency in ultraviolet light can be explained in terms of a relative metal content above the average.

Although the color index  $U - B$  is of importance as an indicator of the heavy element-hydrogen ratio, it can only be used with confidence together with additional photometry, preferably  $c - l$  photometry. I have found the following index obtained through photoelectric narrow-band photometry to be a very useful indicator of the heavy element-hydrogen ratio, namely

$$m = \text{constant} - 2.5 \left[ \log I(a) + \log I(e) - 2 \log I(d) \right]$$
 where  $I(a)$ ,  $I(d)$  and  $I(e)$  are intensities measured through the filters  $a$ ,  $d$  and  $e$  described on p. .  
For F stars the intensities  $I(a)$  and  $I(d)$  are about equally reduced by the total absorption of metal lines in their transmission range, while the reduction is much stronger for  $I(d)$ . Consequently the index  $m$  increases with increasing metal-hydrogen ratio. On the other hand the index is not sensitive to variation in  $T_e$  and  $g$ , and it is insensitive to interstellar reddening.

It was found (cf. (16) ) that the index  $m$  in a sample of 33 well-observed nearby population I F stars varied over a range of  $0.^m.10$ , with only a small fraction of the variation due to differences in  $T_e$  and  $g$ , or to observational errors. The root mean square variation of  $m$  was  $\pm 0.^m.021$ , or  $\pm 0.^m.020$  after correction for errors of observation. The index  $m$  proved to be well correlated with the  $(U - B)_0$  residuals referred to above, as was to be expected. The range in  $m$  is about 60 per cent of the range in the  $(U - B)_0$  residuals.

The extreme population II F stars studied by Miss Roman (cf. (17)) have a very low metal-hydrogen ratio. Miss Roman has shown that they have an ultraviolet excess of about  $0.2^m$  in comparison with population I F stars, while measures of  $m$  give a value that is over  $0.1^m$  lower than the value for population I F stars.

It is interesting to compare the  $m$ -value and abundance ratios obtained from quantitative spectral analysis for two extreme cases. For the extreme population II F star HD 19445 the metal-hydrogen ratio can be estimated from an investigation by Chamberlain and Aller (18). For the population I F star 20 C Vn, which has an abnormally high  $m$ -value, Savedoff (19) has derived a value of the iron-hydrogen ratio. We use this as an indicator of the metal-hydrogen ratio. The results are given below.

	$m$	Abundance ratio metal-hydrogen
20 C Vn	$0.19^m$	$4 \times 10^{-4}$
Average population I F star	$0.14^m$	$1 \times 10^{-4}$
Extreme population II F star, MD 19445	$0.02^m$	$1 \times 10^{-5}$

These abundance differences, and the corresponding photometric effects are certainly not small, but the cases of 20 C Vn and HD 19445 are of course extreme. However, even for a normal sample of population I F stars the root mean square deviation from the average of the metal-hydrogen ratio is not very small, perhaps  $\pm 40$  per cent. It is interesting to note that  $m$ -photometry indicates that the variations of the metal-hydrogen ratio among stars belonging to the same cluster are much smaller.

In concluding this lecture I would like to comment on a problem concerning the extreme population II F stars, speaking not about work that has been done, but of work that might be done in the near future.

Theoretical work already carried out concerning the internal structure and evolution of extreme population II F stars (cf. (20), and work on model atmospheres corresponding to these stars (cf. (21) indicates that it is now possible to compute theoretical curves of equal age in the  $c - l$  diagram for this stellar category. If  $c - l$  photometry can be obtained for globular cluster stars in the absolute magnitude

range  $2^m - 4^m$ , then an age determination of considerable accuracy might be possible. Furthermore,  $c - l$  photometry of nearby extreme population II F stars in the absolute magnitude range  $4^m - 5^m$  could lead to a determination of their helium-hydrogen ratio. Finally, if the photometry could be pressed to yield  $c-$  and  $l-$ values for the very faint globular cluster stars in this absolute magnitude range, their helium-hydrogen ratio might be determined, without accurate knowledge of the distances of the clusters.

It would seem that photoelectric narrow-band photometry can lead to information on the initial composition of stars and their ages which may help our efforts to form a picture of the evolution of the galaxy.

## An Introduction to Astrophysics

Notes of a Lecture Series by Professor Bengt Strömberg  
The Institute for Advanced Study, Fall 1957

Notes taken by Jeremy Bernstein

### Lecture 2

#### Our Galaxy. Interstellar matter

In lecture 1 we referred to the fact that high luminosity population I stars are all very young and must have formed out of interstellar matter  $10^6 - 10^8$  years ago; i.e. a time interval that is relatively short compared to the age of our galaxy. Now we shall turn to a discussion of the properties of interstellar matter in our galaxy.

Observational data pertaining to interstellar matter. The discussion of interstellar matter is based on observations of the following categories

	<u>Spectral region</u>	<u>Produced by</u>
Emission lines	Optical	<u>Interstellar gas</u>
Absorption lines	Optical	"
Emission lines	Radio, 21 cm.	"
Absorption line	Radio, 21 cm.	"
Thermal radio emission	$\sim 10 \text{ cm.}$	"
Non-thermal radio emission	$\sim 1-15 \text{ m}$	Relativistic electrons moving in a magnetic field
Cosmic rays, presumably of galactic origin		
Interstellar extinction and reddening	Optical	Interstellar particles
Interstellar polarization	Optical	Aligned interstellar particles
Reflection nebulae	Optical	Interstellar particles illuminated by stars

Interstellar emission lines. The Orion nebula ( a well-known diffuse nebula at a distance of about 500 pc and located in the nearest spiral arm) has a spectrum consisting of a number of emission lines super-imposed on the back ground

of a faint continuum. The most prominent emission lines are the following,

Balmer lines $H_{\alpha}$ , $H_{\beta}$ .....	HI
3726, 3728	OII
4959, 5007	OIII
6584, 6548	NII
6731, 6717	<del>R</del> II

The Orion nebula has an angular diameter  $\sim 1^{\circ}$  and a linear diameter  $\sim 10$  pc. . About 100 similar diffuse emission nebulae are known. Most of them much fainter than the Orion nebulae, (down to about 1000 times fainter than the center of the Orion nebulae), but all objects of this class show on plates taken in the ordinary photographic wave length region. With special techniques it is possible to obtain photographs in a narrow band around a strong emission line, in particular  $H_{\alpha}$  . If this is done then the intensity ratio between the emission nebulae and the terrestrial air glow background is considerably enhanced. Hence long exposures with fast cameras can be made and much fainter emission nebulosities photographed. With this technique faint  $H_{\alpha}$  emissions have been recorded which cover much larger areas of the sky than the diffuse emission nebulae. For example, a faint  $H_{\alpha}$  emission region in Vela-Puppis is  $15^{\circ} \times 22^{\circ}$ , and many are known that cover areas  $\sim 10$  square degrees. About 10-20 per cent of the Milky way (galactic latitude  $< 10^{\circ}$ ) shows faint  $H_{\alpha}$  emission. Where spectra of the faint emission region have been obtained, they show the emission lines most prominent in the Orion nebula(cf. above).

Clearly interstellar emission lines can yield very important information concerning the density, distribution and chemical composition of interstellar matter.

Let us consider first the interpretation of the observed interstellar emission lines of hydrogen. When the Balmer lines  $H_{\alpha}$  ,  $H_{\beta}$  , . . . .

are observed to be emitted by inter stellar gas then one can conclude that there is a corresponding population of excited states  $n=3,4, \dots$  of inter stellar hydrogen atoms. The following mechanisms of population of the states in question have been considered: a. capture of a free electron by the hydrogen ion into one of the given states, b. capture of a free electron into some higher state followed by one or more transitions leading to the state in question, c. Lyman line absorption of stellar radiation by hydrogen atoms in the ground state yielding either the electron state under discussion or some higher state from which it is reached by transitions, d. excitation from the ground state by electron collisions.

The analysis of these processes shows, first, that excitation of the higher states of hydrogen ( $n=3,4, \dots$ ) will be negligible unless the hydrogen is appreciably ionized. We start out, then, by considering the ionization equilibrium of hydrogen in inter stellar space.

#### Ionisation of interstellar hydrogen.

Let us consider the idealized situation of a star embedded in an interstellar medium consisting of a pure hydrogen gas of uniform density, say,  $N$  hydrogen atoms and ions per cm. The radius of the star is  $R$ , and we assume that the radiation emitted per unit area of the stellar surface in the wave length region relevant for hydrogen ionization ( $\lambda < 911 \text{ \AA}$ ) can be described with sufficient accuracy by Planck radiation corresponding to a temperature  $T$ . We want to compute the degree of ionization  $x$  of hydrogen ( $xN$  ions and  $(1-x)N$  atoms per  $\text{cm}^3$ ) at a distance  $S$  from the star.

The predominant process of ionization is photo ionization through absorption

by neutral hydrogen in the ground state of a Lyman quantum ( $\lambda < 911 \text{ \AA}$ ) of stellar radiation. Recombination takes place through the capture of a free electron by a hydrogen ion into the ground state or an excited state (in interstellar space capture into an excited state is almost always followed by transitions leading to the ground state). The ionization equilibrium is determined by the condition that the number per second and  $\text{cm}^3$  of photo ionizations and recombinations are equal.

Let us consider the recombination process first. After an electron has been set free by absorbing a quantum  $h\nu$  ( $h\nu > I$  where  $I$  is the ionization potential of hydrogen 13.6 e.v.) it will start its travel through interstellar space with a kinetic energy  $h\nu - I$ . Knowing the frequency dependence of the hydrogen absorption coefficient in the Lyman continuum and the spectral distribution of the ionizing radiation (Planck temperature  $T$ ) we can compute the distribution of the initial kinetic energies of the free electrons and the average initial kinetic energy. The latter turns out to be close to  $3/2 kT$  (1.3 e.v. for  $T=10,000$  deg.) The free electron will suffer a large number of elastic collisions with electrons and protons before it is finally re-captured by a hydrogen ion, since the elastic collision cross sections are the order of  $10^6$  times larger than the capture cross sections. A Maxwell distribution of velocities will therefore be established. Also, in spite of the small energy exchange per collision between electrons and ions, equipartition will be established. The velocity and energy distribution is thus given by the kinetic temperature defined by the average kinetic energy ( $\bar{E} = 3/2 K T_K$ ). We shall presently discuss the relation between  $T$  and  $T_K$  in more detail. In a pure hydrogen medium  $T_K \sim T$ .

The calculation of the number of recombination processes per second per  $\text{cm}^3$  is now simple. The velocity distribution of the electrons is given, and the capture cross sections are known functions of the energy and the state into which capture takes place.

The necessary quadratures can be performed numerically, and the results tabulated.

As an example; for the capture into the state  $n=3$ , the number is to a good approximation (in the relevant temperature range)  $4 \times 10^{-12} T^{-\frac{1}{2}} N_i N_e$  captures per second and per  $\text{cm}^3$ , where  $N_i$  and  $N_e$  are the number of hydrogen ions and free electrons, respectively (in a pure hydrogen gas  $N_i = N_e$ ).

In order to compute the degree of ionization in equilibrium, we compare the matter at a distance  $s$  from the ionizing star to matter in thermodynamical equilibrium at a temperature  $T$  and a hydrogen density of  $N$  atoms or ions per unit volume. The number of photo ionizations in interstellar space is reduced in comparison to thermodynamical equilibrium by, 1. , a factor that is given by the solid angle that is subtended by the star at a distance  $s$ , namely  $w = \frac{R^2}{4s^2}$  and 2. a factor that measures the reduction of the ionizing radiation through continuous absorption by neutral hydrogen, namely  $e^{-\tau_n}$  where  $\tau_n$  is the effective mean optical depth in the relevant wave-length region of photo ionization ( $\lambda < 911 \text{ \AA}$ ). If  $a_n$  is the effective mean continuous absorption coefficient per neutral H atom, then  $\tau_n = a_n(1-x)Ns$ .

The number of recombinations is the same as for thermodynamical equilibrium at temperature  $T_n$ , which is the number at temperature  $T$  multiplied by

$\left(\frac{T_n}{T}\right)^{-\frac{1}{2}}$ , very nearly (cf. the example for  $n=3$ , considered above). The total

correction factor to the equation valid for thermodynamical equilibrium thus becomes

$$w e^{-\tau_n} \left(\frac{T_n}{T}\right)^{\frac{1}{2}}$$

and the equation is

$$\frac{x^2 N}{1-x} = \frac{(2\pi m_e)^{\frac{3}{2}} (kT)^{\frac{3}{2}}}{h^3} e^{-\frac{I}{kT}} w e^{-\tau_n} \left(\frac{T_n}{T}\right)^{\frac{1}{2}}$$

For illustration, consider a point at  $s=10^{20}$  cm.  $\sim$  30 pc. For an ionizing star with  $R=6R_{\odot}=4 \times 10^{11}$  cm.,  $w=4 \times 10^{-18}$ . Since  $a_n=6 \times 10^{-18}$  we find

$\tau_n = 600N(1-\bar{x})$ , where  $\bar{x}$  denotes an average degree of ionization along the path

from the star to the point considered.

The factor  $w$  is referred to the dilution factor. Dilution greatly reduces the degree of ionization. On the other hand the density in interstellar space is extremely small compared to that in stellar atmosphere, and this works in the opposite direction.

Close to the ionizing star, interstellar hydrogen will be highly ionized if  $T > 10,000$  deg. . Thus  $x \sim 1$  and  $1 - x$  is small and we have  $e^{-\tau_u} \approx 1$  hence

$$1 - x \sim s^2$$

$$\tau_u \sim s^3 \quad (s \text{ small, but } \gg R)$$

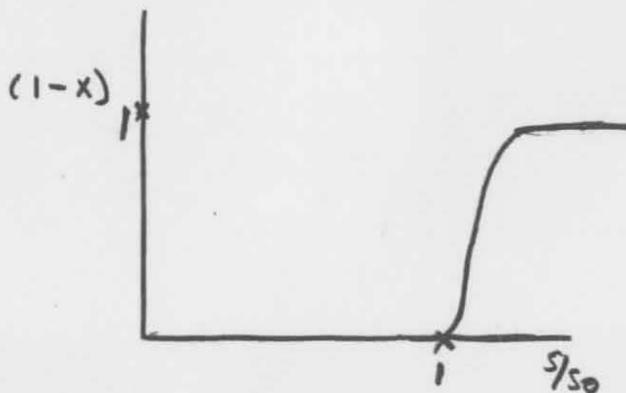
As  $s$  increases the factor  $e^{-\tau_u}$  will become significantly smaller than 1, while  $w$ , of course decreases and correspondingly  $1-x$  increases. As the proportion of neutral atoms goes up  $e^{-\tau_u}$  becomes small and this leads to an accelerated increase in the proportion of neutral atoms. Numerical integration of the equation

$$\frac{d\tau_u}{ds} = a_u N(1-x)$$

taken together with the equation of ionization give  $x$  and  $\tau_u$  as functions of  $s$ . They confirm the expectation that the degree of ionization decreases very rapidly with  $s$ , once  $1-x > 0.1$ . It is found that hydrogen is practically un-ionized outside a sphere with radius equal to (or a few percent in excess of)  $S_0$  where

$$S_0 = N^{-2/3} \left( 3 \frac{(2\pi me)^{3/2}}{h^3} (kT)^{3/2} e^{-\frac{I}{kT}} \left( \frac{\tau_u}{T} \right)^{1/2} R^2 \frac{1}{3 \times 10^{18} a_u} \right)^{1/3}$$

A typical example is shown below:



$s/s_0$	$1-x$	$\tau_m$
0.00	0.000	0.00
0.25	0.001	0.02
0.50	0.003	0.2
0.75	0.01	0.5
0.90	0.03	1.3
0.97	0.06	2.1
1.00	0.15	3.1
1.03	0.67	6.3
1.06	1.00	

The region inside  $s = s_0$  where hydrogen is highly ionized is referred to as an HII region, the outer region of neutral hydrogen is called an HI region.

The radius of the HII region is proportional to  $N^{-2/3}$ . Below are given the values of  $s_0$  for main sequence stars of spectral classes O-Ao. For stars with lower  $T$  the value of  $s_0$  is quite small

	$T$	$s_0$ pc x $N^{-2/3}$
O7	50,000	90
O9	32,000	50
B0	25,000	20
B3	18,600	7
B5	15,500	4
Ao	10,700	0.5

We shall learn that the observations of hydrogen emission lines suggest that  $N \sim 1$  in extended regions, but that there are relatively small regions of high density where  $N$  is  $10 - 10^4$ . Therefore hydrogen ionization is restricted to very small regions around all stars except those of the highest temperature. The high temperature stars are scarce but still nearly all of the HII volume is contributed by high temperature stars of spectral class O-B3.

It should be emphasized that we have been discussing the idealized case of an ionizing star imbedded in hydrogen gas assumed to have uniform density. However, the discussion can be extended to yield quantitative information regarding the extent of HII regions produced by a given distribution of high temperature stars in a hydrogen gas of non-uniform density.

The kinetic temperature in an HII region. Our previous discussion indicated that the kinetic temperature  $T_k$  in an ionized pure hydrogen gas is about equal to the temperature  $T$  (characterizing the Lyman continuum) of the ionizing star. We now wish to analyse this problem somewhat more closely.

Let us consider the life of a free electron in an HII region in interstellar space from the moment it is set free by a photoionization process until the moment it is re-captured. From the known capture cross section it is found that the life time of a free electron is of the order of  $10^{10} \lambda T^{\frac{1}{2}}$  seconds, i.e., of the order of  $10^5$  years. The path travelled during the life-time is of the order of  $10-100R$ . During its life time the electron suffers about  $10^6 > 90^\circ$  deflections in elastic collisions with electrons and protons, and the cumulative effect of small angle scatterings is even more effective in establishing a Maxwell distribution of the kinetic energies.

We may consider the total kinetic energy of the electrons in a unit volume ( $E_\tau$ ). Whenever an electron goes through the cycle of photo ionization and re-combination there is a gain in  $E_\tau$  given by

$\Delta E_\tau$  = The kinetic energy of the electron immediately after photo ionization  
- the energy loss of the electron through free-free transitions (bremsstrahlung) and inelastic collisions - the kinetic energy of the electron immediately before re-combination.

In equilibrium, the average value of  $\Delta E_\tau$  for all electrons must be zero. The average value of the kinetic energy immediately after photo-ionization can be calculated as a function of  $T$  (the temperature of the ionizing radiation), the dependence of the continuous absorption coefficient in the Lyman continuum upon frequency being known. The average kinetic energy immediately before

recombination can be found as a function of  $T_k$  (the temperature which characterizes the Maxwell distribution), since the dependence of the capture cross section upon kinetic energy is known.

If there were no energy losses during the life time of the electron, the two quantities just discussed could be equated and the relation between  $T$  and  $T_k$  found. However it is a fair approximation to state that  $T \approx T_k$ .

The reason for this is that the energy losses of the electron are small. The bremsstrahlung cross section is at least down by a factor of  $\alpha$  over the Rutherford scattering. In an ionized pure hydrogen gas the loss due to inelastic collisions out of the continuum also turns out to be small: when we compare the cross-sections for excitation of the neutral hydrogen states  $n=2,3,\dots$  through inelastic electron collisions with the cross sections for the capture of a free electron by the hydrogen ion, we find that the former are larger by a factor of the order of  $10^3 - 10^4$ . However, only the very small fraction of the electrons which have kinetic energies larger than 10 e.v. can excite even the  $n=2$  level. The captures are further favoured over the excitations because the number of hydrogen ions is much larger than that of neutral atoms. As a consequence inelastic processes for the electron are rather unimportant in an ionized pure hydrogen gas, which is to say  $T \approx T_k$ .

The emission spectra of the HII regions show that the matter is not pure hydrogen but contains important admixtures of other elements, notably O and N. We must now examine the possibility that inelastic collisions with atoms and ions other than hydrogen play a significant role in determining the temperature.

In looking for atoms and ions which might be important in this connection we must consider: 1 Abundance - We base our discussion on relative abundances of the elements derived from stellar atmospheres. As we proceed we shall find the expectation

of a rough similarity in the abundances of stellar atmospheres and interstellar space confirmed. Only the relatively abundant elements are of interest in the present context, namely He, O, Ne, N, C, Mg, Si, Fe. 2. Ionization. We consider only atoms and ions that will be present in HII regions in reasonably large amounts ( $> 10^{-2}$  of the total amount of the particular element). 3. Excitation. We look for low lying levels (excitation potentials  $< 3-4$  e.v.) that can be excited by a fairly large fraction of the free electrons.

Proceeding in this fashion one finds that in HII regions the most important loss of energy of free electrons via the excitation mechanism is by collision with OIII ions.

The stationary states in question are

		E.P.	
OII	$2 p^3$	$^2D_{5/2}$	3.3 e.v.
		$^2D_{3/2}$	3.3
OIII	$2 p^2$	$^1D_2$	2.5

The ionization equilibrium for oxygen can be determined for an HII region along lines similar to the ones pursued in the determination of the ionization of hydrogen. However, reduction of the ionizing radiation through absorption by oxygen is negligible because of the relative smallness of the abundance of oxygen as compared to hydrogen. It is found that nearly all of the oxygen present is in the form of OII or OIII.

Let us consider, as an example, the case of OIII (it is easy to generalise the discussion). The number of inelastic collisions exciting the  $^1D_2$  state of OIII is

$$N(O_{III}) 2 \times 10^{-6} T_e^{-\frac{1}{2}} e^{-\frac{E.P.}{kT_e}}$$

per free electron per second. The radiative transition from the ground state of OIII

$^3P_0$  and the  $^1D_2$  state is forbidden, however the cross section for excitation by electron collision is of the order of  $10^{-16}$  cm.<sup>2</sup> for the electron velocity in question.

We can now write down the equation expressing the equilibrium condition

$\Delta E_c = 0$  considered above: 1. The gain in kinetic energy per photo ionization of an H atom is  $kT$ , 2. The loss of kinetic energy per electron capture by an H ion is  $kT_k$  3. The loss of kinetic energy per electron collision excitation of the  $^1D_2$  state of OIII is 2.5 e.v. Furthermore,

$$\begin{aligned} \text{Number of photoionizations of H atoms per second per unit volume} &= \\ \text{Number of electron captures by H ions per second per unit volume} &= \\ N(\text{HII}) N_e 5 \times 10^{-11} T_k^{-1/2} \end{aligned}$$

and

Number of electron collision excitations of the  $^1D_2$  state of OIII per second and unit volume

$$= N(\text{OIII}) N_e 2 \times 10^{-6} T_k^{-1/2} e^{-\frac{2.5 \text{ e.v.}}{kT_k}}$$

where  $N(\text{HII})$ ,  $N(\text{OIII})$  and  $N$  denote the numbers of HII ions, OIII ions, and free electrons per unit volume. Thus the equilibrium equation is

$$\begin{aligned} N(\text{HII}) N_e 4 \times 10^{-11} T_k^{-1/2} (kT - kT_k) \\ = N(\text{OIII}) N_e 2 \times 10^{-6} T_k^{-1/2} e^{-\frac{2.5 \text{ e.v.}}{kT_k}} 2.5 \text{ e.v.} \end{aligned}$$

or

$$kT = kT_k + \frac{N(\text{OIII})}{N(\text{HII})} 4 \times 10^4 e^{-\frac{2.5 \text{ e.v.}}{kT_k}} 2.5 \text{ e.v.}$$

For any specified value of the abundance ratio  $N(\text{OIII})/N(\text{HII})$  this equation gives the relation between the temperature  $T$  of the ionizing radiation and the kinetic temperature  $T_k$  of the ionized gas. The following table gives

$T_k$  as a function of  $T$  for two values of  $N(\text{OIII})/N(\text{HII})$ . The range of  $T$  chosen corresponds to the fact that HII regions are largely produced by high temperature stars

T	$N(\text{HII})/N(\text{OIII})=800$	$N(\text{HII})/N(\text{OIII})=5,000$
	$T_k$	$T_k$
20,000	6,000	9,000
30,000	7,000	11,000
40,000	8,000	13,000
50,000	8,000	15,000
60,000	9,000	17,000

The two values of the abundance ratio given above are meant to bracket the values in actual HII regions. The last column probably corresponds more closely to actual conditions than the first.

The cooling effect of the OIII admixture is clearly seen. As the ionizing temperature  $T$  goes up, the kinetic temperature increases, however the rise is kept small because the OIII loss increases sharply with increasing  $T_K$ . This thermostat action of OIII is quite pronounced for the relevant oxygen abundance and temperature ranges.

For many purposes it is sufficiently accurate to put the kinetic temperature of an HII region equal to  $10,000^\circ$ .

The kinetic temperature in an HI region. As we have seen hydrogen is practically un-ionized in an HI region, and the intensity of radiation below the Lyman limit at  $911 \text{ \AA}$  is vanishingly small. Only atoms with ionization potentials below 13.6 e.v. become limited. Of the more abundant elements He, O and N are un-ionized, while C, Mg, Si, and Fe are ionized and contribute most of the free electrons.

Let us consider the question of the kinetic temperature in an HI region. As in the HII regions, the average kinetic energy of an electron immediately after photo-ionization will be given by the temperature  $T$  characterizing the ionizing radiation, while the average kinetic energy immediately before capture is determined by the kinetic temperature  $T_K$ . The energy loss during the life-time of the electrons will determine the difference between  $T$  and  $T_K$  as before. The number of electron captures per second, per unit volume will be much less than before, because there are practically no hydrogen ions available for collision. The captures are by CII and metal ions, the total per unit volume of which is probably  $10^3$   $10^4$  times smaller than the number of hydrogen atoms. Captures of electrons by H atoms, through which H formed occur but the capture cross sections are the order of  $10^3$  times smaller than those for the capture of electrons by hydrogen.

ions in the HII regions. On the other hand, analysis shows that the number of collisions causing excitations of atoms and ions is of the same order as in the HII regions, although OII and OIII are no longer present. As a consequence the kinetic temperature is very much reduced. This shifts to lower energies the levels of the states that are important in connection with the calculation of the excitation loss. It is now found that the  $^1D_2$  state of OI (E.P.=0.01 e.v.) and the  $^2P_{3/2}$  state of CII (E.P.=0.008 e.v.) are the most important. The temperature of HI regions is found to be of the order of 100 deg. K.

Loss of energy due to inelastic collisions between H atoms and interstellar dust is of importance also, but is not a determining factor. Very likely excitations of rotational levels of H<sub>2</sub> molecules plays a role as well.

Determination of the hydrogen density in interstellar space from the observed strengths of the interstellar Balmer emission lines. We have already considered the mechanisms for exciting the states n=3,4,... of neutral hydrogen in HII regions. Let us compute, as an example, the excitation of the state n=3 by electron capture and the subsequent emission of H $\alpha$ . If the total number of hydrogen atoms and ions per unit volume is N(H), then in an HII region the number of hydrogen ions and free electrons per unit volume is very nearly equal to N(H). The number of captures of free electrons into the state n=3 is (cf. above)

$$N(H)^2 \cdot 4 \times 10^{-12} T_e^{-1/2} \text{ sec.}^{-1} \text{ cm.}^{-3}$$

It is sufficiently accurate to set  $T_e = 10,000$  deg, so that the number of captures becomes

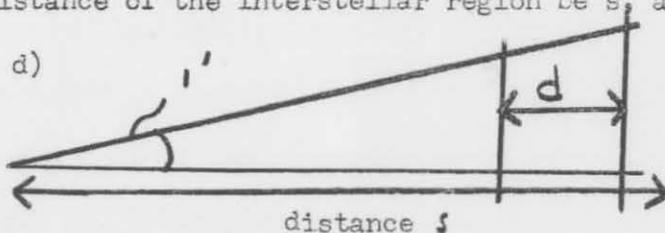
$$N(H)^2 \cdot 4 \times 10^{-14} \text{ sec.}^{-1} \text{ cm.}^{-3}$$

A hydrogen atom in interstellar space in the state n=3 will always remain practically undisturbed making <sup>it</sup> almost certain that a transition to either n=2 or n=1 will take place. The transition probabilities for the two processes being nearly equal, the number of transition to n=2 with the emission of an H $\alpha$  quantum is

$$N(H)^2 \cdot 2 \times 10^{-14} \text{ sec.}^{-1} \text{ cm.}^{-3}$$

Now consider an experimental set-up by which the  $H\alpha$  quanta from an interstellar region of extent  $1' \times 1'$  (one square minute of arc) which are intercepted by a telescope with an aperture of radius  $a$  cm, are collected and counted.

Let the distance of the interstellar region be  $s$ , and its extent the line of sight  $d$  ( $s \gg d$ )



The emitting volume considered is then (since  $1' = 1/3438$  radians)

$$\frac{1}{(3438)^2} s^2 d$$

and the fraction of the emission intercepted by the telescope is

$$\frac{a^2}{4s^2}$$

so that the number of  $H\alpha$  quanta received by the telescope is

$$a^2 d \text{ (cm)} N(H)^2 \cdot 5 \times 10^{-22} \text{ sec}^{-1} \text{ per square minute emission}$$

where  $a$  and  $d$  are measured in cm. If the emission length is measured in parsecs,

$$a^2 d \text{ (pc)} N(H)^2 \cdot 1.5 \times 10^{-3} \text{ sec}^{-1} \text{ per square minute emission}$$

The product of the emission length in parsecs and the square of the number of hydrogen atoms and ions per  $\text{cm}^3$  is called the emission measure, E.M. Hence the number of  $H\alpha$  quanta collected by the telescope with an aperture of radius  $a$  is

$$1.5 \times 10^{-3} a^2 \text{ E.M. sec}^{-1} \text{ per square minute emission}$$

Measurements of the intensity of interstellar  $H\alpha$  emission can be converted to the number of  $H\alpha$  quanta received per second per square minute of emission, and hence the emission measure  $\text{E.M.} = d \text{ (pc)} N(H)^2$  can be evaluated. It is often possible from the surface extent of the emission to estimate its depth  $d \text{ (pc)}$ , and then  $N(H)$  can be found. (Note that the hydrogen density is found as the square root of  $\text{E.M.}/d \text{ (pc)}$  and that the inaccuracy of the derived  $N(H)$  is correspondingly decreased.)

In addition to the mechanism of direct capture into  $n=3$ , other mechanisms contribute to the  $n=3$  population, as we have seen. This can be taken into account, but the corresponding change in the hydrogen density is not large, a reduction by a factor which is less than 2.

As an example, let us consider, first, the center of the Orion nebula. The emission measure is found to be  $1 \times 10^7$ . The intensity maximum at the center is sharp, and most of the intensity corresponds to emission along an emission path  $d \sim 0.1$  pc. This gives a hydrogen density  $N(H) = 10^4$  atoms and ions per cm. A typical faint diffuse emission nebula has E.M. =  $1 \times 10^4$ ,  $d \sim 10$  pc,  $N(H) \sim 30 \text{ cm}^{-3}$ . Finally a typical faint extended  $H_\alpha$  emission region has E.M. = 500,  $d \sim 400$  pc.,  $N(H) \sim 1 \text{ cm}^{-3}$ .

The analysis of interstellar hydrogen emission indicates the general presence of ionized hydrogen in the spiral arms whenever high temperature stars producing HII regions are found. The occurrence is generally limited to distances from the galactic plane  $< 250$  pc. Because of interstellar extinction (see below) it is very difficult to record hydrogen through Balmer emission when the distance is over 3000 pc. The extinction then reduces the  $H_\alpha$  intensity below the threshold of detection.

Interstellar emission lines of elements other than hydrogen. Forbidden lines. It has been mentioned already that the emission regions show emission lines of OII, OIII, NII and SII. We shall discuss briefly the derivation of densities for these elements from the observed strengths of the emission lines.

We have seen that in an HII region the higher states of neutral hydrogen are excited largely through electron capture. Excitation by electron collisions is less important because of the factor  $e^{-E.P./kT}$  (with  $T \sim 10,000$  deg. and E.P. = 12 e.v. for  $n=3$ ,  $e^{-E.P./kT} \sim 10^{-6}$ ). For elements with low lying levels the situation is different (cf, the discussion of kinetic temperature). For instance the  $^2D$  levels of OII will be excited by electron collisions more often than through electron capture by OIII ions. For the  $^1D$  level of OIII excitation

from the ground state by electron collision dominates even more, because the relative number of O IV ions is low (the ionization potential of OIII is 55 e.v.).

The stationary states of OII and OIII just referred to are metastable. However densities in interstellar space are so low that excitation even of a metastable state is practically always followed by a transition down with a corresponding line emission. The cross sections for collision excitation of metastable levels are of the same order of magnitude as those for ordinary levels.

When the cross section for excitation by electron collision from the ground state to the upper level of an interstellar emission is known as a function of the electron velocity, and when the kinetic temperature  $T_k$  of the interstellar gas is also known, then a quantity equal to the density of the ion in question times the length of the emission path can be derived from the measured emission line strength. If an estimate of the emission path can be made, and the ionization equilibrium determined, then the density of the given element in interstellar space is known.

Let us consider the case of O III emission. The value  $T_k = 10,000$  deg. was sufficiently accurate for the discussion of the Balmer line intensities, but for O III this is not the case, since excitation is proportional to  $e^{-2.5 e.v./kT}$ . In fact, referring to our previous discussion of the kinetic temperature problem, we see that the kinetic temperature depends on the relative oxygen abundance, and that it is not possible to derive both  $T_k$  and the relative oxygen abundance from the measure of one OIII emission line.

In bright emission nebula like the Orion nebula, however, it is possible to observe and measure a faint O III emission line at  $4363 \text{ \AA}$  ( $^1S_0 \rightarrow ^1D_2$ , E.P. for  $^1S_0 = 5.3$  e.v.) in addition to the two strong lines at 5007 and 4959 already discussed ( $^1D_2 \rightarrow ^3P_2$ ,  $^1D_2 \rightarrow ^3P_1$ , E.P. for  $^1D_2 = 2.5$  e.v.). The

intensity ratios  $4363/5007$  and  $4363/4959$  depend upon known collision excitation cross sections, and upon the kinetic temperature  $T_K$ , but are independent of the oxygen density. The value of  $T_K$  can thus be found, and subsequently used in the determination of the oxygen density. For the center of the Orion nebula  $T_K = 13,000$  deg. is the number which is derived. The abundance ratio  $N(O)/N(II)$  is then found to be  $1 - 2 \times 10^{-4}$ . With this abundance ratio the correct temperature  $T_K$  is predicted. (cf. the table given above) with the temperature  $T$  of the ionizing radiation being of the order of  $40,000$  deg.

The He I line  $5876 \text{ \AA}$  is seen as a relatively weak emission line in the spectrum of the Orion nebula. It has been recorded in two other emission nebulae, but is too faint to be seen in the spectra of the faint extended  $H\alpha$  emission regions.

The chief mechanism of the upper level of the  $5876 \text{ \AA}$  HeI line is electron capture by HeII ions. The abundance ratio  $N(\text{He})/N(\text{H})$  is found to be  $\sim 10^{-1}$ .

For N the abundance relative to H is found to be  $\sim 10^{-4}$  for  $S \sim 2 \times 10^{-5}$ . These relative abundances pertaining to interstellar space agree with the corresponding values derived for stellar atmospheres within the uncertainty of the determination (involving factors of about 2-3).

We shall presently consider the determination of relative abundances from interstellar absorption lines.

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