Chronologi

I

Ancient Chronology.

An Introduction to Methods and Problems of Astronomical Chronology.

by

O. Neugebauer

Addition to "Chronology"

Pogo, Corrections to Opp. Canon
Parker-Dubberstein, Bab. Chr.

Civil hours in Bab. eclipses
e.g. Almagest V, 14
(Man. p. 307)

Additions

E. von Haerdtl, Astronomische Beiträge z. Assyrischen Chronologie Denkschr. d. math. nat. Kl. d. K. Akad. d. Wiss. Wien 49 (1884)
conjunction from 757 to 606 B.C.

Martin: Mars occultation -356 V, 4

---

give tables for comput. with Eg. cal. using zero dates

I	Thoth	0
II	Phaophi	30
III	Athyr	60
⋮	⋮	⋮

---

14 years Roman census cycles, beginning 61/62 AD

Weidner, Assyr. Eponymen AfO 13 (1941) p. 308 ff

A. Ungnad, Mari letters AfO 13 (1941) p. 145 ff.

Examples from Papyri and Tablets for use of P.V. tables of planetary phases. Introduce Greek letter terminology

---

Chronology

[a]rrange the P.V.-tables according to my scheme beginning with $a_1\ a_2\ a_3\quad m_1\ m_2\ m_3\ \ldots\quad \odot_1\ \odot_2\ \odot$

add table

♈	0	360
♉	30	390
♊	60	420

Mention Macedonian financial years ≠ regnal years.

cf. Skeat [1] JEA 34

Wilcken's error in assuming midnight epoch in Egypt in P. Lond. 130 (Horoscope) because of ἐπιφωσκοντος)

Note about Francis Baily

cf. Clerke, Hist. of Astron. p. 77

"archaeology of practical astron."

Remark about Schroeter's Kanon SF

very inconvenient limitation of chart area. Consequently unreliable for Arabic astron. Only such $\odot$ eclipses are mentioned which were visible in his area. Thus the eclipses Oppolzer 5789/5790 are omitted, observed by Ibn Yunus and used for secular accel. by Dunthorne [1] p. 168

NEUG, Anc. Chron

p. 153. to show $q = \frac{p}{p-1}$ $(p>1)$, write the expression as

$\frac{1}{q} = 1 - \frac{1}{p}$. But this says the heliocentric angular velocity of the planet in revolutions/year subtracted from the (hel.) ang. vel of the earth (1 rev/year) equals the heliocentric relative ang. vel of the planet w. resp to the earth. Hence, etc.

---

momentaneous for instantaneous

continuous fractions for continued fractions

See also corrections in Carbon Copy

Preface.

παρ' οἷς ἀσυνάρτητός ἐστιν
ἡ τῶν χρόνων ἀναγραφή, παρὰ τούτοις
οὐδὲ τὰ τῆς ἱστορίας ἀληθεύειν δύναται *)

Tatianus - Scaliger

---

*) Those whose chronology is not exact cannot make history speak the truth. [From the title page of Scaliger, De emendatione temporum, Paris 1583.]

These lectures attempt to give an answer to the very natural question: How can one establish the exact date of historical events in periods far remote from our own time ?

The system of dates of the main events of a ~~certain~~ period is called its "chronology." To obtain such a chronology, one usually distinguishes between two essentially different methods, which lead to an "absolute" or a "relative" chronology. The setting up of the relative chronology of a period is considered to be one of the main objectives of the historian, or, at least, as a necessary condition for all of his further studies; it implies the establishing of the order of succession of events and tries to determine as accurately as possible the time intervals between these events. The basic material for this "relative chronology" consists in archeological evidence (e.g. succession of strata in excavations), or king-lists, inscriptions, historical reports, etc. The accuracy of the results is necessarily dependent on the accuracy and reliability of this combined source-material. By way of contrast, "absolute" chronology is based on single events, which, although isolated, by their astronomical character (e.g. reports of a solar eclipse during a battle) permit exact dating. The combination of these absolute dates with the results of relative chronology gives the final chronological scheme of the period in question.

These lectures deal exclusively with absolute chronology and are mainly of a methodological character. Their aim is to illustrate methods but not to give a complete account of the results obtained. They are therefore essentially different from the extant "handbooks" of chronology and are, moreover, of an introductory character. Their goal is to give the reader some impression of the complexity of the problems involved and to help develop his independent judgment as to the degree of "absoluteness" of the results of the application of astronomical methods to elements provided by the historian. In the cooperation between historian and astronomer, the former is inclined to accept the results of astronomical calculation as not permitting any flexibility; on the other hand, the astronomer has the tendency to use the elements given to him no differently than the results obtained through instruments of the highest exactitude, without necessarily knowing how many more or less plausible additional assumptions have been made in interpreting the original source. These lectures are an attempt to broaden from both sides the surface of contact and mutual understanding.

No knowledge of mathematics and astronomy which goes beyond the most elementary facts is assumed. On the other hand, I do not accept the assumption which is usually made in books about astronomical methods for the use of historians, namely, that historians are not able to operate with such simple concepts as negative numbers or remainder of division. As to the historical facts, a certain familiarity with ancient history is necessary in order to understand within the framework of the history of the ancient world the rôle of the examples discussed here. The original sources as well as recent literature for further information are consistently cited. The first chapter gives a short introductory description of the main concepts used in chronological discussions, ~~the~~ e.g. calendaric~~al~~ systems and various ~~different~~ units and methods applied to time reckoning. The subsequent chapters are arranged according to the astronomical character of the problem. The moon's movement is the basic element in all discussions involving moon calendars and eclipses (Chapter II): the positions of the planets determine the dates of horoscopes, and the movement of Venus is especially of great interest in determining ~~the~~ Old-Babylonian chronology (Chapter III); while the heliacal rising of fixed stars plays an important rôle in Egyptian chronological concepts (Chapter IV). ~~The final chapter discusses briefly certain problems of astronomical "orientation."~~ Each chapter contains at the end a short bibliography; all abbreviations used are explained ~~collected~~ in the general bibliography at the end of the book. Much effort has been exerted to make the index as useful as possible.

Providence R.I.
~~1939/40/41~~
1941/42

Table of contents.

Chapter I. Introduction.

Chapter II. The Moon.

Chapter I. Introduction.

Ἄγουσι δὲ (sc. Αἰγύπτιοι)
τοσῷδε σοφώτερον Ἑλλήνων ....*)

Herodotus II,4.

*) And the Egyptians reckon to this extent more wisely than the Greeks.

§ 1. Notation.

1. Years and months.

We take our point of departure from very unhistorical ground. We shall give definitions of different forms of "years" on a purely formal basis, completely disregarding the historical background in which these variants of the fundamental chronological unit of the year actually originated. In other words, we treat the concept "year" as a purely arbitrary unit of time measurement which more or less suits the practical needs of "dead reckoning."

In all writings referring to medieval or ancient history the word "year" means "Julian year." A Julian year is a time interval of $365\frac{1}{4}$ days; in order to dispose of the fraction of a day, inadmissible in any civil calendar, four Julian years are grouped into cycles of three "ordinary" years of 365 days each and one "leap year" of 366 days. Using the Christian era, all Julian years whose number is divisible by 4 are leap years, so far as "A.D." years are concerned; for the "B.C." years, however, the years 1 B.C., 5 B.C., 9 B.C. etc. are leap years.

These Julian years are subdivided into twelve months, conventionally named, as today, January, February, etc., which contain the same number of days as in our present calendar, inserting at the end of the month of February the additional day in leap years. We abbreviate these month names to I, II, ..., XII, respectively, adding (j) if necessary for purpose of distinction from other calendars. Leap years will be distinguished by an asterisk if we wish to emphasize this fact. A formula like

$$\text{Diocl. } 33 \text{ I(e) } 28 \approx 316^{*} \text{ IX(j) } 25$$

should be read as "The 33rd year of Diocletian, first month of the Egyptian calendar, 28th day, corresponds to the leap year 316 A.D., September 25th."

The inconvenience of distinguishing between leapyears and ordinary years is avoided by using the "Egyptian years" of 365 days each, without any intercalation. These years are subdivided into twelve months of 30 days each (designated here by I(e), II(e),..., XII(e)) and five "epagomenal" (i.e. "additional") days.[0] We shall omit the (e) if no misunderstanding is possible, as we shall do in all analogous cases. These months are frequently quoted by their late-Egyptian names in conventionalized Greek pronounciation as follows[1]:

0) These epagomenal days belong to the end of a year, at least in the later periods of Egyptian history. In earlier periods, however, they seem to have been considered as preceding the year. Meyer (Ed.) Aeg. Chron. Nachtr. p. 5 f. and Sethe, Beitr. p. 30 ff.

1) In Roman time exist in Egypt also honorary names like Ἁδριανός for IV or Γερμανικείο for IX (cf. Wilcken, Ostraka I p.809 ff.).

	I	Thoth	V	Tybi	IX	Pachon
(1)	II	Phaophi	VI	Mekhir	X	Payni
	III	Athyr	VII	Phamenoth	XI	Epiph
	IV	Khoiak	VIII	Pharmouthi	XII	Mesorē

These names are very rare, however, in Egyptian documents, which usually date according to seasons (three seasons of four months each), e.g. instead of Athyr "month 3 of the inundation"[2]. In Greek papyri, however, and in Greek (and even

2) Cf. Gardiner Gr. p. 205 and Sethe, Zeitr. p.30 ff. and p. 294 f. The earliest appearance of the above-given ~~names~~ order seems to be the Elephantine papyri (Persian period, i.e., fifth cent. B.C.; cf. Cowley Aram. P. and Ginzel II p.45 ff.) For the earlier history of these names and their arrangement cf. Sethe, Zeitr. p.30 ff and the literature quoted there.

Arabic as well as Western in medieval[1]) astronomical literature, the twelve names quoted in (1) are exclusively used in references to the Egyptian calendar.

The Egyptian years are especially convenient for astronomical computation and are therefore the time scale of the ancient astronomers, as, for example Ptolemy[3]) and his commentators[4]), the Christian[5]) and Mohammedan astronomers[6]) and finally Copernicus, who expresses himself as follows[7]): "In the computation of the celestial movements, we will con-

3) About 150 A.D. Examples passim in the Almagest.

4) E.g., Beda, De temporum ratione (written 725 A.D.) ch.XI (De mensibus) Migne P.L. 90 col.341 ff.

5) E.g., Pappus (4th cent. A.D.) ed. Rome p.3.

6) E.g., Al-Battani (about 900 A.D.), ed. Nallino p.41,17.

7) De revol. (1543) ed. Thorn p.172 f.

sistently use the Egyptian years, which are the only ones, among all civi[l] years, which are of equal length. The measure should ~~be congruent to~~ agree with the measured quantity[7a]), which is not the case in the years of the Romans, Gre[eks] and Persians, in which the intercalation is not made in a uniform way bu[t] as it pleases somebody. The Egyptian year, on the contrary, contains no ambiguity .... The Egyptian years are therefore especially fitted to th[e] counting of uniform motion ..."

The purely practical attitude of the astronomers is very diffe[r]ent from the factors which actually influenced the history of the calend[ar]. Because the Julian year is more closely related to the length of the so[lar]

7a) "Congruere" i.e. the measure should be also of constant length, as the length of the year

year than the Egyptian year, the Julian calendar was introduced in Egypt by Augustus[8)] in the following form: the names of the months and their

---

epoch adopted for

8) For the ~~four ...~~ this new order ~~...~~ cf. Ginzel I p.225 f., Wilcken, Ostraka I p.789 ff. and below p. ~~...~~. ~~(with caution) Borchardt [2] p.5 ff. The question seems to me now to be decided by Colson Week p.52 f according to whom the new era begins with -29 VIII 31 which is the first of Thoth in this year~~ (cf. below p. ~~...~~)

---

length are the same as in the Egyptian calendar, but in each fourth year a 6-th epagomenal day is inserted. This calendar was in commom use among the Greek and Roman population in Egypt during the Roman empire; the native population called it the "Greek year"[9)]. We prefer to use the name "Alex-

---

9) Cf. AZ 10,27 and Sethe Zeitr. p.310. For double datings given both in the Alexandrian calendar (καθ' Ἕλληνας) and in the Egyptian calendar (κατὰ δὲ τοὺς ἀρχαίους or Αἰγυπτίους), cf. e.g. Grenfell-Hunt, Ox.Pap.II p.139 and Grenfell-Hunt-Hogarth, Fayûm p.294. An interesting combination of three

---

andrian calendar" and abbreviate the names of the months in this calendar by I(a), II(a),..., XII(a). The coexistence of the Egyptian and the Alexandrian calendar is a very typical example of the source of difficulties connected with the astronomical evaluation of dates found in documents of this period. There is no strict rule as to the kind of calendar used in a particular document. Thus we meet both types of calendar in Demotic astronomical texts of Roman times with no visible reason why papyri written about 50 A.D. or 150 A.D. should use the Egyptian calendar while texts from the same type written about 140 A.D. should refer to the Alexandrian calendar[10)]. The application of exact astronomical calculation to dates

---

10) Cf. Neugebauer ~~...~~ p.229ff. (o.) [1]

calendars is given in a ~~n~~ horoscope (P.London Nr.130), dated Titus year 3 (= 81 A.D.), equating VIII(a) 6 with IV(j) 1 (ὡς δὲ Ῥωμαῖοι ἄγουσι καλάνδαις (Ἀπριλλίας) and with IX(e) ~~night from 1st to the~~ (κατ' ἀρχαίους δὲ παχὼν νεομηνίᾳ εἰς τὴν δευτέραν).

... ους κα ... 5 P.Ox ... (Horoscope + 14 (?))

of this period should therefore always be preceded by the investigation of the problem as to which of the two calendars is meant[11].

11) Another modification of the Egyptian calendar is the Persian("Young-Avestan") calendar which inserts one complete month of 30 days after 120 Egyptian years, in this way again reaching agreement with the Julian calender. Cf. Taqizadeh [1] esp. p.17 and 36.

The year forms mentioned up to now could have been defined without introducing any astronomical concept at all. They merely consist of a certain number of days, whose number may be variable but eventually follows a definite rule of intercalation. The same holds for the smaller parts of these years, the months. The directly opposite divice is followed by the Babylonian calender and its derivations, e.g. the Jewish calendar. Here the month in a strictly astronomical sense is the basic unit of time. Such a "month" is defined as the time between two subsequent moments of reappearance of the moon's crescent after the moonless nights around new moon. These "lunar months" were grouped into "years" of sometimes 12 or 13 months. For the greater part of Babylonian history there was no definite rule as to when a year should be long or short. Only during the last centuries B.C. were definite cycles adopted according to which certain specified years were to contain a 13-th month. The final and historically most important cycle is a 19-years cycle which we shall meet at later occasions, when more details about its basis and its application will be given.[12] Here it is sufficient to remark that this cycle six times inserts

12) Cf. below p.***.

a second twelfth month (in the following denoted as XII_2) and once a second sixth month (VI_2). The length of the single months, however, apparently was never regulated by a definite rule but was always determined according to the actual appearance of the new crescent. This gives to the Babylonian calendar a very complicated character. On the other hand, this calendar is in very close relation to the actual movement of the moon, thereby permitting direct checks through modern calculation. An isolated date like January 1 is astronomically absolutely valueless. On the contrary, if we read in a cuneiform list of favorable and unfavorable days[13] that a sun eclipse

13) KAR 178 obv.V,60. Cf. Labat, Hémérol. p.88/89.

on the 22-nd of Sivan is an unlucky day, we then know that no real sun eclipse can be meant because they are necessarely bound to new moons (which, in a moon calendar, have only dates like 28, 29 or 30). The text thus belongs to a period (not yet have been distinguished) where atmospheric eclipses from astronomical eclipses.

As in the case of the Egyptian calendar, conventional names of the Babylonian calendar are in common use in the literature. They are as follows:

	I	Nisan	V	Ab	IX	Kislev
(2)	II	Ayar	VI	Elul	X	Tebit
	III	Sivan	VII	Teshrit	XI	Shebat
	IV	Tammuz	VIII	Arahsamna	XII	Adar

We shall avoid burdening our discussion with these names as much as possible and replace them by the simple numbers, distinguished (if necessary) by (b) from other calendars.

2. Counting of years.

Modern historical writing reckons Julian years according to the "Christian era." This era actually depends upon an ancient era counting the regnal years of Diocletian. The defining relation [illegible] eras is

(3) Diocletian 248 $\approx$ 532 A.D.

This kind of definition of the Christian era corresponds to its actual introduction by the monk Dionysius exiguus.[14] He computed Easter tables in continuation of previous ones by Bishop Cyril of Alexandria[15] which ended with the year Diocletian 247.[16] In the year 525 Dionysius sent

---

14) i.e., "the little one", "the insignificant one" added to his name as a sign of modesty. See RE 5, p.998 f., Cassiodorus, De inst.div.litt. c.23 = Migne P.L. 70 col. 1137.

15) Died in 444 A.D. Cf. CMH 1 p.500 ff.

16) This is the usual version, based on the statement of Dionysius in his letter to Petronius (Migne P.L. 67 col.20) that the tables of Cyril began in Diocl.153 ($\approx$ 437 A.D.) and ended in Diocl.247 ($\approx$ 531 A.D.). But from the letter of Cyril himself (adressed to Theodosius II, published from an Armenian manuscript by Conybeare, Cyril, esp. p.221) it follows that his tables covered the years from Diocl.115 ($\approx$ 399 A.D.) to Diocl.228 ($\approx$ 512 A.D.). I do not see how this contradiction can be reconciled.

---

his new tables to Bishop Petronius of Alexandria expressing the opinion that the era of Diocletian, a "tyrant more than a prince", should more appropriately be replaced by the counting of years "ab incarnatione domini"[17]. The arguments which ~~xxxxxxxxxx~~ brought Dionysius to the assumption of the equivalence (3) are not known: the most plausible theory seems to be the assumption of certain speculations, typical for this period, about the parallelism of the creation of the world and the resurrection of Christ, combined with Easter cycles - used by Cyril and others.

---

17) The text of the "epistola ad Petronium" is published Migne P.L. 67 col.19 ff. For the manuscript, cf. Ideler II p.366.

This theory has been worked out in detail by G.Oppert[18]. The exactly oppo-

18) Oppert [1]. This theory is summarized by Ginzel,III p.179.

site point of view is taken by Van Wijk, who thinks that the Dionysiac era does not pretend to give any information about the actual date of the birth of Christ, because the number 532 is a number in itself important from the point of view of cyclic calculation[19]. And it is of course true

19) Van Wijk p.17. That the Easter dates are repeated after 532 years will be shown below p.***.

that the Christian era cannot be historically correct because Herod died in the year 4 B.C. On the other hand, the name of the era speaks strongly against Van Wijk's hypothesis. The importance of the number 532 for the cyclic Easter calculation might have been <u>the</u> essential argument for Dionysius, who hardly had historical sources at his disposal to determine a date 500 years before his time within a margin of 5 years - not to mention his lack of understanding for so modern a formulation of the problem. The purely speculative reconstruction of historical facts, therefore, seems to be the most plausible explanation of Dionysius' procedure. That he nevertheless came to a result not too remote from the truth is undoubtedly due only to the fact that he used a well established ancient era for half of the interval to be brigded.

From our present point of view, the only interest in the Christian era lies in the fact that therewith a definite habit was finally developed to count years without the interruptions and ambiguities inevitably connected with systems based on regnal years, dynasties etc. But in order to make full use of any era one must usually extend it backwards beyond the arbitrary point of departure, the so-called "epoch" of the era. In doing so, it is only selfevident that one must avoid unnecessary complications and thus eliminate sources of errors, which would arise if one would forbid the application of the rules of ordinary arithmetic in the counting of years.[20] Hence we shall treat year numbers exactly like ordinary num-

20) Such a rule is e.g. that two odd numbers are always separated by an even number; this fundamental principle is violated by making "1 B.C." and "1 A.D." neighbours.

bers and call the year which precedes the year 1 of any era the year 0, the preceding year the year -1, etc. The time elapsed between a year a and a year b is then always b - a , regardless of which signs the year numbers a and b have.

If one, however, adopts a notation like years A.D. and B.C. where the year "1 B.C." is immediately followed by the year "1 A.D." then three different rules are necessary according to the three cases: (a) both years are A.D.-years, (b) both years are B.C.-years, (c) one year is A.D. but the other B.C. A simple example might illustrate this. Ancient historiography frequently uses the "Olympiads," i.e., groups of four years called the first, second, third, fourth year of the first, second, ..., Olympiad. The usual statement is that the first year of the first Olympiad corresponds to the year 776 B.C. or, written as a formula,

Ol.1,1 = 776 B.C.

If one wishes to know which year of the Christian era corresponds to the k-th (k = 1,2,3,4) year of the n-th Olympiad (abbreviated by Ol.n,k), one has to give three different rules, requiring careful consideration of the position of the beginning and the end of the Olympiad in question with respect to the epoch year of the Christian era.[21] If, on the contrary, we

21) Such rules, frequently incomplete or wrong, can be found in many textbooks. It is worth mentioning that the long rule given in Ginzel II p.357/

/358 is correct, but the example on p.353, last paragraph, is wrong (11 A.D. instead of 13 A.D.).

---

introduce "Ol.0,0" (i.e., the year preceding the first year of the tetrad preceding the first Olympiad), we have only one formula

$$\text{(4a)} \qquad \text{Ol. } 0,0 \approx -780$$

and consequently

$$\text{(4b)} \qquad \text{Ol. } n,k \approx -780 + 4 \cdot n + k$$

e.g.,

$$\begin{aligned} \text{Ol. } 195,1 &\approx -780 + 4 \cdot 195 + 1 \\ &= -780 + 780 + 1 = 1 \end{aligned}$$

showing without any special consideration that the year 1 A.D. corresponds to the first year of the 195th Olympiad.

This principle of adapting our notation to the simple rules of arithmetic will be strictly followed in our discussions. Of course, it is meaningless to replace loose expressions like "around 1500 B.C." by "around -1499" if we do not wish to guarantie an exactitude of one year. We will correspondingly use expressions like "first century B.C." because there is no need for actual calculating with centuries which would make it advisable to introduce the number zero here too.

3. Years and seasons.

If "year" were only an arbitrary unit of time (as, e.g., "mile" is a unit of length), then constancy of this interval would be the only essential requirement for its usefulness. From this point of view there would therefore be no reason to replace Egyptian years by Julian years or to adopt our present "Gregorian" instead of the simpler Julian calendar.[22] Histori-

---

22) The Gregorian rule of intercalation is given below p. [illegible]

ally, however, the year has been created to represent the periodic renewal of the seasons and in this respect the Gregorian year represents by far the best solution of the problem.

As mentioned above, "years" in chronological discussions usually mean Julian years. Occasionally, however, problems occur in which one wants to connect chronological dates as accurately as possible with the seasons. This is the reason for extending the Gregorian calendar backwards from its introduction in 1582. The results thus obtained are not absolutely exact from the astronomical point of view but the deviation between Gregorian calendar and the astronomically defined seasons are so small as to be negligible for all historical periods. The following little table will give an impression of the minuteness of the differences even between the Julian and Gregorian calendars.

Gregorian			Julian		
1500	I (g)	1	1499	XII (j)	22
0	I (g)	1	0	I (j)	3
-1000	I (g)	1	-1000	I (j)	10
-2000	I (g)	1	-2000	I (j)	18
-3000	I (g)	1	-3000	I (j)	25

The difference between the two calendars amounted to ten days at the date of its introduction in 1582 by Gregory XIII's decree that the relation

$$(5) \qquad 1582\ \mathrm{X}\ (j)\ 5 = 1582\ \mathrm{X}\ (g)\ 15$$

should be accepted.[23] The divergence disappears, of course, for the time of the Roman empire but increases to about one month in the fourth millenium

23) The text of the papal bull is published, e.g., in Clavius, opera V (1612) p.13-15. Cf. Ideler II p.302.

B.C. [24]. The divergence between Julian and Egyptian year is much more cons-

24) For precise transpositions one can use P.V.Neugebauer HTCh. p.173/174 table 15 or Schram (cf. the instructions on p.XVI).

picuous because the Egyptian year falls behind one day every fourth Julian year. In 365.4 = 1460 Julian years a given Egyptian date therefore falls behind one complete Julian year; or, in other words,

(6) 1461 Egyptian years = 1460 Julian years.

This interval of 1460 Julian years is known as the "Sothic period" for reasons which will be discussed later.[25] During this period, a point in the

25) p.???.

Egyptian year, e.g. New Year's day, will have occupied every place in the Julian year (and hence of the seasons). One therefore speaks about a "revolving" ~~calendar~~ or "wandering" year.

The maintenance of such a revolving year has frequently been considered as the consequence of a specifically Egyptian innate conservatism.[26] As a matter of fact, this conservatism is by no means greater than

26) E.g. Sethe, Zeitr. p.310.

that shown by the European culture for the last thousand years in calling the 10 th month the "eighth" (October), the 11-th the "ninth" (November) etc., or in the bloody revolts against the Gregorian reform of the calendar, accepted, e.g. in Switzerland only after 200 years of stubborn resistance. Even in modern astronomy a very unconvenient mixture of sexagesimal and decimal notation[27] reflecting the history of the past 5000

27) E.g. numbers like $125°15'31.52''$.

years, is still used. Analogous curiosities may be pointed out in our daily use in measuring time, space, or related magnitudes. And as a matter of fact the Egyptians abandoned their calendar in much shorter a time than the Gregorian calendar was generally adopted in Europe.

4. "Congruences."

We must now introduce another mathematical concept which is especially adapted to discussions typical for all chronological problems; or better: we must introduce a convenient generalisation of the well known concept of the "remainder" of divisions of integers by integers.

The definition in question is as follows: let all the letters a, b, m, ... etc. represent integers; supposing m to be a given integer, we call any pair of numbers a and b "congruent" with respect to the given "modul" m if their difference is divisible by m. This is usually written

$$a \equiv b \pmod{m} \tag{7a}$$

(read: "a congruent b modulo m") and means according to the given definition nothing more than

$$a - b = km \qquad k \text{ being any integer.} \tag{7b}$$

The connection of this concept[28] with the divisibility of integers becomes clear if we consider the special case $b = 0$. From (7a) and (7b) it follows that the case

$$a \equiv 0 \quad \text{mod.} m \tag{8a}$$

means

$$a = km \tag{8b}$$

i.e., a is a "multiple" of the given modul m or a is "divisible" by m.

---

28) Introduced by C.F.Gauss in his "Disquisitiones arithmeticae" (1801); cf. Gauss, Werke, I p.9 f.

Correspondingly, if we divide a number $\underline{a}$ by the modul $\underline{m}$ and find that $\underline{m}$ is contained $\underline{k}$-times in $\underline{a}$ but leaves a remainder $\underline{b}$, in formula

$$\underline{a} - k\underline{m} = \underline{b} ,$$

then we say that

$$\underline{a} \equiv \underline{b} \quad \text{mod.}\, \underline{m} .$$

The importance of the concept "congruence" lies in the fact that in many cases one is not interested in how many times (our $\underline{k}$) a number $\underline{m}$ is contained in another number $\underline{a}$ but that one only needs to know what number $\underline{b}$ remains if multiple of the given number $\underline{m}$ are disregarded. This may be illustrated by the following examples.

(1). Let us suppose that a celestial body moves around a fixed center with constant velocity. We assume furthermore that a certain point on its circular orbit is defined as the origin for counting angles ($\lambda = 0$) and that the angular distance of the celestial body from this point is called its "longitude" (λ). During one revolution, the longitude of the moving body increases by 360°, after two revolutions by 720°, etc., but in order to characterize the place of the body on its orbit only one number λ between 0° and 360° is sufficient, regardless of how many times a multiple of 360° has been added during the preceding revolutions. In other words, it is sufficient to consider only the longitudes "modulo 360."

If we take the movement of the moon around the earth and suppose a uniform daily increase in longitude by 13;10,35 degrees,[29] let us investigate the longitudes of the moon every 30 days. Now

$$30 \cdot 13;10,35 \equiv 35;17,30 \quad \text{mod. } 360$$

or the longitude of our moon increases by $35;17,30^\circ$ after 30 days. Let us

29) We consistently use the notation $13;10,35^\circ = 13^\circ 10' 35''$ etc.

furthermore consider the zodiacal signs, i.e., the twelfths of the orbit, containing 30 degrees each. We now have

$$35;17,30 \equiv 5;17,30 \quad \text{mod.}30$$

If the point $\lambda = 0$ coincides with the beginning of one of the zodiacal signs, the points occupied after 30, 60, 90 etc. days will therefore be

5;17,30	21;10	6;52,30
10;35	26;27,30	etc.
15;52,30	1;35	

degrees distant from the beginnings of consecutive zodiacal signs.

(2). The concept of "congruence" is not restricted to positive integers. Obviously all numbers

$$\ldots\ 88,\ 58,\ 28,\ -2,\ -32,\ \ldots$$

are different from each other by multiples of 30, exactly as are

$$\ldots\ -88,\ -58,\ -28,\ 2,\ 32,\ \ldots$$

Making the same assumption as to the movement of the moon as before and remarking that

$$29 \cdot 13;10,35 \equiv 22;6,55 \quad \text{mod.}30$$

we know that after 29 days the longitudes will increase by 22;6,55 degrees modulo 30. But because

$$22;6,55 \equiv -7;53,5 \quad \text{mod.}30$$

the longitudes in successive zodiacal signs will decrease by $7;53,5^{\circ}$ after 29 days, thus yielding the following values:

0	6;20,45	12;41,35
22; 6,55	28;27,45	etc.
14;13,50	20;34,40	

obtained by continued substraction of 7;35,5 mod.30 .

(3) The fact that the longitudes ... $\lambda - 360$, λ, $\lambda + 360$, ... etc. all correspond to the same point on the circle can be expressed by saying that a certain point on the circle "determines the longitudes only

modulo 360°." Analogously, if we know a year in which a certain date in the Egyptian calendar corresponds to a certain ~~date~~ Julian date, so that, e.g.,

$$(9) \qquad I(e)\ 1 = VIII(j)\ 29$$

is true in the year -25, then we can say that this coincidence holds in all years which are congruent to -25 modulo 1460 Julian years, i.e.

$$\ldots, \quad -1485, \quad -25, \quad 1435, \quad \ldots$$

Or, more generally, any equivalence of Egyptian and Julian dates determines the year only "modulo the Sothic period."

(4) The rule of intercalation of the Gregorian calendar can be expressed as follows: Leap years are all years whose number $\underline{n}$ is $\equiv 0$ mod. 4 <u>except</u> the years $\underline{n} \equiv 100 \quad \equiv 200$ or $\equiv 300$ mod. 400.
Hence: Because $1900 \equiv 300$ mod.400 1900 is no leap year but 1904 or 2000 are leap years. In the Julian calendar, however, <u>all</u> years $\equiv 0$ mod.4 are leap years e.g. the years ..., -4, 0, 4, ... etc. There is no difference in the rule between negative and positive Julian years as it is necessary with A.D. and B.C. years.

5. The Julian days.

We now proceed to explain a group of concepts which play an important rôle in medieval calendarical art, thus becoming the basis of a certain kind of era which is much used in modern works on astronomical chronology under the name of the "Julian period." This new instrument of scientific chronology was introduced by Joseph Justus Scaliger in his work "De emendatione temporum", the first edition of which appeared in Paris in 1583, the year after the Gregorian edict about the new calendar.[30] Scaliger was known

30) For complete title and new editions (1598 and 1629) cf. Bernays, Scaliger, p.283.

to the scholars of his time as one of the leading philologists and is one of those who established the fame of the University of Leyden, where he tought for 16 years, indirectly influencing the development of humanistic

~~Closely~~ ~~five~~ Cycles.

Closely related with the concept of ~~congruen~~ congruences is the "cyclic" arrangement of numbers or symbols in general. If we, e.g., consider integers only mod 5 then any integer is congruent either to 0, or to 1, or to 2, ..., or to 4. In other words, we need only five symbols to characterise ~~the~~ every integer modulo 5. This can be described also geometrically by the following process (cf. fig. 0): we write the numbers 0 to 4 ~~the~~ beside ~~the~~ ~~equidistant~~ five points which divide the circumference of a circle into ~~5~~ five equal parts; ~~and~~ then we continue to write all following ~~and preceding~~ numbers in their natural order around the circle and do the same in opposite direction with the preceding negative numbers. All the numbers (positive and negative) written beside one of our ~~5~~ five points are then congruent to each other modulo 5. We call such an arrangement a cyclic arrangement of ~~the~~ all integers.

The counting of "longitudes" ~~in ...~~ mod. 360°, mentioned in the preceding section, is a special example of this cyclic arrangement.

~~As analogous~~ The procedure of cyclic arrangement ~~can be followed with~~ is not restricted to numbers. ~~another sequence of symbols~~ Suppose we have three letters at our disposal (say b, k, and o) and repeat these letters ~~in arbitrary fashion~~ infinitely many times, e.g. writing ...obbkokookobobokooookbb..... Among such infinite sequences one specific type is of great interest namely the case where the ~~same group of p letters is always repeated, e.g.~~ whole sequence is created by repeating one single group of, say, p letter periodically, e.g. ...bookkbookkbookkbookk.... (p=5). We call such sequences periodic sequences and the number p the "length of its period". If we again write ~~this~~ such a "periodic" sequence around the circle, the ~~the~~ same letters will always near beside a point of the circle. We say that such periodic sequence defines a "cycle" of letters. owing the letters in *one* cycle of length p we ow all letters in the infinite sequence. Such a cyclic angument has, of course, no distinguished beginning point; cycle bookk defines exactly the same infinite uence as the cycle ookkb or okkbo etc. The only ential requirement is that p consecutive tters are ~~uniquely~~ determined; all the preceding and lowing letters are then also fully determined by the cyclic angement.

The most important example ~~application~~ of this concept the "*intercalation cycle*". Let o designate an ordi- ry year, i an leap year. An "intercalation cycle" is n a cyclic arrangement in the above defined sense, based on a cle composed by a certain number of symbols o and i. ample:

.... i o i o o i o o i o i o o i o o i o i o o .....

an intercalation cycle whose period has the length $p = 8$ d where the cycle

i o o i o o i o

ght serve as basic cycle; but o o i o o i o i or o i o o i o i o fines exactly the same intercalation cycle[1]. As before is essential for this concept of intercalation cycle at the ~~idea~~ arrangement of all of the o's and i's is absolutely termined. In other words: the concept "intercalation cle" *includes* the statement that the character of [illegible] uniquely determined[2]

, beginning with 1 A.D.,

2) It is therefore not a sufficient description of the Julian intercalation cycle to say that each tetrade of years contains one leap year. The sequence |o o o i | i o o o | o i o o | o o i o |... would fulfill this definition but does not determine uniquely the character of every future year.

1) Another example is the Julian intercalation cycle, mentioned above p. [illegible], which is defined by o o o i (or by o o i o or o i o o or i o o o).

studied at Oxford.[31]

31) His biography is given by Bernays, Scaliger.

a. Indictio.

"Indictio" means tax-declaration[32] delivered at regular inter-

32) Greek ἐπινέμησις or ἰνδικτιών (RE 9, 1327).

vals to the proper governmental authorities. In the later Roman empire a 15-year indictional cycle used in dating contracts and similar documents was developed. The first year of this dating according to the indiction is 313 A.D.,[33] the year after Constantine won the Western Roman empire.

33) This date is given as early as in the "Chronicon paschale," a chronological work of the 7-th century (ed. Dindorff I p.522, ed. Migne PG 92, col. 700). The statement usually found in modern books that the indictio originated 15 years before 313 in Egypt has been fully disproved by Kase, PRP ~~[illegible]~~ p.25-31.

Fifteen years later the indictio again became 1, etc.; in this way a serie of short consecutive eras always running from 1 to 15[34] were established.

34) Because of the overlaping of the fiscal year (which begins in Septembe and our present Julian years, one usually finds 312 quoted as the beginnir of the cycle. There are also differences between Egypt and other parts of the empire (cf. Bickermann, Chron. p.36.).

This kind of dating seems very inconvienient to modern man but undoubtedl constituted a great improvement compared with dating according to consuls whose only rôle consisted in lending their names to the years.[34a]

It is very simple to determine the indictio of a given year. W extend this dating to the period before 313 and need then only [illegible] the

34a) cf. below p. [illegible].

313 ≡ −2 mod.15 in order again to reach a year with indictio 1. If $\underline{n} \equiv \underline{a}$ mod.15, $\underline{a}$ being an integer between −2 and +12, then the year $\underline{n}$ has the indictio $\underline{a}$ + 3.

b. Solar cycle.

The solar cycle ("circulus solaris") is connected with the institution of the seven-day week. Because $365 \equiv 1$ mod.7 and $366 \equiv 2$ mod.7, a tetrade of three ordinary and one leap year results in a change of the weekdays by $5 \equiv -2$ mod.7. Hence seven such quadruples, i.e. 28 years, move the weekdays by $-14 \equiv 0$ mod.7, which means that after 28 years the same dates fall on the same weekdays as they did 28 years before. This cycle of 28 years is of importance for the calculation of Easter and is therefore always indicated in medieval calendarical works. A year in which January 1 is a Monday gets the "circulus solaris 1." Starting from the weekdays in the time of Dionysius exiguus, one finds that the year −8 has the solar cycle 1 and hence a year $\underline{n}$ the solar cycle $\underline{a}$ + 9 if $\underline{n} \equiv \underline{a}$ mod.28, $\underline{a}$ being an integer between −8 and +19.

c. Golden number.

Of much ~~older~~ later origin is the concept "golden number," which appears first in the "Massa compoti" of Alexander de Villedieu in 1200 A.D.[35] The golden number of a year is its ordinal number in a 19-year cycle having year No.1 in 532 A.D., the year of the introduction of the Christian era.[36] The importance of this cycle lies again in the Easter calculation, because 19 years return the new moon or full moon to the same date.[37] Because

35) Cf. Van Wijk [1] p.31.

36) Cf. above p.###.

37) Cf. ~~xxxxx~~ below p.###.

$532 \equiv 0$ mod.19, the golden number of the year 0 is 1. The golden number of a year $\underline{n}$ is therefore $\underline{a}$ + 1 if $\underline{n} \equiv \underline{a}$ mod.19, $\underline{a}$ being an integer between 0 and 18.

Knowledge of the three concepts of indictio, solar cycle, and golden number is necessary in order to understand the era introduced by Scaliger. He intended to begin an era in the most convenient way for calendarial computations, namely with indictio = solar cycle = golden number = 1, keeping of course the already adopted relationship between the Christian era and the three cycles.

Let us suppose that the first year of the new era is the year $-\underline{n}$. According to the rule (established above) of calculating the indictio of a given year, the condition to be fulfilled in order to give to this year the indictio 1 is $-\underline{n} \equiv -2 \bmod. 15$. Correspondingly, solar cycle = 1 requires that $\underline{n}$ in addition to $\underline{n} \equiv 2 \bmod. 15$ fulfills $\underline{n} \equiv 8 \bmod. 28$; finally, from golden number = 1 it follows that $\underline{n} \equiv 0 \bmod. 19$. It is a simple problem of elementary number theory to find a number $\underline{n}$ obeying these conditions, but it is sufficient for our present purposes to verify that $\underline{n} = 4712$ actually solves the problem because $4712 = 314 \cdot 15 + 2 = 168 \cdot 28 + 8 = 248 \cdot 19$. This shows that the year -4712 is a year with all three characteristic numbers = 1 as required by Scaliger. It is evident, however, that this is not the only possible solution because if we add to $\underline{n}$ a number $\underline{p}$ divisible by all three periods 15, 28 and 19, then $\underline{n} + \underline{p}$ will also be $\equiv 2 \bmod. 15$, $\equiv 8 \bmod. 28$, and $\equiv 0 \bmod. 19$. The smallest number $\underline{p}$ with this quality is obviously

$$\underline{p}_o = 15 \cdot 28 \cdot 19 = 7980$$

All the years

$$\ldots\ldots \quad -12692, \quad -4712, \quad 3268, \quad \ldots\ldots$$

distant from each other by $\underline{p}_o = 7980$ years have the same quality of making indictio = solar cycle = golden number = 1. The number $\underline{p}_o = 7980$ is called the "<u>Julian period</u>" and the year -4712 is adopted as the beginning of the "<u>Julian era</u>."

From the definition of the solar cycle it follows that the first of January of the year -4712, is a Monday. This day is called "Julian day 0" and all following days are simply counted as 1, 2, 3, ..., etc.

We can now forget about the motives which brought Scaliger to his definition of the "Julian period" and restrict ourselves to the simple agreement that we introduce an "era" of days, beginning with

(10) $\qquad -4712 \text{ I } 1 = \text{Julian day } 0$.

This arbitrarely introduced era became of enormous practical importance for all kinds of chronological calculations. The basic idea is very simple: instead of computing tables for all possible combinations of different eras, e.g., Mohammedan era, Christian era, Gregorian calendar, Julian calendar, etc., one reduces all of these eras to Julian days. The comparison between any two eras say A and B, is then always reduced to determining first the equivalence in Julian days of era A and then comparing the obtained Julian day with era B. In other words, the "Julian days" are used as a common time scale for all chronological calculations.

The concept of "Julian days" represents a very typical situation in chronology in general. Its introduction was based on considerations absolutely heterogeneous to our modern direction of thought. In order to compute chronological tables, any other day could have been used as the point of origin for continous counting. But radical innovations both in methods and terminology are extremely rare in all fields of science, and the main process of "progress" consists in the unconscious changing and abandoning of the heavy burden of historical traditions.

6. Examples of calculations with Julian days.

In the following it is not necessary to know anything about the historical background of the "Julian days"; everything is reduced to the purely mechanical use of computed tables such as are to be found in Schram, "Kalendariographische und chronologische Tafeln."

Examples.

(A) Find the Julian day corresponding to 1941 X 4. Schram, p.81, contains the following section:

year	I		II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1935		803	834	862	893	923	954	984	$\bar{0}15$	$\bar{0}46$	$\bar{0}76$	$\bar{1}07$	$\bar{1}37$
1936	2428	168	199	228	259	289	320	350	381	412	442	473	503
1937		534	565	593	624	654	685	715	746	777	807	838	868
1938		899	930	958	989	$\bar{0}19$	$\bar{0}50$	$\bar{0}80$	$\bar{1}11$	$\bar{1}42$	$\bar{1}72$	$\bar{2}03$	$\bar{2}33$
1939	2429	264	295	323	354	384	415	445	476	507	537	568	598
1940		629	660	689	720	750	781	811	842	873	903	934	964
1941		995	026	$\bar{0}54$	$\bar{0}85$	$\bar{1}15$	$\bar{1}46$	$\bar{1}76$	$\bar{2}07$	$\bar{2}38$	$\bar{2}68$	$\bar{2}99$	$\bar{3}29$
1942	2430	360	391	419	450	480	511	541	572	603	633	664	694
1943		725	756	784	815	845	876	906	937	968	998	$\bar{0}29$	$\bar{0}59$
1944	2431	090	121	150	181	211	242	272	303	334	364	395	425

Looking at the line containing 1941 and column X, we find the number $\bar{2}68$. The bar on the $\bar{2}$ means that this number is not to be attached to the 2429 given in column "I", but to the 2430 of the following group. The number thus found is 2,430,268. This represents the Julian day number of the "day 0" of the month in question (X); in order to get the 4th of this month, we must simply add 4. Hence

2430272 is the Julian day corresponding to 1941 X 4 .

(B) How many days have elapsed between 1938 IX 15 and 1941 X 4 ? From the table given in the preceding example we find 1938 IX 0 Jul.day 2,429,172 and ~~correspondingly~~ consequently

1938 IX 15 Jul.day 2,429,187 .

Using the result obtained above, we find for the elapsed time

2,430,272 - 2,429,187 = 1085 days.

(C) Find the Julian day corresponding to -746 II 26. Schram, p.1[illegible], contains the following table:

year: $\begin{cases} -(700 + \tau) \\ -(4700 + \tau) \end{cases}$

I		II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	τ
1447 9986	120	151	179	210	240	271	301	332	363	393	424	454	50
	485	516	544	575	605	636	666	697	728	758	789	819	49
	850	881	910	941	971	$\bar{0}02$	$\bar{0}32$	$\bar{0}63$	$\bar{0}94$	$\bar{1}24$	$\bar{1}55$	$\bar{1}85$	48
1448 9987	216	247	275	306	336	367	397	428	459	489	520	550	47
	581	612	640	671	701	732	762	793	824	854	885	915	46
	~~946~~	~~977~~	~~$\bar{0}05$~~	~~$\bar{0}36$~~	~~$\bar{0}66$~~	~~$\bar{0}97$~~	~~$\bar{1}27$~~	~~$\bar{1}58$~~	~~$\bar{1}89$~~	~~$\bar{2}19$~~	~~$\bar{2}50$~~	~~$\bar{2}80$~~	~~45~~
~~1449 9988~~	~~311~~	~~342~~	~~371~~	~~402~~	~~432~~	~~463~~	~~493~~	~~524~~	~~555~~	~~585~~	~~616~~	~~646~~	~~44~~

Being interested in -746, we must take the year $-(700 + 46)$, i.e., $\tau = 46$ in the last column. For the day II 0 we then find the number 612 in column II; hence, $612 + 26 = 638$ for II 26. As the first part, column I gives two numbers, 1448 and 9987; this corresponds to the two years indicated at the top of the table: $-(700 + \tau)$ and $-(4700 + \tau)$. Because we are dealing with the first case, we must take also the upper number in I. Therefore

$$(11) \qquad -746 \text{ II } 26 = \text{Jul.day } 1,448,638 \ .$$

Remark: The second year number at the top of the table is greater than the first by 4000; now 4000 Julian years amount to $(365 + \frac{1}{4})\cdot 4000 = 1,461,000$ days. Therefore the last three places are not affected by any change modulo 4000 years and accordingly the same tables can be used by adding the corresponding multiple of 1461 to the first number. In the center, we have 9987 + 1461 = 11448; the actual number given is only 1448 because a further addition of 10,000,000 days is historically irrelevant.

(D) What is the equivalent of the Julian day 1,728,053 in the Alexandrian calendar? Schram, pp.108 ff., gives tables headed "Alexandrian years;" the number 1728. ... appears on p.110 as follows

$$\begin{matrix}-17\\-4017\end{matrix}$$

t	I(a)		II(a)	III(a)	IV(a)	V(a)	VI(a)	VII(a)	VIII(a)	IX(a)	X(a)	XI(a)	XII(a)	epagom.
35	1728	872	902	932	962	992	$\overline{0}22$	$\overline{0}52$	$\overline{0}82$	$\overline{1}12$	$\overline{1}42$	$\overline{1}72$	$\overline{2}02$	$\overline{2}32$
36	267	238	268	298	328	358	388	418	448	478	508	538	568	598
37		603	633	663	693	723	753	783	813	843	873	903	933	963
38	1729	968	998	$\overline{0}28$	$\overline{0}58$	$\overline{0}88$	$\overline{1}18$	$\overline{1}48$	$\overline{1}78$	$\overline{2}08$	$\overline{2}38$	$\overline{2}68$	$\overline{2}98$	$\overline{3}28$
39	268	333	363	393	423	453	483	513	543	573	603	633	663	693

Because 1728 is the upper number of the pair $\begin{matrix}1728\\267\end{matrix}$, we must combine it with the upper number in the year numbers $\begin{matrix}-17\\-4017\end{matrix}$ given at the top of the table,[38] i.e., with −17. The second part (053) of our given number does

38) For the meaning of this difference of 4000 years, see the remark at the end of the preceding example.

not appear but the closest smaller number is 052 in the line t = 35. The year in question is therefore −17 + t = 18 A.D. The month into which ....053 falls is VII(a), the 0-day of which has the number ... 052. Therefore the date in question is

(12) Jul.day 1728.053 = 18 A.D. VII(a) 1 .

In Egyptian documents of this time, this year would have been called the fourth year of Tiberius.

one line empty !

This is not the place to go into further details of the technique in using chronological tables which can, at all events, only be acquired by actual experience. Moreover, typical examples are given in the introductory chapters of Schram and analogous works; cf. also ~~One of the most useful applications of the concept of "Julian day" consists in solving the important problem of determining the date in one era (say A) which corresponds to a certain date in another (say B). The basic idea consists in transforming date A into Julian days and proceeding to B through Julian days. In this way, the task of transforming any era into any other era is reduced to the tabulation of each era into Julian days instead of computing tables for all possible pair of eras. Some examples of this simple method will be given at~~ the end of th next paragraph.

§ 2. Ancient Eras.

We have already mentioned that the Christian era, the era used in all modern historical works, was introduced as late as 525 and only ver slowly gained ground in the following two centuries until its reception in the chronological works of Beda.[39] An important reason for this fact can

39) Cf. Poole [1] p.7 f.

be found in the existence of other eras, e.g., the "Spanish era" which is known since the fifth century A.D.[40] or the "world-era" of the Alexandri

40) For the epoch of this era holds Span.era $0 \approx -38$. Cf. Ginzel III p 175 and Poole [1] p.36.

Additions

.25: Byzantine World Era:

A.M. 0 Sept. 1 = − 5508 Sept. 1

with Byzantine years beginning on Sept. 1.

Bibliography: V. Grumel, Traité d'études byzantines La Chronologie 1958 (tables: p. 239 ff.)

V. Gardthausen, Griechische Palaeographie (2) II 1913 (tables p. 488 ff) monk Panodoros (about 400 A.D.) who attempted to determine the date of the creation of the world in order to obtain an absolute chronology in the fullest sense of the word. That his attempt was not very successful is shown by the results which can be expressed by the following equivalence

(13) Panodoros 0 $\approx$ −5493

although such an era could have become exactly as useful for historical sciences as numbering of the Julian days. The coexistence of this and other attempts at creating eras in agreement with the Christian doctrine of the time and the competition with surviving ancient eras made the final victory of one of these systems more or less a question of mere accident. (*) Byz. World Era: A.M. 0 Sept. 1 = −5508 Sept. 1

For the modern scholar, this multitude of chronological systems is one of the main sources of difficulties. The problem of establishing the correspondence is by no means solved if we know that a certain event has the year number $\underline{n}$ in one era, the number $\underline{m}$ in the other. In the majority of cases, the difference in numbering the years is accompanied by a difference in the New Year's Days, with the resultant creation of an overlapping such as is represented by fig.1. We shall formulate such a relationship by

(14a) $\underline{m}(\text{era A}) \approx n/\underline{n}+1(\text{era B})$

or by its equivalent

(14b) $\underline{n}(\text{era B}) \approx \underline{m}-1/\underline{m}(\text{era A})$.

One must, moreover, keep in mind that the degree of overlapping between the years of different eras might be subjected to change, either by arbitrary changes in the New Year's Days adopted[41] or in consequence of differ-

41) In medieval documents, ~~xxxxy~~ New Year's Days appear, e.g., in September, December, January, March etc., officially because of dogmatic differences, actually, however, as remnants of different ancient customs. Cf. Ginzel III p.156 ff. or Poole [1] p.1 to 27.

ent year forms like Julian years on the one hand and Egyptian years on the other. The situation can be still worse if one era is calculated in solar years, the other in a lunar calendar like the Mohammedan calendar; only 33 Julian years are sufficient to add one more lunar year of the latter. In all such cases, however, it is still easy enough to find the corresponding dates in each era because the deviations between the various calendar systems follow definite rules. There exist, however, many instances of historical importance where no general rule exists, e.g. if dates according to regnal years should be equated with years of any other era. Such questions require a special discussion in each individual case and are therefore outside of the framework of this book. What we are going to do is to mention only some of the most important eras of antiquity and omit all details which can be found in the literature quoted in the handbooks listed at the end of this paragraph.

7. Year numbering in the Roman Empire.

The official Roman custom of dating is the same which we shall meet again in discussing Greek or Assyrian chronological methdds, namely by eponyms: each year is characterized by the name of high officials, not numbered according to some era. In Rome the consuls gave their names to the year, even during the times of the emperors, when the power of the consular office became only a weak shadow of its past importance. In fact, the chief function of the "consules ordinarii" was to lend the year their names; for the rest of the year, new consuls, the "consules suffecti", took over the remaining duties of the consular office.

As in all cases of eponymic dating, annalistic lists recording the names of the years became a necessity. In Rome, these lists are called "fasti consulares," and were not only recorded in the governmental archives but also published by inscriptions at important places as we know, e.g., from fragments excavated in Ostia, the harbor of Rome.[42] The restoration of

42) Cf. Calza [1], [2].

these fasti is, of course, an important problem of modern Roman history but has practically no connection with astronomical chronology discussed here. Lists representing our present knowledge of the eponymic consuls can be found in Liebenam, Fasti consulares imperii Romani (1909) and in Cagnat, Cours d'epigraphie latine, 4th edition (1914).[43]

43) Cf. also the article "Fasti" by Schön in RE 6, 2015-2046. (1909); moreover, the additional material from Ostia, quoted in the preceding note and further literature mentioned by Ginzel II § 182.

A real "era," although mainly restricted to literary documents, is the counting of years of the city of Rome. Different attempts have been made to connect the foundation of the city with a definite date. The version which is usually accepted when one speaks of years "ab urbe condita" (= a.u.c.) is the chronology given in M.Terentius Varro's book "De gente populi Romani," written in the middle of the first century B.C.[44] On the basis of astrological speculation[45] the foundation of Rome is dated as falling in

44) Cf. RE 1, 623.

45) Cf. Leuze, Die röm. Jahreszählung p.242 and RE 4 A, 2408.

the third year of the sixth Olympiad (on April 21st). Because the Olympiad era is related to the Christian by[46]

46) Cf. p.*** (4a) where we have replaced -780 by the more precise -780/779 because of the overlapping of the two eras.

$$(15) \qquad \text{Ol. } 0,0 \approx -780/779$$

we have

$$(16a) \qquad \text{a.u.c. } 0 \approx -753/2 \quad .$$

The overlapping, however, between the years of the city and the Julian calendar is usually disregarded by simply identifying

$$(16b) \qquad \text{a.u.c. } 0 = -753 \quad . \quad .$$

This is the usually accepted definition of the "Varronic" era.

Another important Roman era was developed in Egypt. We already mentioned the Egyptian custom of dating according to regnal years. This method was adopted during the latest periods of Egyptian history, also by the Greeks and Romans; ~~and~~ thus we find dates in Egypt an document of Roman times, usually expressed by the regnal years of the emperors. This numbering goes in perfect agreement with the civil calendar, such that civil years and regnal years both begin with the first of Thoth. If a new ruler ascended the throne, the civil year which began with the first New Year's Day falling in the reign of the new ruler was called his second year. The "first year" of an emperor is therefore only the fraction of a year left from the last year of his predecessor. As a specific example, the following case might be mentioned. The 11th of August 117 is the "dies imperii" of Hadrian,[47] i.e., the day when he officially began to rule the

47) Cf. RE $\underline{1}$, 499.

Roman empire. The next New Year's Day of the Alexandrian calendar in Egypt is I(a) 1 = August 29, and therefore the "second" year of Hadrian in Egypt begins only 18 days after his ascent to the throne.[48] This is a very

48) This is proved by a Greek ostracon (Wilcken, Ostraka II p.786) and by the Demotic Stobart tablet C_2 reverse (Neugebauer, [0] [1] ~~Egypt. Texte~~ p. 226).

typical case showing how careful one must be in comparing the Egyptian regnal years of Roman emperors and their years according to Roman sources.

The counting of the regnal years of an emperor in Egypt was undoubtedly continued long after his death, as is shown, e.g., by an astronomical papyrus which calls the 15th year of Trajan the 34th of Titus.[49] This

49) Pap.Tebtunis 274 (Grenfell-Hunt-Goodspeed, Tebtunis II p.24 and Neugebauer (0)[1] p.242). Cf. also the horoscope P.Brit.Mus.130 (Kenyon I p.133). For "provincial eras" cf. Gardthausen Pal.II p.445 f.

procedure is easy to understand in astronomical texts where systematic calculations are involved. Towards the end of the Roman Empire a real era came into wider use, continuing the regnal years of Diocletian according to the Alexandrian calendar. The corresponding relation to our era is

(47) Diocl. 0 = 283/4

The

(

50)

of

Gre

bec

era

ord

thi

to

Rom

no

therefore became the era of the Alexandrian Easter calculations. We have already mentioned the Easter tables of Cyril and their continuation by Dionysius exiguus[51] who replaced the Diocletian era of his predecessor

51) Cf. p. ... and note — there.

typical case showing how careful one must be in comparing the Egyptian regnal years of Roman emperors and their years according to Roman sources.

The counting of the regnal years of an emperor in Egypt was undoubtedly continued long after his death, as is shown, e.g., by an astronomical papyrus which calls the 15th year of Trajan the 34th of Titus.[49] This

49) Pap. Tebtunis 274 (Grenfell-Hunt-Goodspeed, Tebtunis II p.24 and Neugebauer (0)[1] p.242). Cf. also the horoscope P.Brit.Mus.130 (Kenyon I p.133). For "provincial eras" cf. Gardthausen Pal.II p.445 f.

procedure is easy to understand in astronomical texts where systematic calculations are involvéd. Towards the end of the Roman Empire a real era came into wider use, continuing the regnal years of Diocletian according to the Alexandrian calendar. The corresponding relation to our era is

$$\text{(17)} \qquad \text{Diocl. } 0 \approx 283/4 \quad .$$

(More precise: Diocl. I I(a) 1 = 284 VIII(j) 29.

The years of this era are also ~~unitto~~ used in astronomical documents[50] and

50) The two earliest instances known to me of this era are two horoscopes of the years Diocl.31 and 33 (P.Soc.Ital.VII p.53, No.765, and Grenfell [1] Grenfell's statement that Theon uses the era Diocl. seems to be unfounded because Ideler (I p.164) remarks that there is only one place where this era appears in Theon, quoting the edition of the commentary to the Almagest printed in Basle 1538 p.284 (Ideler I p.142 note 1). The modern edition of this commentary, however, shows that the passage in question does not belong to the original text, which makes no reference to the era Diocletian (ed. Rome p.174); ~~and~~ incidentally ~~moreover~~, ~~that~~ the author of this commentary is in reality Pappus, not Theon, as Rome discovered).

therefore became the era of the Alexandrian Easter calculations. We have already mentioned the Easter tables of Cyril and their continuation by Dionysius exiguus[51] who replaced the Diocletian era of his predecessor

51) Cf. p. ??? and note — there.

29

an reg-
es.
as un-
strono-
9) This

Neu-
.133).

ic cal-
ra came
to the

50) and

scopes
ell [1];
nded
this
lmagest
ion' of
t belong
(ed.
ppus,

The origin of this era seems to be a rather unexpected consequence of Diocletians reform of the organisation of the Roman empire, by which Egypt lost its exceptional position as an imperial province. Up to that date Egypt was allowed to continue the old habit of counting years as regnal years of the ruling emperor. Now Diocletian introduced the ordinary Roman dating by consuls also into Egypt, interrupting thus the counting by rulers. As a consequence, however, the regnal years of Diocletianus himself remained the only basis for continuous chronological reckoning, and became thus the basis of a real "era".

have
by
sor

Position: 24
Operator: EndChar
Error: IllegalOperatorSequence
Subsystem: KERNEL
PCL XL error

by years "ab incarnatione domini." The Christians in Egypt, the so-called Copts, continued the counting of the regnal years of Diocletian, substituting in Arabic times[52] the name "era of the martyrs" for the name of Diocletian,

(Cf. Chaîne, ChrEE p. 14f.

52) Gardthausen, Pal.II p.446 quotes as the latest instance of the era Diocletian the double date Diocl.451 = "year of the Saracens" 111 on a Greek papyrus.

in memory of the prosecutions during his reign. In this Christianized form, the Diocletian era continued in Egypt the counting of the regnal years of this emperor for more than a thousand years.[53]

53) Gardthausen mentions (Pal.II p.446) the year 1181 A.D. for the latest use of this era in Greek inscriptions, as late as the 19th century for Coptic texts.

8. The Seleucid Era.

One of the most important eras of antiquity is the "Seleucid" era, beginning 312 B.C., when Seleucus, one of Alexander's generals, ~~captur~~ occupi-ed Babylon, thus founding an independent kingdom. In contrast to most of the ancient eras, the Seleucid era is not restricted to astronomical or literary use but is the generally adopted method of dating found in countless documents[54] from public and private life in Mesopotamia and Syria. This era survived the collapse of the Seleucid Empire and was still in use in Parthian and even in Arabic times[55]. There are, however, local differenc

54) For dates on coins cf. e.g. McDowell, Coins.

55) Cf. Ginzel I p.136 ff. and III p.40 ff.

with respect to the beginning of the years. The two most important variances are the Babylonian style, beginning the year near the spring equinox, and the type which follows the Macedonian calendar with New Year around the autumn equinox. The relative position of these two forms of the Seleucid era is indicated by fig.2 and consequently by

(18a) Sel. Babyl. $0 \approx -311/310$

(18b) Sel. Maced. $0 \approx -312/311$.

Because both types of the Seleucid era are based on a real lunar calendar which requires the knowledge of the dates of the new crescents ~~more or less irregular intercalation of a thirteenth month~~, no simple - general rule for a day-to-day correspondance can be given. A very close relationship, however, can be established between the Babylonian branch of the Seleucid calendar and the Julian calendar because, besides many cuneiform business documents, we have from this period many astronomical tablets which make comparison by modern calculation possible for about two centuries (ca. 250 to 50 B.C.). This period, therefore, constitutes one of the best defined parts of ancient chronology.[56)]

56) A comparative list of Seleucid and Julian dates is given by W.Dubberstein .... A previous table composed by Cavaignac ([1] p.73 ff.) is very inconveniently arranged for practical use.

A variation of the Babylonian form of the Seleucid era is the "Arsacid" era of the Parthians. If $\underline{n}$ is the year number in the Seleucid era, $\underline{m}$ in the Arsacid, then

(19) $\underline{m}(\text{Ars.}) = \underline{n}(\text{Sel.}) - 64$

holds.[57)]

57) Cf. Kugler SSB II p.443 ff.; furthermore Debevoise, Parthia p.48 note 74 and p.157 note 56.

9. Era Nabonassar and Ptolemaic Canon.

Exclusively restricted to astronomical use is the era Nabonassar which can be defined by

$$\text{Nab. } 0 \approx -747/746 \tag{20}$$

with

$$\mathrm{I}(e)\ 1 \approx \mathrm{II}(j)26 \tag{21}$$

as the day of departure for the first year. It is, however, very important to keep in mind that "years" of this era as used by the Greek astronomers are Egyptian years of 365 days only. The correspondence (20) between years of Nabonassar and the Julian years of the Christian era can therefore not be directly extended to far distant times because the Egyptian beginning of the year continuously falls back in the Julian calendar. For example, Nab. 1068 is not -747/46 + 1068 - 321/22 but overlaps 320/1.[58)]

---

58) Cf. the example calculated below p. 38.

The name Nabonassar is the first in a king-list usually known as the "Ptolemaic canon." This name is somewhat misleading. Ptolemy uses the era Nabonassar very frequently in his Almagest and elsewhere; all his tables are based in this counting of Egyptian years. But the king-list is not contained in the Almagest and neither king-list nor the era Nabonassar can be proved to be Ptolemy's invention. It is, on the other hand, evident that he had such a canon of regnal years at his disposal; one of his works, of which only the introductory chapters are preserved, contained a "ἀπὸ τῆς ἐποχῆς βασιλέων χρονογραφία"[60)] and such tables are actually preserved in Theons

---

59) The "handy tables" (πρόχειροι κανόνες); Ptolemy opera ed. Heiberg II p.159-185.

60) Ptolemy opera II p.160,8. But as Ptolemy remarks (p.160,20 ff.), the planetary tables in the handy tables were based on years beginning with Nabonassar.

commentary, written about 200 years later. This "Ptolemaic canon" was first printed in 1820 by Halma,[61] and a modern edition was given by Usener;[62] the canon is now reproduced in most of the modern works on chronology.[63]

(61) Halma, Comm. de Théon I, p.139-143.

(62) In Mon.Germ.hist., Auctores antiquissimi, 13 (1898) = Chronica minora saec. IV-VII, vol.III ed. Mommsen, p.359 ff. and p.438 ff.

(63) E.g. Kubitschek GAZ p. 61 ff. or Ginzel I p.139.

The Ptolemaic canon is an idealization for purely chronological purposes achieved by identifying exactly regnal years and Egyptian years and omitting short reigns. But within these obvious limits this list has been proved very reliable and undoubtedly based on authentic annalistic documents. Why Nabū nāṣir, an unimportant Babylonian king, ruling from -746 to -733, was chosen to be the first name in the canon is unknown. It may be that the reason will be found in an older historiographic tradition. Berossos, who wrote his Babylonian history about 275 B.C.,[64] ~~began~~ concludes his ~~third~~ second book with Nabonassar's son, discontinuing ~~and beginning~~ with the third book the subdivision into "dynasties" characterizing the preceding books.[65] Berossos seems to be the first historian known to the Greeks who used original source material[66] and books like his "Babylonian history" must have been the basis for the composition of the Babylonian part of the king list. Moreover, it must not be forgotten that the Ptolemaic canon was mainly intended for astronomical

(64) Cf. Reall.d.Ass. II p. 3 b.

(65) Schnabel, Ber. p.24 f.

(66) Cf. Dougherty, Nabonid p.10.

use. We have a very important remark of Ptolemy,[67] telling us that "the

67) Almagest III,7 ed. Heiberg I,1 p.254, 9-13.

ancient observations are preserved almost completely since then (the reign of Nabonassar) up to the present date." This indicates that the choice of this point of departure for an astronomical era ~~was also~~ might have been determined by the available material of observational records.

Whether we assume that the cause for the era Nabonassar lay in a purely practical consideration or in the continuation of an older tradition, the result remains that the mere existence of a well defined system of counting years was of the greatest importance for all astronomical calculations. It is therefore not surprising that we find both the canon of kings and the years of Nabonassar continued far beyond the limits of antiquity. Their usefulness for chronological computations can be compared with the rôle of the Julian days.

10. Tables. Examples.

The problem of passing over from one era to another can be solved to a large extend by using special tables given in the larger chronological handbooks. At any rate, Schram's tables[68] will make it possible to deter-

68) Cf. the bibliography at the end.

mine all equivalences needed in practice by using the Julian days as a common time scale. There are, however, other tables which save even this small amount of calculation for the cases most frequently occuring. The tables of Wüstenfeld and Mahler[68] e.g. give directly the equivalences between the Mohammedan and Christian eras. Tables for the reigns of the Ptolemies in Egypt have been computed by Ernst Meyer[69] and T.C. Skeat[70].

69) Meyer (Ernst) [1].
70) Skeat [1].

Very useful tables for 59 different eras are given in P.V.Neugebauer's HTCh.[68] It must, however, be emphasized that all tables are forced to make assumptions as to the uniformity of the calendar in question. If, therefore, a calendaric system like the Babylonian depends upon empirical elements (observations of new moons), then small deviations of one or two days cannot be avoided. Moreover, in moon calendars without a definite rule of intercalation the difference can amount to one month because the tables are necessarily compiled on the assumption of a regular intercalation rule. In such cases the obtained equivalences can only be considered as averages. Thus again the Egyptian calendar shows its superiority for all practical computation.

Examples.

(A) Find the Julian date corresponding to Nab.1068 V(e) 17.[71]

71) This date occurs in Pappus' commentary to Ptolemy, ed. Rome p.180,10 f. p.181,15 f. and p.XI. The date is characterized as "κατ' αἰγυπτίους Τυβί" in order to avoid the interpretation of "Tybi" as a month of the Alexandrian calendar. Cf. example C.

P.V.Neugebauer HTCh table 2[illegible] A gives the number M of every day in the Egyptian calendar beginning with Thoth 1 $\approx$ 0 up to the fifth epagomenal day $\approx$ 364. Thus V(e) 17 corresponds to M = 136. Now table 2[illegible] contains the following information

Nabonassar	A.D.	N	D
1067	319	VI 5	156 s
1068-70	320-22	4	155
1071	323	4	15[illegible] s
1072-74	324-26	3	154
1075	327	3	154 s

N means the Julian date of the I(e) 1, D gives its equivalent in days counted continuously from I(j) 1 ≈ 1 in ordinary years, from I(j) 1 ≈ 0 in leap years, marked by "s". Adding to D = 155 the number M = 136, we obtain 291 as the day number counted from I(j) 1 ≈ 1. On table 51 the day numbers are compared with the Julian calendar whence one gets directly 291 ≈ X(j) 18. Hence[72]

72) Cf. above p. 32.

(22) 1068(Nab.) V(e) 17 ≈ 320 A.D. X 18 .

(B). Find the equivalent of Nab. 287 III(e) 16. As in the preceding example, we find that III(e) 16 has the day number M = 75. Furthermore table 2[illegible] gives

Nab. 287 I(e) 1 = −461 XII 17 and D = 351"s".

Therefore M + D = 426 "s". Because this number is greater than 365 we must add one year, i.e. we obtain −460. The "s" indicates that −460 is a leap year, but this is automaticaly provided for in table 51 which gives separately numbers with and without "s". In the present case, we must use the number 426 s and get III 1 (instead of III 2 in the case of an ordinary 426). Hence

287 (Nab.) III(e) 16 ≈ −460 III 1.

(C). Find the date in the Alexandrian calendar which corresponds to Nab. 1068 V(e) 17.[73] Schram p.184 gives as the first part of the Julian

73) Cf. example A.

day number of Nabon. 1000 + t , t = 68 , the number 1838: the second part follows from the opposite page under year t = 68 V(e) 0 + 17 as 212 + 17 = 229 . Hence

(23) 1068 (Nab.) V(e) 17 ≈ Jul.day 1838.229 .

Now we go to Schram's table "Alexandrian year" and find on p.116 the number 1838.208 in the line $t = 37$ and in the column II(a). The day is therefore 229-208 = 21 i.e. II(a) 21. The year heading this table is 283; hence in our case $283 + t = 283 + 37 = 320$ A.D. and

$$1068 \text{ (Nab.) } V(e)\ 17 \approx 320 \text{ A.D. } II(a)\ 21.$$

In the civil calendar of this time, the year 320 A.D. would have been dalled according to (17) p.[illegible] year 36 of Diocletian. The complete answer to our problem is therefore

$$(24) \qquad 1068 \text{ (Nab.) } V(e)\ 17 \approx 36 \text{ (Diocl.) } II(a)\ 21\ .$$

From (23) we can immediately verify the result obtained in the first example. Schram p.40 "Julian calendar" contains the Julian day 1838.211 in line $t = 20$ column X(j). The date is therefore $229 - 211 = 18$, i.e., X(j) 18. The year is $300 + t = 320$. This is the same result as expressed in (22).

In general, P.V.Neugebauer's tables can always be replaced by Schram's tables (but not vice versa), which reduces all transformations to finding the Julian day. For systematic chronological computations, however, (e.g., comparison of ancient astronomical tables with modern calculations) every step which can possibly be avoided means considerable saving of time and moreover reduces the probability of errors. In such cases, tables which permit direct transformations between the two eras in question are very valuable.[74]

---

74) Inconvenient in P.V.Neugebauer's HTCh. is his counting of years "B.C." especially because his tables for computing astronomical phenomena are based on "negative" year numbers.

§ 3. Other ancient methods of dating.

As mentioned in the preceding paragraph, dating by continuous counting of the years of some arbitrary era is by no means the normal case in antiquity. We must therefore give a short sketch of the development of some of the most important forms of the dating of years which can be considered as typical for other local developments.

10. Egypt.

We are fairly well informed about the history of dating in Egypt, mainly thanks to the investigations of K.Sethe.[75] The earliest method of

75) Sethe, Unters. 3; a summary is given in Gardiner Gr. p.203 ff. For Egyptian time reckoning in general see Sethe, Zeitr.

characterizing a certain year consists in mentioning an event of importance which took place in the year, e.g., a victory over enemy tribes, the erection of an important building, etc.[76] We shall call this kind of dating

76) For examples see Breasted AR I p.58 ff.

a dating by "year formulae". Its usefulness depends on the existence of annalistic records giving year by year the events considered as important. Such a document is actually preserved in the famous "Palermo stone"[77] wh

77) Cf. Breasted AR I p.51 ff. and Ed.Meyer, Aeg.Chr., pl.VI, VII.

shows that beginning with the Fifth Dynasty the events recorded became standarized in counting years of the census. The recording of taxation, e.g., "year of the third occurrence of the census" here plays the same r for chronology as the "indiction" of medieval Europe.[78] The earlier form

78) Cf. above p. [illegible].

of the census consisted in "numbering" every second year; shortly thereafter (Sixth Dynasty), the census became a yearly institution, and the year formulae degenerated to a simple counting, e.g., "third occurrence," without mentioning the self-evident event of the taxation. The the word "occurrence" ḥ³t sp,[79] acquired a meaning like "regnal year," i.e., year in an or-

79) The translation "occurrence" is not literal; ḥ³t means "beginning," sp "occasion" (used e.g. in sp 3 = "three times").

dered sequence, in contrast to the word rnp.t "year" in the undetermined sense of the word. This distinction is still clearly visible in texts of the latest period of Egyptian history.[80]

80) Cf. the Demotic Pap.Carlsberg 9 (Neugebauer - Volten [1]).

The above described counting of repetitions of the census thus resulted in the counting of "regnal years," in which form Egyptian chronological notices are given. We have already mentioned the fact that this custom was not abandoned as late as Roman times. This led to the counting of regnal years of Roman emperors in Egypt and thus finally to our present era. We have noted on the same occasion that these regnal years were identified with the years of the civil calendar. The same principle of coordination of regnal years and calendar years appears also in cases where a calendar different from the Egyptian was used. The Jewish garrison on the island of Elephantine in upper Egypt during the Persian domination of Egypt used in addition the Egyptian calendar also the Jewish-Babylonian calendar. Papyri from this site have been found with dates like "on the 24th of

Shebat, year 13, that is, the 9th day of Athyr, year 14 of Darius the king",[81] which in our notation represents the following correspondence:

Darius 13 XI(b) 24 = Darius 14 III(e) 9 .[82]

---

81) Cowley, Aram.Pap. No.28 (p.104). Another example No. 25 (p.85): Darius 8 IX(b) 3 = Darius 9 I(e) 12 .

82) The Julian equivalent is -409 II 10 .

This shows that the number of the regnal year depends on the calendar, which must be known in order to identify a regnal year without ambiguity.

The grouping of Egyptian years into "Dynasties" is based on Manethos' history of Egypt (about 280 B.C.).[83] The fragments of his work are

---

83) Cf. the introduction to the English translation of Manethos' writings in the Loeb Classical Library (edited and translated by W.G.Weddell; the same volume contains Ptolemy's Tetrabiblos).

---

mainly preserved through notations in Josephus' history of the Jewish people (first cent.A.D.) and consequently by Christian chronographers like Eusebius (ca. 300 A.D.)[84]. Although more or less artificial, this grouping into

---

84)

---

dynasties has proved to be so convenient that it is used by all modern historians. The general scheme may be given here for the sake of references in the following. The basis for the dates given (here abbreviated to round numbers) will be discussed below in chapter IV.

Predynastic

Old Kingdom: Dynasties I to VI. 3200 to 2300.

Intermediate period.

Middle Kingdom: Dynasties XI to XIII. 2100 to 1700.

Hyksos

New Kingdom: Dynasties XVIII to XX. 1600 to 1100

Late period, including also Assyrian, Persian, Hellenistic-
Roman rule.

Between the oldest extant annals (the "Palermo stone") and the latest Egyptian history (Manetho) falls one of the most important king-lists of ancient history, the "Turin papyrus", written about 1300 B.C., discovered in 1824 among hundreds of other fragments in the Turin museum by Champollion.[85] These combined sources are of such a character that they yield

85) Cf. for the dramatic history of this discovery Hartleben, Champollion I p.520 ff. and Meyer (Ed.) Aeg.Chron. p.105 ff. Latest edition: Farina, PR (1938).

only a relative chronology. One of the most discussed problems in Egyptian history consisted in evaluating the time to be attributed to the dark ~~peri~~ periods between the Old and Middle Kingdoms and between the Middle and New Kingdoms. The resulting theories diverged by more than a millenium until the "short chronology" finally won general approval. The astronomical questions involved will occupy us in the fourth chapter.

12. Babylonia.

The dating by "year formulae" which appears as the earliest form of the designation of years in Egypt also exists in the older periods of Mesopotamian history in a much more highly developed form and kept in general use during a much longer period than in Egypt; even the common Sumer:

word for "year" is the same as for "name". Year formulae appear in Babylonia during the time of Narām-Sin (ca. 2300 B.C.) and were still the method of dating during the whole so-called first Babylonian dynasty, i.e., to about 1600 B.C. when dating according to regnal years became customary together with Cassite rule.[86)]

86) There are, however, traces of a dating according to regnal years preceding the year formulae. Cf. Langdon [1] p.137 and Reallex.II p.132.

The study of the calendaric systems of Mesopotamia faces far more complex problems than the Egyptian calendar, a reflection of the much more eventful history of Babylonia and the neighboring countries. We know practically nothing about the beginnings of the Sumerian calendar. Influence from Egypt has been assumed[87)] but is by ne means proved, although the possibility cannot be denied. The period about which we are best informed today is the time of the third Dynasty of Ur.[88)] The problem of designating

87) Langdon [1] and [2].

88) This is mainly due to the work of N.Schneider, ZWV.

a year seems to be have been solved by the method of calling the first part of a year "year following the year of ..." until a new important event justified giving the year its definite name "year of ..." such and such a happening.[89)] Here as in Egypt, the collection of these year-names became a necessity and such lists are actually preserved,[90)] but in insufficient numbers to establish a complete relative chronology,[91)] not to mention the

89) This is the result of Schneider ZWU p.60 in contradiction to Reallex. Ass.II p.132 b.

90)

91) The results based on material known up to 1938 are published in Reallex. Ass.II p.131 ff. and p.256 f.

difficulties originating in the many local variations in this early period of Babylonian history.

Also the third type of dating, the "eponymic",[92] is represented

92) This name is taken from the analogous Greek institution. For the origin of the word ἐπώνυμος see RE 6, 244. For the Athenian list of archons see Dinsmoor, Archons [1] and [2]; for the ephors in Sparta, RE 5, 2860 ff., for Hellenistic Egypt Otto PT I p.137 ff. and Thompson [1].

in Mesopotamian cultures. Just as the years in Rome were called after the consuls, the years of Assyrian documents were called after high officials the "limmu" of the year. The list of these limmu can be restored from about 900 B.C. to -647[93] and is one of the most important elements in the relative chronology of this period in the history of the ancient Near East.

93) Cf. Reall.Ass. II p.418-428 and below chapter II p.***.

In summary, it must be said that our knowledge about the development of the calendaric systems of Mesopotamian still deserves a great deal of investigation and is far behind the corresponding studies concerning Egypt and the Greek-Roman world although there can be little doubt that the Babylonian calendar exercised the greatest influence on all later documents, partly direct, partly indirect, as, e.g., through the Jewish calendar. Only few but important questions are treated in detail; thus, the religious aspects of the older calendars in Babylonia are discussed by Landsberger in very important but unfinished study.[94] Langdon's lectures "Babylonian

94) Landsberger KK.

Menologies" contain much material of great interest but require caution because of preconcieved doctrines. The best known period is, of course, the latest phase when the calendar had lost many of the arbitrary elements with respect to intercalations, local variations, etc.[95] The natural difficul-

95) Also for this period much is known about the religious background, mainly from Thureau-Dangin, Rit.Acc.

ties involved in the tremendous complexity of three millenia of Mesopotamian history were greatly increased by the theories promoted by the so-called "panbabylonistic" school, which succeeded in making a large proportion of the existing literature in this field misleading and almost useless for further attempts to bring order and understanding in the actual facts. A large and very important field is here still open for systematic research.

§ 4. The smaller time units.

Months, days, and hours are so familiar to us today that almost nobody outside of the small group of people who deal with astronomical problems realizes how many purely arbitrary elements are involved in the definition of these fundamental concepts of time measurement. Each of these units is a "natural" or simple concept only so long as one does not attempt to give any kind of precise definition and so long as one does not compare one of these units with another. Such a comparison, however, is required by the practical life; to overcome the resulting difficulties, several thousand years passed before a clear understanding of the basic astronomical phenomena was reached. It lies outside of the plan of this book to relate the history of these astronomical concepts, but a few details must be explained to get a proper understanding of ancient time measurement and its relationship to chronological problems.

lements with
al difficul-

ound, mainly

f Mesopo-
by the so-
arge propor-
lmost useless
actual facts.
atic research.

y that almost
tronomical pro-
ed in the defi-
ch of these
oes not attempt
es not compare
, is required by
several thou-
astronomical
book to relate
ls must be ex-
rement and its

It will be useful for our following discussions to have a short list at hand which gives in round numbers the dates of the main periods of Babylonian history.

Old Akkadian	2300 – 2100
Dynasty of Ur III	2100 – 2000
First Dynasty of Babylon	1900 – 1600
Cassites	1600 – 1200
Neo-Babylonian and Persian period	600 – 330
Seleucid, Parthian, Roman period	from 311 onwards

13. Months.

The concept "month" is obviously taken from the periodic change in the appearance of the moon. The movement of this satellite presents the most difficult problem in the study of our planetary system. Its use for calendarial purposes (the real "lunar months") therefore requires a more detailed discussion which will be given in the next chapter, where we shall discuss also the closely related problem of lunar and solar eclipses. The institution of lunar calendars and the interest in the phenomenon of the eclipsed moon or sun has undoubtedly been one of the main forces in developing a theoretical study of the irregularities in the moon's movement. Until the last century B.C., however, nobody was able to predict with sufficient accuracy the number of days from new moon to new moon. Festivals falling on a definite date in the lunar month were therefore dependent upon actual observations. On the other hand, the development of economic life (the conclusion of contracts, delivery of materials, payment of taxes, etc.) created the necessity of determining dates much farther ahead than the irregularities of the lunar calendar could be estimated, The logical consequence of this situation was the introduction of a purely artificial calendar consisting of months and years of round numbers of days. We might call this calendar a "bus~~s~~iness" or "fiscal" calendar. In other words, the natural tendency to coordinate the calendaric months as closely as possible to the actual appearance of the moon led to a second form of the civil calendar with no relation to the moon at all.

The most far-reaching consequence of this process can be seen in Egypt. The real lunar calendar plays only a very secondary rôle as the religious calendar of certain lunar festivals. The real civil calendar, however, exclusively used in datings, was based on twelve months invariably 30 days each, and five "epagomenal" days at the end in order to keep the "year" in harmony with the seasons?[96]

96) For this later point see chapter IV (below p.***).

The essential point in this explanation of the Egyptian year as a "fiscal" year lies in the fact of the coexistence of real lunar months and the schematic 30-days months. Many theories of the Egyptian calendar have been proposed, all operating with the assumption of different more or less developed year forms gradually approching the year of 365 days. The basic error in this kind of argument lies in the assumption of successive improvement of a single year form at a time, presupposing the purely astronomical form of the problem of determining step by step with continously increasing exactitude the length of the "true" solar year. There exists, however, no evidence whatsoever of such a tendency of essentially astronomical character. Neither the Egyptian nor the old Babylonian calendar shows any interest in the "solar" year; we have already mentioned[97] that the Egyptian "seasons" have no relationship at all with the astronomical seasons, and we shall later[98] discuss the old Babylonian methods of intercalation which also clearly shows the absence of any attempt at approximating the solar year. Interest in such a problem is entirely restricted to a highly developed astronomy, non-existant in any part of the ancient world before the last millenium B.C. Practical life, however, does not refrain from the most obvious contradiction. For some purposes the month can be considered as being strictly lunar, for others as a round interval of 30 days, and nobody thought about "improving" this situation.

97) Cf. p. ***.

98) Cf. p. ***.

The existence of a real lunar calendar in Egypt can be proved from the earliest times to the latest periods; from the latter we possess evidence even of a simple device for the approximate calculation of lunar phenomena which will be discussed in chapter II.[99]. Here we need only

99) Cf. below p.***.

briefly mention some well-known data for the twelve fiscal months of 30 days each - the epagonal days, in accordance with their name, are considered as being "on the year", or, as we would say, outside the year. The offerings in the great calendaric inscription on the walls of the temple of Ramses III (about 1200 B.C.) at Medinet Habu are considered as consisting of the offerings "for the year and the five days",[100] showing clearly that

100) Medinet Habu III and Meyer (Ed.) Aeg.Chron. p.9.

"the year" contains only 360 days. This can be followed back into the beginning of the Middle Kingdom: a contract between the nomarch of Siūt and and the priests of ~~KKK~~ two temples says[101] "See, a temple day is 1/360th

101) Reissner [1] p.84, 85.

of a year. You shall divide all the daily rations which enter this temple consisting of bread, beer, and meat; for a temple day is reckoned at 1/360t of bread, beer, and everything which enters this temple ..." Here and in analogous occasions it is evident that the business year consisted only of the 12 months of the civil calendar.

It is not surprising that this simple scheme was also applied in matters which we usually call "astronomical" but which are much too complicated to be described accurately by the very primitive mathematical method existing in Egypt. Therefore, the natural way of expression consists in adopting the same simplification of the facts and to describe the changing appearances of the star configurations in a schematic way. Hence we find in the Royal tombs of the New Kingdom representations of the rising and setting of stars (the "decans") which are based on a year of 360 days.[102] There is

102) Cf. Lange - Neugebauer [1] p.69 ff.

no doubt that also this astronomical use of the idealized calendar goes back much farther, at least to the beginning of the Middle Kingdom.

Turning now to Babylonia, we find the exactly parallel phenomena as in Egypt: a fiscal or business year of 360 days, consisting of twelve months of 30 days each and the extension of this schematic calendar into purely astronomical problems. For the fiscal year, we can point to Old-Babylonian mathematical tablets (about 1800 B.C.) which state explicitly that in calculating interests the year should be counted as 360 days;[103] correspondingly 30 days are assumed as the length of a month.[104] We know,

103) Neugebauer MKT I p.360 and III p.59 f.

104) Neugebauer MKT III p.63.

furthermore, that dates in contracts for delivery or payment of agricultural products at a later time must not necessarely mean the actual date of the calendar month but merely the season.[105] Finally, we know that equinoxes

105) Thureau-Dangin [4] p.188 ff. Rawa + ZA 15 (1900) 412 2).

and solstices were simply called I 15, VI 15 and IV 15, X 15, although the actual dates in the lunar calendar could deviate from these dates by more than one lunar month.[106] The same scheme of 30-day months is applied in the oldest known texts describing the disappearances and reappearances of Venus, which we shall discuss in more detail in chapter III.[107] This

---

106) This is the case in all texts which deal with the variable length of the shadow during the seasons. Oldest example Mul Apin (ca. 700 B.C.) first tablet II 42,43 III 7,9 (Bezold [1] p.26; the translation of II 43 and III 9 on p.27 is, however, incorrect). Moreover Virolleaud Babyloniaca 6 and Weissbach BM p. 150 f. and Kugler SSB Erg. p.88 ff.

107) Cf. p.[illegible]; see the next section p.[illegible] for the latest form of the Babylonian planetary theory.

---

clearly shows the correctness of our basic assumption: the schematic calendar becomes a necessity as soon as dates in the future are concerned becaus it is not known, how to extrapolate the exact lunar calendar over a period of some months.

To summarize: Babylonian and Egyptian calendars are certainly as different as possible in their final form; the Babylonian calendar is strictly lunar and hence dependent on astronomical facts, the Egyptian calend purely schematic with no astronomical relation at all. And yet, both systems originated in very analogous situations, namely, the coexistence of both types of months, the real lunar month and the convenient business month of 30 days. The difference in emphasis, which finally led to such different results, is, of course, easily understood from the general historical background. The numerous small city-states of Mesopotamia did not develop a common fiscal calendar which in Egypt was the natural consequenc of the centralized Pharaonic regime. The basic concepts, however, are the same in both cultures, but there is no need to assume any direct influence simple analogy is sufficient.

The civil calendar of the Egyptians is, of course, much more ovement than a real lunar calendar which is affected by all the regularities caused by the complication of the movement of the oon. The departure of the Roman calendar from the pure lunar ype resulted also in months independent of the moon but did ot reach the simplicity of the Egyptian calendar. Also the unting of the days in the single months is very peculiar in e Roman calendar, being counted backwards from the nones the 5th or 7th day), from the idies (13th or 15th day) and from e first day of the following month. The details of this cheme are given in the following list:

	I VIII XII	III V VII X	IV VI IX XI	II		
1	Kalendis N	kalendis N	Kalendis N	Kalendis Februarii		1
2	4 ante nonas N	6 ante nonas N	4 ante nonas N	4 ante nonas Februarii		2
3	3 " " "	5 " "	3 " " "	3 " " "		3
4	pridie " "	4 " "	pridie " "	pridie " "		4
5	nonis N	3 " "	nonis N	nonis Februarii		5
6	8 ante idus N	pridie " "	8 ante idus N	8 ante idus Februarii		6
7	7 " " "	nonis N	7 " " "	7 " " "		7
8	6 " " "	8 ante idus N	6 " " "	6 " " "		8
9	5 " " "	7 " " "	5 " " "	5 " " "		9
10	4 " " "	6 " " "	4 " " "	4 " " "		10
11	3 " " "	5 " " "	3 " " "	3 " " "		11
12	pridie " "	4 " " "	pridie " "	pridie " "		12
13	idibus N	3 " " "	idibus N	idibus Februarii		13
14	19 ante Kalendas N+1	pridie " "	18 ante Kalendas N+1	16 ante Kalendis Martii		14
15	18 " " "	idibus N	17 " " "	15 " " "		15
16	17 " " "	17 ante Kalendas N+1	16 " " "	14 " " "		16
17	16 " " "	16 " " "	15 " " "	13 " " "		17
18	15 " " "	15 " " "	14 " " "	12 " " "		18
19	14 " " "	14 " " "	13 " " "	11 " " "		19
20	13 " " "	13 " " "	12 " " "	10 " " "		20
21	12 " " "	12 " " "	11 " " "	9 " " "		21
22	11 " " "	11 " " "	10 " " "	8 " " "		22
23	10 " " "	10 " " "	9 " " "	7 " " "		23
24	9 " " "	9 " " "	8 " " "	6 ante Kal. Mart.	bissextum	24
25	8 " " "	8 " " "	7 " " "	5 " " "	6 ante Kal. Mart.	25
26	7 " " "	7 " " "	6 " " "	4 " " "	5 " " "	26
27	6 " " "	6 " " "	5 " " "	3 " " "	4 " " "	27
28	5 " " "	5 " " "	4 " " "	pridie " "	3 " " "	28
29	4 " " "	4 " " "	3 " " "		pridie " "	29
30	3 " " "	3 " " "	pridie " "			30
31						31

14. Days.

In all modern discussions about ancient time reckoning, the problem of determining the "epoch" in counting days involves the greatest difficulties. No complete agreement, e.g. , has yet been reached as to whether the Greeks began the day with the morning ("morning epoch") or with the evening ("evening epoch"), although apparently evening epoch is the right solution, at least for the classical periods in Athens.[108] Many of the

108) The morning epoch was assumed mainly by Bilfinger in his numerous writings on the subject (cf. the bibliography). An excellent summary of the present situation is given by Sontheimer in the article "Tageszeiten" RE 4 A, 2011-2033 (1932).

contradictory statements in ancient literature which are used in support of widely diverging opinions might be explained as resulting from the fact that the problem of epoch in the exact sense of the word does not play an important rôle outside of astronomy and special legal cases. The question whether the night should have the same date as the preceding or the following day could have been disregarded by most of the people. For special reason, the "day" could naturally have been considered as beginning at sunrise as is the case in Egypt.[109] On the other hand, special religious reasons might require a midnight epoch, as in Rome.[110] The evening epoch is closely

109) Sethe, Zeitr. p.130 ff. Cf. note 3.

110) Cf. RE 4 A, 2012.

connected with the lunar calendar, which begins the new month with the reappearance of the crescent at sunset after the new moon; if one counts the first day of a month from sunset according to this rule, then logically all following days must also begin in the evening. This is the case in the Babylonian calendar[111) as well as in the Jewish[112) and Mohammedan[113) calendar.

---

111) For details see the following section (p. ***). Morning epoch in Egypt and evening epoch in Babylonia contradicts the statement of classical writers (collected in Bilfinger [1] p.15/16), the oldest of whom seems to be Varro (first cent. B.C.). This shows how unreliable ancient reports about Egypt and Babylon can be; cf. p. 65 note 146a.

112) Ginzel II p. 2 f.

113) Ginzel I p. 256.

The coexistence of morning and evening epochs can have had an influence on chronological problems because it introduces an incertitude of one day in the correspondence of dates between calendars using different epochs. The same, is, of course, true in comparing dates according to midnight- and noon-epoch. Although there was hardly any civil calendar using noon epoch, this definition is convenient for astronomical purposes, not only because a midnight epoch involves the change of date during the time of observation but also by reasons which will be explained at the beginning of the next section. At the moment it is sufficient simply to accept as a fact that Ptolemy in the Almagest uses noon epoch for his calculations and that modern astronomical tables until 1924 December 31 followed the same principle. Beginning with 1925 January 1 the astronomical epoch coincides with the civil (midnight) epoch.[114) The procedure adopted in chronological works is

---

114) The exact definition is 1924 XII 31.5 = 1925 I 1.0 .

---

not uniform; dates in Schram's and P.V.Neugebauer's tables are to be understood in our familiar civil midnight epoch. Ginzel, however, based his tables on astronomical time in which the first half of the civil nth day i still called the n-1th. Ginzel, moreover, refers to Greenwich time, in which noon is 3 hours later than in Babylon, 2 hours later than in Alexandria, about 50 minutes later than in Rome etc.

This situation can be illustrated by an example like the beginning of the era Nabonnassar on -746 II 26 (cf. p.***). The same 48 hours around the beginning of the era are shown four times on Fig.3. The heavy dark parts represent night, the rest day time; midnight is indicated by ●, noon by ○. The first line gives day and night at Greenwich, the three following lines at Alexandria, Greenwich noon being two hours later than noon in Alexandria.

Before leaving the discussion of the days, we must mention an interesting concept developed by Babylonian astronomy of Seleucid times which brings the concept "day" into relation with the lunar calendar. As emphasized in the preceding section, the irregularity of the change between lunar months of 29 and 30 days led to the introduction of months of equal length. This holds, of course, also for astronomical calculations, in which real lunar months are very inconvenient. The Babylonian astronomers in calculating planetary positions therefore used not the real lunar calendar but lunar months of constant average lengths (i.e., abot 29 $\frac{1}{2}$ days). Each such average month was subdivided into 30 equal parts, which we might call "lunar days." Such a lunar day is obviously shorter than a real day, but the deviation between average dates obtained by this method and the real d dates as determined by the movement of the real moon amounts to so little that its use is fully justified in practice.[115] It is, however, interesti

115) This procedure was discovered by Pannekoek([1]) and independently by Van der Waerden ([1] p.28 ff.).

to see to what consequences the institution of a lunar calendar leads wher ver its extension into the future becomes necessary.

A parallel to these "lunar days" can be found in Hindu astronomy, the so-called "tithi", which might even be related to the Babylonian lunar days. One tithi also amounts to 1/30th of one lunar month, but, in contrast to the Babylonian concept, not to 1/30th of an average lunar month but to 1/30th of a <u>real</u> lunar month. Consequently, the length of the tithi varies in proportion to the length of the lunar month, the limits being about 21 ½ and 26 hours. The introduction of this unit is therefore not a simplification but a strong complication of the situation. This could be considered as an argument for its importation from the outside into India without a real understanding of the original purpose of its introduction in the Babylonian planetary theory. This would become even more possible by considering the fact that direct contact between Babylonian and Hindu astronomy is historically excluded, and, consequently the assumption of a Greek or Near - Eastern medium could easily account for the misinterpretation of the original problem.

15. Hours.

To the modern man a concept like "hour" seems perfectly simple: the 24th part of a day. It is therefore not surprising to find frequently remarks in modern historical literature considering the complication in the ancient definitions of hours as a sign of primitiveness in time reckoning. In order to understand these ancient definitions, it is necessary to appreciate the astronomical facts which will show how far from simplicity our concept "hour" actually is and how long a road of discoveries was necessary to reach a really convenient definition of this fundamental unit.

a. Astronomical concepts.

The "sky" is a sphere of arbitrary radius on which an observer projects the visible fixed stars. Because of the tremendous distance of the fixed stars from the earth, no change of appearance can be recognized if the observer changes his place on the earth. The sphere of reference of one observer can therefore be considered as identical with the sphere of reference of all other observers, or, in other words, one can assume that the center of this common "celestial sphere" lies in the center of the earth.

A place on the celestial sphere can be recognized by the constellation of fixed stars in this region. Within periods of interest for human history, the relative distances between the fixed stars can be considered as constant. "Distance" between points on the celestial sphere here (and always in the following) means, of course, angular distance, i.e., the magnitude of the angle (expressed in degrees and their fractions) subtended at the center of the earth by the straight lines connecting two points on the sky with the center.

Given a certain horizon, the observer has the impression of continuous rotation of the sky around a fixed axis of constant inclination towards his horizon. We call this movement engaged in by all fixed stars in one day once around a complete circle the "daily rotation" of the celestial sphere. The plane perpendicular to the axis of rotation is called the "celestial equator" (or shortly "equator"). The inclination of the given horizon towards the equator is called the "geographical latitude" of the observer, usually denoted by φ. The line of intersection between the plane of the horizon and the equator is the line from "east" to "west;" perpendicular to this line in the horizon is the line from "north" to "south." The great circle with its diameter north-south and in a plane perpendicular to the horizon is called the "meridian" of the place of latitude φ. The meridian obviously passes not only the zenith of the observer but also the north and south pole of the axis of rotation of the celestial sphere (cf. fig.4).

We now disregard the uniform rotation of the sky from east to west and locate night after night the place of the moon or of one of the five planets with respect to the fixed stars. We shall find that these bodies move more or less regularly on the sky, the moon continuously from west towards east, the planets generally in the same direction, yet sometimes moving for a short time again westwards ("retrograde"). A movement of the sun with respect to the fixed stars cannot be directly observed because the stars are not visible during daylight. If one, however, notices evening after evening the configurations near the western horizon visible shortly after sunset one will recognize that a configuration still high above the horizon today is after a few days much lower at sunset and finally disappears completely, obviously setting together with the sun. This shows that also the sun has a movement with respect to the fixed stars from west to east. This movement proceeds continuously eastwards and marks an orbit in a great circle on the celestial sphere called the "ecliptic". The time necessary for the sun to travel once around this circle is the "year."

The plane of the ecliptic is inclined towards the equator by an angle, called ε , of about 24 degrees in antiquity, today about ½ degree less. Suppose the sun at a given moment exactly at the point of intersection between equator and ecliptic. The daily rotation moves this point on a great circle (the equator) which is exactly half above and half below the horizon, consequently making day and night of equal length. These points are therefore called the "equinoxes." Because the sun travels the 360 degrees of the ecliptic during one year, the daily movement almost exactly amounts to one degree. Three months after equinox, the sun will therefore be at a point 90 degrees distant from the equinoxes. This point has the maximum distance of ε degrees from the equator; it is moved by the daily rotation in a circle parallel to the equator ~~and~~ which is therefore divided into unequal parts by the horizon, the difference being as great as possible during the year. These positions of the sun and the diametrically opposite position are called the solstices (cf. fig.5). The time intervals which the sun must travel in orde to come from one of these four characteristic points of the ecliptic to the next, 90 degrees more advanced, are called the "seasons."

It is one of the most far reaching discoveries of ancient astronomy - whether Babylonian or Greek[116] - that the seasons are of unequal

116) The fully developed consequences of this discovery are visible in the Babylonian theory of the moon, which covers at least the two last centuries B.C. The earliest appearance in Greece seems to be with Enktemon and Meton (ca. 450 A.D.). Cf. Kugler BMR p.83 ff. and Böckh, Sonnenkreise p.46 f.

length although the arcs travelled by the sun are alqays 90 degrees. In other words, the discovery of the inequality of the seasons is equivalent to the discovery of the inequality of the sun's movement.[117] Such an irregularity in the movement of the sun obviously influences the definition of "day" and all its parts because the time from noon to noon or from midnight to midnight will be of different length in different parts of the year.

117) It does not matter whether this inequality was interpreted as real or apparent. The latter explanation was given by Hipparchus (Almagest III, 4) in assuming uniform but excentric movement of the sun, which finally led to the discovery of the elliptic orbits by Kepler.

There is, however, another effect which makes the time between subsequent meridian passages of the sun of unequal length even if we disregard the small differences in the sun's movement. We already know that the sun travels about one degree per day in the ecliptic. Let us suppose that the sun at noon of a given day stood exactly at one of the two points of intersection between ecliptic and equator, say in the "vernal point" where the movement of the sun in the ecliptic passes from the southern hemisphere to the northern (cf. fig. 6a). Suppose that S_1S_2 is the arc of about one degree which the sun travels during one day. At noon of the following day, the sun will be at S_2', corresponding to a rotation of the equator of $360^o + S_2S_2'$. If we, secondly, consider the analogous case at summer solstice (fig. 6b), the daily movement $S_1^*S_2^*$ is now on the highest point of the ecliptic practically to the equator which hence must rotate by $360 + S_2^*S_1^*$ degrees in order to bring the sun from one meridian passage to the next. But $S_2^*S_1^*$ is the full amount of the sun's daily movement, while S_2S_2' is only a fraction of it. The "true solar day", i.e., the time from noon to noon, is therefore longer at the solstices than at the equinoxes. In other words, the movement of the sun, even disregarding its inconstancy, does not yield "days" and hours of equal length during the year.

The "hours" which we use today are therefore based on the following definition: we introduce a "mean sun" which ~~coincides with the real sun at the vernal equinox and~~ completes one revolution with constant angular velocity in the time which the real sun takes in making one revolution in the ecliptic: moreover, this fictitious body travels not in the ecliptic but in the equator. The time between two consecutive passages of the meridian of the mean sun is called "mean solar day", and its 24th part is "one hour."

Astronomical tables frequently count hours from 0 to 24 and indicate smaller parts not as minutes, seconds etc. but by decimal fractions: thus 22.5 means 10^h30^{min} at night.

b. The 24 hours.

After the preceding discussion of the discrepancies between "hours" of equal length and the actual movement of the sun, it will not be surprising to find a time reckoning in ancient civil life which is different from our present system. Moreover, it is not only the necessity of astronomical knowledge which stands in the way of the introduction of hours of constant length but the additional practical difficulty of constructing instruments which are reliable enough to show equal time intervals. All sun-dials require astronomical theory of their construction and correct adjustment if one wishes to obtain more than a rough estimate of time; water-clocks involve great inaccuracies by physical reasons and their gradation again requires astronomy. The use of exactly defined hours is therefore necessarily restricted to astronomical purposes in all periods before the invention of time-recorders like the pendulum-controlled clock.

The oldest type of time unit smaller than a day undoubtedly consists in simple fractions of the night, like the four vigiliae of the Romans[118)] and the φυλακαί of the Greeks;[119)] both day and night are divided into three maṣṣartu in Babylonia[120)] and in four sa in Egypt[121)]

---

118) RE 4 A, 2021, 52 ff.

119) It is generally accepted that the φυλακή is the fourth part of the night. This is based on a statement of Suidas (ca. 1000 A.D.) s.v. προφυλακή and φυλακή (ed. Adler IV p.244 and p.772) and is supported by Hephaistion (fourth cent. A.D.) who divides Apotel.I,21 the night into four τρίωροι, presupposing the division into twelve hours; the division into four parts is also assumed by Pollux (ca. 200) Onom. I,70 and Euripides, Rhesos, 5 (concerning Pollux see the remarks in Macan, Herodotus I,2 p.702/703). There is, however, a scholion to Euripides Rhesos 5 (ed. Dindorf p.19 f.) telling us that "the ancients (quoting Homer) counted only three watches but that Stesichoros (about 500 B.C.) and Simonides (before Stesichoros) assumed five parts of the night (πεντεφύλακόν φησιν ὑποτίθεσθαι τὴν νύκτα). The question deserves further study.

120) Delitzsch BK 2, 284 ff.; ZA 18 and in the first tablet of the series "mul-apin" (ca. 700 B.C.) II,43 and III,9 (cf. p. note 106).

121) Sethe AZ 54 p.3 note 5 and Sethe Zeitr. p.127.

(all these expressions mean exactly the same as English "watch").

Egypt further developed the division of day and night by the creation of real "hours", namely, twelfths of day and night, respectively. The origin of this institution is unknown but undoubtedly belongs to an evidence[128] is an interesting passage in Geminus' "Introduction to astronomy", written in the first century B.C.,[129] in which he quotes Pytheas of

128) Sethe Zeitr. p.113 quotes a doubtful fragment of Aristotle. Certainly genuine is the passage in Aristotle, Ἀθηναίων Πολιτεία 30,6 but indecisive because nothing is said about the numbers of hours but only that the members of the council should appear in time (ὥρα ἡ προρρηθεῖσα).

129) VI,9 ed. Manitius p.70/71. Cf. also Hennig, TI I p.120 ff., IV p. 406 ff.

Massilia (time of Alexander), who travelled to northern regions where the shortest day lasted only two or three hours. This expression is usually considered to be evidence for the use of hours of constant lengths because in seasonal hours every day would have twelve hours, no matter how long it would be.[130] This argument, however, is not conclusive because one cannot

130) E.g. Kubitschek GAZ p.179.

assume that Pytheas used in the far north clocks properly calibrated for these latitudes. His expression can just as well be understood as the statement that the shortest day corresponded to about two or three hours which he and his readers would have expected at this time of the year. Outside of strictly astronomical use exists no evidence of another hour as the seasonal hours (ὧραι καιρικαί[131]).

131) Litterarely "timely" or "appropriate" hours.

Only twice a year, at the equinoxes are the seasonal hours of the day of the same length as the hours at night. Greek astronomers therefore called hours of equal length ὧραι ἰσημεριναί i.e. "equatorial hours" The probelms connected with this concept are discussed by Ptolemy in the early period of Egyptian history. Hours are mentioned in the Pyramid texts,[122] and the division of the night into twelve parts is the basis of the so-called "diagonal calendars" on coffin lids of the XIIth dynasty.[123] The procession of the twelve deities of the day and of the night is represented in the funeral temple of queen Hatshepsut[124] and frequently thereafter[125].

122) Sethe, Zeitr. p.110.

123) For these star lists cf. chapter IV below (p.###).

124) Naville, Deir el Bahari IV, 114, 116.

125) Cf. e.g. Sethe, Zeitr. p.111 and Brugsch, Thes.I, p.185 ff.

These Egyptian hours, however, afe hours of unequal length, called "seasonal" hours because they are the twelfth part of the actual lengths of days or nights which vary during the seasons. This follows, for example, from the divisions of the scales in Egyptian water-clocks;[126] each

126) Borchardt [1]; cf. also Pogo [1].

month has a scale of its own, divided into twelve parts by small holes drilled into the stone. The variability of these seasonal hours, however, is not very great in Egypt because even in the northern-most parts of the country the longest day is only 2/5 longer than the shortest.

The same type of seasonal hour is found in Greece, but only in comparatively late periods. The word ὥρα (whence Latin "hora" and our "hour") originally means any definite period of time. The division of the day into twelve parts is first mentiones in Greek writings by Herodotus (5th cent. B.C.), although with special reference to Babylon.[127] The next

127) Herodotus II,109. This passage has been declared to be an interpolation, most recently by Powell ([1] p.69); but this view is based on unconvincing and even false arguments (e.g., that there is no other evidence for the twelve hours in Greece before Plutarch). Cf. also Kubitschek, GAZ p.178 note 1.

evidence[128] is an interesting passage in Geminus' "Introduction to astronomy", written in the first century B.C.,[129] in which he quotes Pytheas of

128) Sethe Zeitr. p.113 quotes a doubtful fragment of Aristotle. Certainly genuine is the passage in Aristotle, Ἀθηναίων Πολιτεία 30,6 but indecisive because nothing is said about the numbers of hours but only that the members of the council should appear in time (ὥρα ἡ προρρηθεῖσα).

129) VI,9 ed. Manitius p.70/71. Cf. also Hennig, TI I p.120 ff., IV p. 406 ff.

Massilia (time of Alexander), who travelled to northern regions where the shortest day lasted only two or three hours. This expression is usually considered to be evidence for the use of hours of constant lengths because in seasonal hours every day would have twelve hours, no matter how long it would be.[130] This argument, however, is not conclusive because one cannot

130) E.g. Kubitschek GAZ p.179.

assume that Pytheas used in the far north clocks properly calibrated for these latitudes. His expression can just as well be understood as the statement that the shortest day corresponded to about two or three hours which he and his readers would have expected at this time of the year. Outside of strictly astronomical use exists no evidence of another hour as the seasonal hours (ὧραι καιρικαί [131]).

131) Litterarely "timely" or "appropriate" hours.

Only twice a year, at the equinoxes are the seasonal hours of the day of the same length as the hours at night. Greek astronomers therefore called hours of equal length ὧραι ἰσημεριναί i.e. "equatorial hours". The probelms connected with this concept are discussed by Ptolemy in the

Almagest[132] exactly along the same lines as given in the introduction to this section.[133] We have already mentioned that noon was chosen as epoch for these astronomically defined solar days.[134]

132) Book III chapter 9.

133) Cf. above p.***. The details belong to a history of ancient astronomy and do not interfere with chronological problems.

134) Cf. above p.***.

c. Babylonian "hours".

The history of the 24-hour system is not exhausted by the statement that the division of day and night into twelfths originated in Egypt and was transferred to the Greeks around the fourth or fifth century B.C. The last-mentioned type of hours, the astronomical hours of constant length used by the Hellenistic astronomers, is undoubtedly more closely related to Babylonian astronomy than to Egyptian tradition. The foundation of our present system of time reckoning is formed only through the combination of both influences.

We have already mentioned[135] the division of day and night into

135) Above p.***.

three watches each. This partition of the whole day into sixths was combined with another form of expressing time which obviously originated in measuring marching distances not only by lengths (say miles) but also by time, as we do in saying that a place is "only one hour distant." This very natural type of expression creates a parallelism between distance and time measuring. If we assume that the average distance travelled during a day (or, better, during one night) was supposed to amount to about 60 km (36 miles)[136], the distance corresponding to one "watch" would be about 2 bēru, expressed in Babylonian units.[137] Assuming this equivalence between watches

136) Albright [1] p.25 gives the estimate for caravan travel of 50 km (30 miles) per day. Also in Hittite texts occur measures for time intervals (cf. Otten [1]).

137) The Sumerian name of this unit is danna, read KAS-BU in antiquated literature. The reading danna was found by Thureau-Dangin [1] p.223.

and bēru, the length of a complete day would be 12 bēru. This relation

$$1^d = 12^b \tag{25}$$

is indeed the relation on which the Babylonian time reckoning with "bēru" rests.

Because of (25) the "bēru" is frequently called "double hour" in the older literature.[138] This is correct in so far as one bēru amounts to two of our hours; the name is, however, misleading in so far as (a) the word bēru does not contain the element "double" and (b) the bēru is originally not a measure of time like "hour", but of distance. The emphasis on

138) E.g. Bilfinger [5] or Ginzel I p.122.

this latter point is very essential because it has far reaching consequences. As we have seen all ancient time measures are on inconstant length, depending on the variability of the seasons. By introducing according to (25) units which are originally measures of distances, a <u>uniform</u> measure of time has been created long before any astronomical theory existed.

Some additional consequences must be mentioned which will underline the historical importance of using units of non-astronomical descendency. The unit bēru has well defined relations to smaller units of lengths whose basic unit is the GAR (about 5 m or 15 feet). Sixty GAR make up one uš (literally simply "length", and 30 uš are one bēru. We have therefore according to (25)

$$(26) \qquad 1^{d} = 12^{b} = 360^{\text{uš}} \qquad \cancel{\text{uš}}\ 1 \neq 60\ \text{GAR} = 1^{\text{uš}} \quad 30^{\text{uš}} = 1^{b}$$

or in other words: in consequence of expressing time by lengths, the day has been subdivided into 360 parts, or "degrees", each of which is again divided into 60 "minutes". Because one day corresponds to one complete revolution of the equator, the relation (26) has also been applied to measure the equatorial circle by 360 degrees and their sexagesimal parts, and consequently all circles on the celestial sphere. This parallelism is clearly expressed in the first passage in Greek literature where the "degrees" appear, namely in Hypsikles' "Anaphorikos".[139] Therein "degrees in space" are distinguished from "degrees in time",[140] corresponding to the later "parts" and "times" e.g. in Ptolemy.[141] The method used in modern, medieval

139) "Ascension" (of the ecliptic above the horizon); written ca. 200 B.C.

140) Greek μοῖρα τοπική and μοῖρα χρονική. The passage is translated in Heath, Hist. II p.214; the text is given Manitius, Hypsikles, p.XXVI.

141) The τμήματα, e.g., in Almagest I,10 (Heiberg I p.31), the χρόνοι in II,7 (Heiberg I p.130), opposed to μοῖραι.

and Greek astronomy to express time by arcs of the equator measured in degrees is nothing but the use of bēru and uš by Babylonian astronomers.

A remark of general historical character may be appropriate here. It is well known how deeply Babylonian mathematical astronomy of the last centuries B.C. influences Greek astronomy and herewith the later development of Hindu, Arabic and European theories of the movement of the celestial bodies; but it is rarely recognized what conditions made the origin of a mathematical astronomy in Babylonia possible. One is the development of sufficiently advanced mathematical methods which require on their part the existence of convenient methods for numerical calculations. Here lies the basic importance of the place value notation constantly used in the mathematical texts of the Old-Babylonian period and in the astronomical texts of the latest period. The second element is the existence of methods of measuring time and angles by units which do not depend on astronomical concepts. This is the essential point in the introduction of the "degrees" as discussed before. Number system and time reckoning, previously developed and independent of astronomy, removed obstacles which elsewhere kept mathematics and astronomy on a much lower level than in Babylonia. It is therefore not surprising that these Babylonian methods were adopted wherever astronomy was further developed. The 24 hours of constant length, the "equinoctial hours" of the Greek astronomers, and the decimal place value notation created in Hindu astronomy and transferred to Europe by the astronomers and mathematicians of Islam are only variations of the Babylonian system.

We can now return once more to the question of "epoch" of the day in Babylonia. Moments of the day are frequently expressed with reference to sunrise or sunset, both in bēru and in uš ("degrees"). Examples of these expressions have been collected by Thureau-Dangin beginning with the Persian period.[142] The same kind of terminology is still used in the mathematical astronomical tablets of the Seleucid period but with the interesting restriction that the calculations themselves consistently use midnight epoch and translate the result into "civil" epoch, i.e. either evening epoch,[143] or sunset and sunrise.[144] This can be illustrated by the foll

142) Thureau-Dangin [2] p.124.

143) This is the method followed by the older system (system II in Kugler terminology, now called A).

144) So in the more advanced theory (Kugler's "system I" = B).

ing example taken from a tablet which gives the new moons for the years 208 to 210 Sel.era (written 209 IX(b) 18 = −102 XII(j) 22).[145] The first

145) The text is discussed by Kugler SSB 9 ff. and Schaumberger Erg. p. 375 ff.

line of the reverse gives the calculation of the new moon which separates the VIIth and VIIIth month of the year 209. The moment of the calculated conjunction is given[146] as VII 28 355°; the following column gives the

146) Col. XI line 1: the numbers are here destroyed but can be restored with absolute certainty from preceding or following numbers. The number 355 is in the text of course written in sexagesimal notation as 5,55 (or rather 5,55,42,50 but we here disregard the fractions of degrees).

same moment as "VII 29 95° after sunset." This equivalence is to be explained by the following consideration (cf. fig.7). From a preceding column, it is already known that the duration of the day at this time of the year is 160°; the corresponding night therefore contains 200°, and 100° elapse from sunset to midnight. Counting 355° in astronomical epoch ing example taken from a tablet which gives the new moons for the years 208 to 210 Sel.era (written 209 IX(b) 18 = −102 XII(j) 22).[145] The first

145) The text is discussed by Kugler SSB 9 ff. and Schaumberger Erg. p. 375

line of the reverse gives the calculation of the new moon which separates the VIIth and VIIIth month of the year 209. The moment of the calculated conjunction is given[146] as VII 28 355°; the following column gives the

146) Col. XI line 1: the numbers are here destroyed but can be restored with absolute certainty from preceding or following numbers. The number 355 is in the text of course written in sexagesimal notation as 5,55 (or rather 5,55,42,50 but we here disregard the fractions of degrees).

same moment as "VII 29 95° after sunset." This equivalence is to be explained by the following consideration (cf. fig.7). From a preceding column, it is already known that the duration of the day at this time of the year is 160°; the corresponding night therefore contains 200°, and 100° elapse from sunset to midnight. Counting 355° in astronomical epoch from midnight is therefore the same as 5° before midnight, and hence the same as 100° − 5° = 95° after sunset. Moreover, the 28th in astronomical epoch begins at midnight and ends at midnight following the sunset when the 29th of the civil calendar begins. This example shows the coexistence of astronomical midnight epoch and civil evening epoch for the dates. The same text, however, also gives the time with reference to sunrise and sunset ("before" and "after"), obviously because this type of expression was used in practice. This shows clearly that an expression "2 Uš after sunrise" doe not stand in contradiction to the evening epoch of the dates, as is necessary in a moon calendar.[146a)] It is very likely the same coincidence of apparent

146 a) As confirmation of an alleged morning epoch in Babylonia (Thureau-Dangin J.As. 10 sér.14 (1909) p.341 note 4), the statement of Varro, mentioned above p. 51 note 111, has been cited (Ed. Cuq RA 7 (1910) p.89/90 note 3). This Babylonian morning epoch has found its way, of course, also

same moment as "VII 29 95° after sunset." This equivalence is to be explained by the following consideration (cf. fig.7). From a preceding column, it is already known that the duration of the day at this time of the year is 160°; the corresponding night therefore contains 200°, and 100° elapse from sunset to midnight. Counting 355° in astronomical epoch from midnight is therefore the same as 5° before midnight, and hence the same as 100° − 5° = 95° after sunset. Moreover, the 28th in astronomical epoch begins at midnight and ends at midnight following the sunset when the 29th of the civil calendar begins. This example shows the coexistence of astronomical midnight epoch and civil evening epoch for the dates. The same text, however, also gives the time with reference to sunrise and sunset ("before" and "after"), obviously because this type of expression was used in practice. This shows clearly that an expression "2 uš after sunrise" does not stand in contradiction to the evening epoch of the dates, as is necessary in a moon calendar. It is very likely the same coincidence of apparent-[146a] ly contradictory expressions which caused difficulties in determining the epoch of the day in Greek calendars.

146 a) As confirmation of an alleged morning epoch in Babylonia (Thureau-Dangin J.As. 10 sér.14 (1909) p.341 note 4), the statement of Varro, mentioned above p. 51 note 111, has been cited (Ed. Cuq RA 7 (1910) p.89/90 note 3). This Babylonian morning epoch has found its way, of course, also into modern handbooks, e.g., Jeremias HAOG(1) p.166 note 5 and HAOG(2) p.280 note 3 or Sethe, Zeitr. p.121 (with wrong quotations in note 3). The question started with the discussion of a Sumerian expression (ba-zal) used in connection with time intervals and believed to mean sun-rise. This etymology, however, has been proved to be wrong (S.Langdon RA 28 (1931) p.14 ff.); moreover, the supposed numerical basis is non-existent because all problems concerning time-intervals are necessarily independent of the problem of epoch as long as both ends of the interval are given according to the same epoch. Actually, there is no trace of a morning epoch in Babylonia at all.

The preceding example also shows clearly why the astronomical epoch is different from the civil epoch: the use of sunset as the point of departure in counting time requires the knowledge of the variable length of the days or nights and thereby makes the calculations dependent on season and geographical latitude.[147] The use of the meridian instead of the horizon is therefore necessary in order to avoid unnecessary difficulties in calculation. This is a typical example showing that many "natural" and "primitive" concepts are actually much more complicated than the notations introduced by systematic science. A large proportion of all scientific progress consists in nothing but the analysis and removal of concepts considered as "obvious".

147) This second argument is explicitly mentioned by Ptolemy (Almagest III,9, Heiberg I p.261) but hardly played a role in Babylonian astronomy.

In contrast to Egypt and Greece, no Babylonian instruments which could tell us something about the actual technique of time measuring are preserved. We know only from textual references that sundials and water-clocks existed. The second method results in another expression of time measurment, namely, by weight of water (mena, Greek mina). The discussion of the rather complicated details of this problem goes beyond the scope of this book. It might be mentioned, however, that the outflow of the water was not considered as proportional to the time and that this experience explains the apparently embarassing fact that the longest day contains twice as many "mana" as the shortest.

This concludes our short and far from complete description of th involved history of the origin of the fundamental units of time reckoning. The complexity of ancient time reckoning should always be kept in mind by the historian when dealing with ancient dates. The more precise an ancient record describes a certain moment the more care must be applied as to how

§ 5. Concluding remarks. Bibliography.

At the end of this chapter, which has attempted to give a short survey of the characteristic concepts of ancient calendars, some general remarks might be appropriate. The complexity of the history of the calendar might make us forget the sole purpose of every calendar: to determine dates in an unmistakable and simple way. The unnecessary entanglement of this problem with numerous problems of absolutely different character, like religious, social or political doctrines, soon created difficulties - hopeless difficulties but highly typical for the so-called development of human culture. It seems to me that the study of the history of the calendar has its special attraction for the historian because it is one of the very rare cases where human reactions can be studied while the facts themselves are indisputable.

The historian has another reason to be grateful to the generations who invented all the complications of the historical calendars; to these complications we owe documents of the highest interest, from the Palermo stone and limmu lists down to the medieval easter chronicles, undoubtedly "products of ignorant assiduity"[148] but still of inestimable value for the reconstruction of the past.

148) Schwartz in RE 6, 1384.

16. Bibliography to chapter I.

Only such works are mentioned in the following which deal with the calendar and chronology in general. For all special questions, references have already been given in the notes to the preceding text. The same principle will be followed in the bibliographies to the subsequent chapters

The history of the calendar attracts remarkable much public attention, mainly in connection with proposed "improvements." There exists therefore a rich literature of a dilettantic character, repeating and expanding long antiquated errors and statements.[149] In contrast to this

149) A good example of this kind of literature is the book of E.W.Wilson, The romance of the calendar, New York, W.W. Norton, 1937.

flourishing production is the rarity of scientific works on chronology. This might be the result of the existence of the large work of Ginzel which considers the calendaric systems of all periods and all nations (3 vols., 1906 to 1914). This work will be the standard work for a long time to come although it is already now incomplete as far as oriental history is concerned. In addition to Ginzel, there is the preceding analogous work, the chronology of Ideler (2 vols., 1825/26), still very useful especially because of the full discussion of details incorporated in abbreviated form in Ginzel.

As introductory works might be mentioned Philip, "The Calendar" (1921, mainly medieval and modern) and the article "Calendar" in Hastings Encyclopaedia of Religions (1911),[150] in the eleventh edition of the Encyclopedia Brittanica and in the Nautical Almanac 1938 by Fotheringham. Also the article 'Chronology' in the fourteenth edition of the Enc. Britt. should be mentioned. The

150) This large article (vol.III p.61-141) is written by different authors The chapter on the Babylonian calendar (by Hommel) was written under the influence of the "panbabylonistic" doctrine and is therefore very misleading in its general statements.

great German handbooks contain special articles on chronology of the Greco-Roman period: by Unger (1886) in the first edition of the "Handbuch der Altertumswissenschaft", by Kubitschek (1928) in the new edition, which is the best reference work, and by Bickermann (1933 in "Gercke-Norden"; insignificant). The chapters in Gardthausen, Palaeogr. and Wilcken, Ostraca are especially designed for use in Hellenistic studies; Hohmann, Chron. (1911) is now out of date. Very important for medieval calendariography is Van Vijk, Le nombre d'or (1936). For the Jewish calendar see the extensive studies of J. Morgenstern [1], [2], [3].

As to tables for chronological computations, Schram is the standard work, equally useful is P.V.Neugebauer's HTChr.[151] Moreover, most of the articles in the handbooks contain various tables and comparative lists; this is especially true of Ginzel. Special tables like Skeat [1] and Meyer (Ernst) [1] for Ptolemaic Egypt, Chaîne ChrEE for Coptic and Ethiopic texts, Wüstenfeld-Mahler for Arabic chronology etc. have been mentiones in the preceding text. Tables for the Athenian calendar are given in Dinsmoor, Archons p.424-440. For the Greek calendar cf. also Beloch GG IV,2.

The length of the seasonal hours in Alexandria, Athens and Rome are tabulated in Kubitschek GAZ p.182 f. The lengths of the watches at Babylon are computed by P.V.Neugebauer KM p.3 according to three different assumptions as to the definition of "night" (a) from sunset to sunrise, (b) from dusk to dawn and (c) as the period of complete darkness. As far as can be seen from the Babylonian astronomical tablets of the Seleucid period, definition (a) is by far the most plausible.

151) Cf. above p. **.

17. History of astronomy.

Although we are here not concerned with the history of ancient astronomy, a few bibliographical remarks about this field might be useful for a reader who wants to obtain information about the historical background of many concepts which we need in our discussions.

The best source for information about Greek astronomy is Heath "Aristarchus" (1913), which contains abundant bibliographical references. The history of the planetary theories is treated by Tannery in his "Recherches sur l'histoire de l'astronomie ancienne" (1893) and by Dreyer, "History of the planetary systems from Thales to Kepler" (1906). Very elaborate is Delambre's "Histoire de l'astronomie ancienne" (1817), now antiquated in many details, but still unreplaced for special studies. Bibliographies for later periods can be found in Wolf "Geschichte d.Astron." and especially in the second volume of his "Handbuch." For Babylonian astronomy Kugler's researches published in his "Mondrechnung" and "Sternkunde" are basic, but difficult to read because of the extensive discussion of special problems. Jeremias' "Handbuch" was written under the influence of the "panbabylonistic" doctrine and is very unreliable. It cannot be used without checking all statements with the original sources. There is no modern survey of Egyptian astronomy. The book of Antoniadi "L'astronomie égyptienne" (1934) is not only dilettantic but disregards all Egyptological results since Champollion. Also Zinner's "Geschichte der Sternkunde" (1931) relies only on second-hand information and gives a very distorted picture of the historical facts.

An introduction to ancient and medieval astrological concepts is given in Boll-Bezold, Sternglaube u.Sterndeutung (1931), which contains rich references to older literature.[152]

---

152) Cf. also the Bibliography of Gundel, Bursians Jahresber. 1934 (covering the period 1907 to 1933).

Chapter II. The Moon.

Quod si adeo quis deses vel hebes est, ut absque omni labore computandi lunae cursum scire voluerit ......

Beda venerabilis, De temporum ratione XXIII: De aetate lunae si quis computare non potest *) ~~[illegible]~~

---

*) If anyone is so lazy and dull as to wish to know the behavior of the moon without the effort of calculation ......

Beda, De temp.rat. chapter XXIII: On the age of the moon if one does not know how to compute. (Migne PL 90, col. 398.)

---

§ 1. Lunar months. Astronomical concepts.

18. The movement of the sun.

In order to describe the sun's movement as seen from the earth, we again consider the celestial sphere as the sphere of reference on which the path of the sun is projected. We assume the observer located at the center of the earth; necessary corrections as to the actual geographic location at a given place on the earth do not involve essential difficulties. We again consider the two fundamental planes of the equator (defined by the daily rotation of the fixed stars) and the ecliptic (defined by the orbit of the sun during the year with respect to the fixed stars). The point of intersection between ecliptic and equator where the sun passes from the southern to the northern hemisphere is called the vernal point (Υ in fig.8). The angular distance of the sun from the vernal point, counted from 0° to 360° in the direction of the sun's movement with respect to the fixed stars, is called its "longitude" (usually denoted by λ).

Let us now ~~xxxxxxx~~ erect a diameter perpendicular to the plane of the ecliptic meeting the surface of the sphere at the points P and P', the "poles of the ecliptic" (fig.9). Let S be an arbitrary point on the celestial sphere (different from P and P'). The location of S can be uniquely determined by the following procedure. We draw a great circle through P , S , and P' which intersects the ecliptic at S' . The position of S can then be characterized by the two arcs $\Upsilon S' = \lambda$ and $S'S = \beta$, called "longitude" and "latitude" of S.[153] If S lies on the same hemisphere as P , the latitude is called positive; if S lies on the ecliptic (and hence S' = S), β is said to be zero; otherwise negative. The latitude of the sun is therefore always zero.

153) We omit here and in the following the additional word "geocentric" when it is self-evident that we consider only geocentric coordinates.

Hipparchus (around 150 B.C.) in comparing such "ecliptic coordinates" of several bright stars measured by himself with the coordinates obtained by older observers, discovered that the longitudes had increased, although the latitudes had remained unchanged. He concluded from this fact that the point from which longitudes were counted, i.e. the vernal point, had moved in a retrograde direction because this would have exactly the observed consequence. Ptolemy 300 years later repeated ~~xxxxxxxxxxxxxxxx xxxxxxxx~~ and confirmed this discovery, which is known as the "precession of the equinoxes." This phenomenon can also be described in the following way. The north pole is perpendicular to the plane of the equator; the precession of the equinoxes means that the plane of the equator moves slowly backwards with respect to the ecliptic, keeping, however, the inclination between these two planes constant.[154] Hence the north pole describes a circle around the pole of the ecliptic[155] (fig.10); if the vernal point

154) Actually small but periodic changes in this inclination (which we, however, disregard) do exist.

155) The pole of the ecliptic is not marked by any bright star. Its position is near the head of the dragon, about halfway between the stars δ and ζ draconis.

moves from Υ_1, to Υ_2 by an angle α , then the north pole moves by the same angle around the axis EP from N_1 to N_2 . In other words, the precession of the equinoxes consists in a slow movement of the axis of the daily rotation EN on a conic surface with its top at E , the axis bein EP , and an angular distance PEN = ε where ε is the inclination betwee equator and ecliptic.

The precessional movement of the north pole takes about 26,000 years for one complete revolution. This is the same as saying the pole moves about 1° in 72 years, or about 50" in one year. In order to give these numbers a more concrete content, it might be said that the diameter of the full moon's disc measures almost exactly ½ degree. If therefore a certain fixed star is at a given moment exactly the "polaris", this star will already 36 years later rotate on a little circle of a radius equal to the moon's apparent diameter. The star " α ursae minoris", today call polaris, actually moves on a circle of about 1° radius. At the beginning of our era this star had an angular distance of 12 degrees from the north pole, at 1900 B.C. of 22°.[156]

156) The position of more than 500 stars for each century from -4000 to +1900 are given in Neugebauer (P.V.) TAChr.I pl.III (p.21-82; the alpha-

betic list of the star names is given on pp.83-85). If one substracts the values indicated in this tables for the declination from 90°, one obtains the angular distance of the star from the north pole of the given time.

The discovery of the precession of the equinoxes is frequently praised as one of the greatest achievements of ancient astronomy and especially of Hipparchus. Actually this discovery played a very modest rôle in ancient astronomy, consisting in nothing more than a small correction to be applied on longitudes of fixed stars measured at sufficiently distant moments. The discovery required nothing more than the existence of older records accurate enough to make the change in longitude and the invariability of the latitude evident. Ancient astronomy contains numerous discoveries and theories which deserve our highest admiration and which by far overshadow the recording of the continuously increasing effect of the precession. The reason why modern authors emphasize this point so much constitutes an anachronism due to the subsequent importance of the precession within the framework of Newton's theory. Newton found that no precession should exist if the earth were a perfect sphere but that the combined attraction of sun and moon on a rotating ellipsoid must result in exactly the precessional movement of its axis which we observe.[157] Thus measurements of the shape

157) About two thirds of the precession is due to the influence of the moon, the rest to the sun and (to very small degree) the planets. Because the members of our planetary system do not rotate in exactly the same plan periodic variations of the precessional movement occur which can, however, be disregarded for our purposes.

of the earth could furnish a direct proof of the general law of gravitatic governing the movements in the planetary system. This explain why precessi became one of the most famous astronomical facts; for ancient astronomy it is only a phenomenon of very restricted interest and for chronological pr blems it plays a still smaller rôle.

Another method of determining a point on the celestial sphere consists in using the celestial equator and the meridians through the nort and south pole to form a system of references in exactly the same way as illustrated in fig.9; these coordinates are called rectascension and declination. Ancient astronomers, however, exclusively used the system of ecliptic-coordinates (longitude λ and latitude β) described above. Yet there exists a difference in notation of measuring longitudes which is sometime of great importance for chronological investigations. Ancient astronomy does not count longitudes from 0° to 360° but considers the eclitic subdivided into twelve equal parts of 30° each, called the "zodiacal sihns", and indicates longitudes by degrees from these signs. The usual notation of these signs is

♈	Aries	♌	Leo	♐	Sagittarius
♉	Taurus	♍	Virgo	♑	Caper
♊	Gemini	♎	Libra	♒	Amphora
♋	Cancer	♏	Scorpius	♓	Pisces .

The usual assumption is that Aries corresponds to the longitudes from λ to $\lambda = 30^\circ$ (hence "♈" as the symbol for the vernal equinox $\lambda = 0$), Taurus to the longitudes from $\lambda = 30^\circ$ to $\lambda = 60^\circ$ etc. Two restriction however, must be made. The first consists in considering precession. The zodiacal signs being configurations of fixed stars but the longitudes bei measured from the vernal point which moves slowly in retrograde direction the section from $\lambda = 0$ to $\lambda = 30$ will be slowly separated from any grpu of stars representing the group "Aries." One must therefore distinguish between the "signs" and the "pictures." The signs are only different names for sections of lengths of 30 degrees beginning with the real vernal point; the pictures, on the contrary, are the configurations on the sky, consisting of certain fixed stars. In the second century B.C. (the time of Hipparchus), signs and pictures coincided; today the ~~XXXXX~~ "sign" Aries almost completely covers the constellation of the fishes; in 2300 B.C. the vernal point lay in the [illegible] ~~beginning of~~ Taurus. Modern tables gives longitudes usually with respect to the true vernal point of the time in question; in discussions of ancient astronomical documents, one must investigate the problem whether a position, say $\Upsilon\ 10^{\circ}$, means 10 degrees distance from the actual vernal point or 10 degrees distance from the beginning of the configuration Υ. In most cases this difficulty can be overcome by calculating according to both possibilities; if one knows the century of a document, say a papyrus, by historical or epigraphical and linguistic considerations, then the influence of the precession is known and an error of one hundred years corresponds to only one degree more or less, which is frequently anyhow the margin of error one must admit in ancient observations or calculations.

There is, however, another difficulty which is much more serious but frequently ignored. This is the fact that many ancient astronomers considered not $\Upsilon\ 0^{\circ}$ as vernal point but $\Upsilon\ 8^{\circ}$, $\Upsilon\ 10^{\circ}$ and other values between $\Upsilon\ 0^{\circ}$ and $\Upsilon\ 15^{\circ}$. The history of these notations is not known in detail; it is only clear that assuming the vernal point at Υ 10 and Υ 8 corresponds to Babylonian tradition and was taught in astronomical and astrological writings for about one thousand years.[158] Hipparchus and

158) Aries 10° is the vernal point of the older Babylonian theory, $\Upsilon\ 8^{\circ}$ of the younger one, both represented in texts of the three last centuries B.C. The most recent evidence known to me is the Syriac letter of Bishop George written in 714 A.D. (Ryssel [1] p.45).

Ptolemy considers $\Upsilon\ 0^{\circ}$ as the vernal point, Egyptian texts of Roman times seem to use some fixed star of about -4° or -5° longitude as the starting point etc.[159] One must therefore not only take into account the effect of

159) Neugebauer (O.) [1] p.231.

precession but also the existence of arbitrary definitions of the distance between the vernal point and the beginning of Aries.

It is obvious that an uncertainty of about 10 degrees in placing the vernal point affects chronological problems to a much higher degree than the slow continuous change by precession. There is no way of eliminating this factor except through intimate knowledge of the doctrine to which the document in question, e.g., a horoscope, belongs - a knowledge which we in many cases do not possess. This is a very typical example of a serious source of error caused by applying naively modern calculation to ancient texts regardless of possible very great differences between modern and ancient definitions.

Before we turn to the discussion of the movement of the moon and its combination with the sun, we must mention the accurate value of the ~~XX~~ length of the period between two consecutive passages of the vernal point by the sun, the socalled "tropical year." This value, which we shall denote in the following by y, is

$$y = 365.2422^{d} \quad . \tag{27}$$

The name "tropical year" indicates the relation of this interval to the seasons, in contrast to the "sidereal year" which is defined by the return of the sun to the same fixed star. Bacause of the precession of the equinoxes, the tropical year is slightly shorter[160] than the sidereal year; for all historical problems, only the tropical year is of importance.

160) By about 20 minutes.

19. The movement of the moon.

In describing the orbit of the moon as seen from the earth projected on the celestial sphere, we again use the system of spherical coordinates with respect to the ecliptic, longitude λ and latitude β . The orbit of the moon is only slightly inclined to the ecliptic (about 5°) and it is therefore possible to disregard the deviation for our preliminary discussions: in dealing with eclipses the moon's latitude is the decisive factor.

The movement of the moon, like the sun's is opposite to the daily rotation. If sun and moon have the same longitude, the two bodies are said to be in "conjunction;" if the longitudes of the two bodies are 180° apart, they are said to be in "opposition." Both conjunction and opposition are called "syzygies".[161] New moon and full moon are the phases of the moon at the syzygies.[162]

161) From Greek ζυγόν "yoke."

162) The symbol for opposition is ☍ , for conjunction ☌ .

Let us consider a certain conjunction, where sun and moon both have the longitude λ_1 (fig.11). During one month the sun's longitude increases by α while the moon performs one complete revolution plus additional α degrees This gives us an estimate of the moon's daily movement; α is obviously about $\frac{1}{12} \cdot 360^\circ = 30^\circ$ and therefore the moon's movement during one mont amounts to about 390°, hence about 13° per day. Because the sun moves about 1° daily, the moon gains about 12° "elongation" over the sun.

These values are only rough estimates; the actual movement of the moon varies from about $11^{\circ/d}$ to $15^{\circ/d}$ and consequently also the time between two conjunctions varies. If one, however, counts the number of days elapsed during a sufficiently large number of months, one obtains average values which will be the basis for our further calculations. The time thus obtained between two conjunctions is called a "synodic month" or one "lunation" and amounts to

$$(28) \qquad m = 29.5306^{d}$$

i.e. slightly more than 29 ½ days. This shows that in lunar calendars the number of months of 30 days must be higher than the number of months of 29 days.

The problem of real lunar calendars, however, involves more than the regular distributions of months of either 29 or 30 days such that the right average length is granted. The actual moment of conjunction is generally difficult to determine because of the invisibility of the moon, except in the case of a solar eclipse which occurs only if the moon has not only the same longitude as the sun but is also exactly (or almost exactly) in the ecliptic. Lunar calendars therefore begin the months not with the astronomical new moon but with the new visible crescent, the so-called "new light." The following will show how this "natural" definition of the beginning of a lunar month introduces further complications.

The main new element introduced by using the new crescent as the starting point of a new month is the horizon. At the moment of conjunction and also shortly thereafter, sun and moon are so near to one another that they set almost simultaneously and thus make a moonless night. The next evening, however, the moon will be about 12° more eastwards from the sun than the evening before and therefore still some degrees above the horizon when the sun is already below the western horizon. It may be that the distance between sun and moon is still not sufficient to allow the fine crescent to be visible before setting in the dusk shortly after the sun; the following evening, however, the crescent is most likely to be visible because the elongation from the sun will be sufficiently great to allow the moon to be seen above the horizon when the sun is already so deep below the horizon that there is full darkness. The visibility of the crescent therefore depends on the various factors which determine the distances of sun and moon from the western horizon on the evenings following conjunction; already this preliminary consideration shows that small variations in the relative positions of the two bodies can result in advance or delay of the moment of first visibility by 24 hours. The reason why real lunar calendars depend upon actual observation is that only a highly developed astronomy is able to follow the movement of sun and moon not only with respect to each other but also with respect to the horizon so accurately as to make a prediction of the visibility of the crescent possible. This will become clear if we now turn to a more detailed discuss of the conditions in question.

Every problem which involves the horizon involves the geographical latitude φ . The setting of sun and moon moreover requires the consideration of the direction of this movement at the western horizon, which is practically the same as the inclination of the equator and parallel circles towards the western horizon. As fig.12 shows, this inclination is $90 - \varphi$; thus sun and moon set much nearer to a vertical direction at southern latitudes like Assum ($\varphi = 24^{\circ}$), Memphis ($\varphi = 30^{\circ}$) or Babylon ($\varphi = 32^{\circ}$) than at Rome ($\varphi = 42^{\circ}$). It is obvious that a given elongation of the moon from the sun will be more sufficient for the visibility of the crescent the nearer the direction of setting is to 90°.

We now consider the geographical latitude as given (the following figures assume $\varphi = 30^{\circ}$) and investigate the influence of the seasons. Let us suppose that we are dealing with a conjunction falling close to the vernal equinox. The sun therefore stands in the equator. Let us assume for the sake of simplicity that the moon moves exactly in the ecliptic; on the evening following conjunction, the moon will therefore be north of the equator. On the contrary, the moon is south of the equator in the analogous situa-ion at the autumn equinox. This shows (fig.13) that the same elongation, i.e., the same "age" of the moon, will bring the moon much higher above the horizon at the vernal equinox than at the autumn equinox; and the higher the moon stands above the horizon the better is the chance of its visibility. An intermediate case exists at the solstices when the direction of the ecliptic is tangential to a parallel circle to the equator; hence the direction of the daily rotation coincides with the direction in which the moon moves away from the sun.

The influence of the variable inclination of the ecliptic with respect to the horizon is to some extent counterbalanced or enlarged by the fact that the moon does not move in the ecliptic but deviates as much as five degrees on both sides. As is evident from fig.14, the influence of the moon's latitude has least effect at the vernal ~~xxxx~~ equinox and greatest at the autumn equinox when the ecliptic inclines as much as possible towards the horizon.

B~~a~~ecause the variation of the latitude is independent of the seasons[163], the resulting influence on the visibility of the crescent is very

163) This movement will be discussed below p.[illegible].

complicated. This makes it very difficult to detect a rule in the variation of the length of lunar months on purely empirical grounds. On the other hand, the problem of predicting the evening of the first visibility of the new moon is one of the most important challenges to develop a theoretical astronomy. It is therefore well to understand that the calculation of the new moon is the central part of Babylonian astronomy[164], from which all

164) It might be explicitly~~s~~ stated that the influence of astrology on the develeopment of theoretical astronomy is practically negligible.

further development has branches. Here again an apparently simple and "natural" notation, namely the counting of the months from crescent to crescent, includes difficulties which can only be removed by a scientific analysis of the underlying phenomena.

20. Cycles.

The first step in every theory of movement of the moon consists in the establishment of "cycles", i.e., numbers of months after which the moon is again in the same (or nearly the same) position with respect to some other periodic phenomena, say the seasons. A typical problem of this kind is the Easter calculation; one tries to find cacles according to which the dates of Easter are repeated in the same order. Also the problem of intercalation in luni-solar calendar leads to the same question. One tries to find a definite rule for the arrangement of years of 12 or 13 lunar months and to determine the minimum number of years necessary in order to repeat exactly the same sequence of ordinary years and leap-years. The following is an explanation of the common basis of all such cacles; it consists in a comparison of the average length os year and month. For the length of the year we take the value

$$y = 365.2422^{d} \tag{29a}$$

mentioned above p.### as the length of the tropical year. For the length of one lunation, we shall use

$$m = 29.5306^{d} \quad . \tag{29b}$$

The actual length of time between two consecutive conjunctions can deviate considerably from this value; if one counts, however, the number of days contained in a sufficiently large number of months (say for 50 years), then the average length of one month will be very close to (29b).

In comparing the two numbers y and m it is evident that we need only consider the difference between y and $12\ m$, which (because of (29)) amounts to

$$(30). \qquad y - 12\ m = 10.8750^d \quad .$$

We now ask how many times this difference must be repeated to give as nearly as possible a complete month. Let us for a moment assume that $y - 12\ m$ is only 10^d and $m = 30$; then we would have $\frac{y - 12m}{m} = \frac{10}{30} = \frac{1}{3}$, or three years would exactly equal 37 months; in this case, an intercalation cycle would consist of two ordinary years and one leap-year. The same kind of argument can also be applied to the more accurate numbers (29) by making increasingly better approximations.

We begin with the statement that the number

$$(31a) \qquad \alpha = \frac{y}{m} - 12 = \frac{108750}{295306} < \frac{1}{2} \quad .$$

This is obviously a very rough evaluation of the proportion $\alpha = \frac{y - 12m}{m}$ $295306 - 2 \cdot 108750 = 77806$ or

$$\alpha = \frac{108750}{295306} = \cfrac{1}{2 + \cfrac{77806}{108750}}$$

If we now replace the fraction $\frac{77806}{108750}$ simply by 1 we obtain

$$(31b) \qquad \alpha > \frac{1}{2 + 1} = \frac{1}{3} \quad .$$

The error we committed in replacing $\frac{77806}{108750}$ ~~simply~~ by 1 is easily determined because

$$\frac{77806}{108750} = \cfrac{1}{1 + \cfrac{30944}{77806}}$$

We therefore get a more accurate result if we write

$$(31c) \qquad \alpha = \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{30944}{77806}}} < \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{2}}} = \frac{3}{8}$$

The replacement of $\frac{30944}{77806}$ by $\frac{1}{2}$ can be improved because

$$\frac{30944}{77806} = \cfrac{1}{2 + \cfrac{15918}{30944}}$$

Hence

$$(31d) \qquad \alpha = \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{15918}{30944}}}} > \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + 1}}} = \frac{4}{11}$$

The replacement of $\frac{15918}{30944}$ by 1 can be improved because

$$\frac{15918}{30944} = \cfrac{1}{1 + \cfrac{15026}{15918}}$$

from which follows

$$(31e) \qquad \alpha = \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{15026}{15918}}}}} < \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + 1}}}} = \frac{7}{19}$$

The replacement of $\frac{15026}{15918}$ by 1 obviously involves only a relatively minute error. We therefore do not continue this process[165] but turn to

165) Called the development of $\frac{y - 12m}{m}$ into a "continuous fraction".

the interpretation of the results obtained. We found in (31a):

$$\alpha = \frac{y}{m} - 12 < \frac{1}{2} \quad \text{or} \quad \frac{y}{m} < 12 + \frac{1}{2} \quad \text{or} \quad 2y < (12 + 1)m$$

and correspondingly in (31b) to (31e) :

$$(32) \qquad \begin{array}{llll} \frac{y}{m} > 12 + \frac{1}{3} & \text{or} & 3y > (3 \cdot 12 + 1)m & = 37\ m \\ \frac{y}{m} < 12 + \frac{3}{8} & & 8y < (8 \cdot 12 + 3)m & = 99\ m \\ \frac{y}{m} > 12 + \frac{4}{11} & & 11y > (11 \cdot 12 + 4)m & = 136\ m \\ \frac{y}{m} < 12 + \frac{7}{19} & & 19y < (19 \cdot 12 + 7)m & = 235\ m \end{array}$$

These inequalities can be considered as approximations of the quotient y/m with values given in (29), and it follows from our procedure that these approximations approach the true value the farther down they were obtained in the process of calculation. If we replace the inequality signs by the equal sign, then each of these equations can be considered as the basic relation for a luni-solar cycle; $3y = 37m$ is the very inaccurate relation we mentioned in the beginning. The three following relations, however,

$$(33) \qquad \begin{array}{ll} 8y = (8 \cdot 12 + 3)m & = 99\ m \\ 11y = (11 \cdot 12 + 4)m & = 136\ m \\ 19y = (19 \cdot 12 + 7)m & = 235\ m \end{array}$$

are cycles which have been used in calendaric practice. Because of their origin from successive approximations, each of these cycles is more accurat than the preceding one. The same fact is expressed by saying that each of these cycles requires a shorter period of observation than the following one. This is historically quite important to emphasize because it shows that such cycles can be established and even improved during the lifetime of one man. All that we actually know from ancient source material speaks for a very rapid development of the basic astronomical ideas, which were the work of a few men and than kept unchanged for a long time. The frequently told story of age-old observational material at the disposal of ancient astronomers is by no means proved for the beginnings; and even in classical periods nothing like modern cooperation and planned work over longer periods existed.

§ 2. Lunar Calendars.

21. Egypt.

Breasted said about the Egyptians:[166] "They, like all other peoples, had suffered from the vexatious fact that the lunar month is not an even divisor of the year." It seems to me that exactly the opposite is true; the Egyptians never made an attempt to create anything like a luni - solar calendar but kept their lunar calendar entirely independent from the civil calendar. Having no astronomical theory whatsoever, they were spared from all troubles; the only relationship between civil year and lunar calendar consists in calling years containing 13 new moon festivals "great years", the others "small years."[167] Which year was great and which small was undoubtedly for very long times decided merely by the events; we shall, however, describe a simple method, found in a Demotic papyrus from Roman times, by which one could decide in advance about the character of a year.

166) Ancient Records I p.25.

167) This is the harmless meaning of an inscription found in Beni Hasan (12th dynasty; published Urk.VII p.29,18 = Newberry Beni Hasan I pl.25,90 f which gave origin to many speculations about all kinds of Egyptian year forms (e.g. Ginzel I p.176 f.) Cf. below p.###.

We must, however, first give a short account about the lunar calendar itself.

The oldest evidence of a lunar calendar in Egypt is found in a papyrus from Kahun (XIIth dynasty) which shows that the schedule of the temple duties was arranged according to a lunar calendar of alternating months of 29 and 30 days. We shall discuss this papyrus in chapter IV because of its importance for the chronology of the twelfth Dynasty.[168a] ~~What we must mention here is the very fortunate accident that the special group of lunar months which this papyrus covers contains an intercalated month of 31 days (between two months of 29 days) at the end of the civil year.[168] This shows that~~

168a) cf. below p.

168) Meyer (Ed.) Aeg.Chron. p.52.

~~that the observation had been made that the assumption of 29 ½ days as the average length of the lunar months results in falling too short with respect to the facts. The most essential point, however, is~~ Important for us, at the moment, is only, the evidence that this lunar calendar is the calendar of the temple service. This purely religious character of the lunar calendar in Egypt is supported by all further evidence. The theological interest in the lunar days, the phases of the moon etc. are amply testified[169] and we know that the days of the lunar month had special names.[170] There even exist double dates characterising a day both in the civil and in the lunar calendar.[171] This shows clearly that the religious

169) Cf. e.g. Brugsch, Thes. I, although mostly from late (Ptolemaic-Roman) periods.

170) Brugsch, Thes.I p.45 ff.

171) These instances are collected by Borchardt, MZ p.39 ff. The influence of the lunar calendar on practical life has been overemphasized by Borchardt (e.g. his theory of coronations only on a full-moon day MZ p.68 has been disproved by Černy ÄZ 72; cf. also Edgerton [1]).

lunar calendar existed independently beside the schematic civil calendar – exactly as Easter depends on a lunar calendar regardless of the merely formal character of the civil calendar. This underlines, on the other hand, our previous statement that the Egyptian civil calendar with its months of invariably 30 days cannot be explained as the degenerated product of a lunar calendar: it was not created in order to replace the lunar calendar but had the purpose of serving for administrative purposes while the natural lunar calendar was never abandoned for religious purposes.[172)]

172) The importance of the lunar calendar in all periods of Egyptian history was discovered and emphasized by Brugsch (e.g. in his Aeg. p.330 ff.). Only the preconceived assumption that this lunar calendar was some kind of predecessor to the civil calendar led to the disregard of its existence (e.g. Sethe, Zeitr. p.301).

We can now turn to the Demotic papyrus, mentioned above, which gives a rule to calculate lunar dates in a simple cyclical way.[173)] The

173) Papyrus Carlsberg 9, published by Neugebauer - Volten [1].

basis is months alternating 29 and 30 days in length, where every 5th yea one more day is inserted in the last month of the year – a procedure wh we already met in the Kahun papyrus. Five such pentades are linked togeth to one 25-year cycle, after which the dates in the civil calendar are repeated again. The details of this method, as given by the papyrus, can b illustrated by the section containing the years from 10 to 15:

		II	IV	VI	VIII	X	XII
year	10	24	23	22	21	20	19
	11	13	12	11	10	9	8
	12	2	1	30	29	28	27
	13	21	20	19	18	17	16
	14	10	9	8	7	6	5
	15	30	29	28	27	26	25
	16	19	18	17	16	15	14

The year numbers are the number in the cycle, the months are the 30 days' month of the civil calendar. Only every second month is mentioned, indicating that the date in the omitted month is undetermined. The dates given decrease by one, as is necessary if a lunar month contains 29 ½ days. The decrease from XII to II amounts in general to 6 days because of the 5 epagomenal days which follow month XII. At the end of the pentade, however, the jump from year 14 XII 5 to year 15 II 30 amounts to 5 only because of the day inserted every 5th year. By extending this scheme over 25 years one will remark that the dates repeat themselves. In order to count the number of lunar month contained in such a cycle, we must remark that usually two lunar month are contained in the interval between two given dates but that <u>three</u> months should be counted in all instances where the numbers begin again with 30, as, e.g., in the year 12: three months elapse from IV 1 to VI 30 , not merely two, as from VI 30 to VIII 29 , etc. Three months are also contained between 14 XII 5 and 15 II 30, and in the same way one finds that 9 years among 25, namely the years

1 3 6 9 12 14 17 20 23

contain 13 lunar dates. These years are called "great years"; the name of the remaining 16 years is not preserved in the papyrus but both "great" and "small" years are mentioned in Banihasan for offerings – which gives us the obvious name for the years containing only 12 lunar festivals. Hence, the total number of months in a 25 year's cycle is $9 \cdot 13 + 16 \cdot 12 = 309$. This lunar cycle is therefore based on the relation

$$(34) \qquad 309 \text{ lunar months} = 25 \text{ Egyptian years} = 9125 \text{ days} .$$

Using the value (29b) $m = 29.5306$ for the length of one lunation we obtain for $309\ m$ the value 9124.95 days, which shows that this cycle of 25 Egyptian years represents a very good basis for the prediction of lunar festivals in the Egyptian civil calendar.[173a)]

22. The Greek moon calendars.

The Greek calendars confront us with so many problems that their treatment has developed into a field of its own, much more related to archeological and historical questions than to astronomical methods. The complication of the Greek calendar systems is a consequence of the splitting of the Greek nation into many independent city-states with local calendars, local eponymic lists and subject to frequent changes according to the eventful history of these small states. Here we see the same situation in the full light of history which we must suppose responsible for the disorder i[n] the Sumerian calendar. There exists, however, another aggravating element in the Greek, or at least in the Athenian, calendar, namely the attempt t[o] regulate the lunar calendar by means of cyclical intercalation. Every suc[h] attempt is necessarely doomed to failure if based only on such primitive astronomical knowledge as existed before the creation of a mathematical astronomy in Hellenistic times. It is therefore not surprising that even such questions as whether νουμηνία ("new moon") means actual conjunctio[n] or the first visibility of the crescent are still open for discussion. W[e]

173a) By using the same process as above in No. 20 we can show that the following cycles exist for Egyptian years (y'): $3y' = 37\ m$; $11y' = 136\ m$; $25\ y' = 309\ m$.

find for example in Thucydides[174] the remark that a solar eclipse[175] occured on νουμηνίᾳ κατὰ σελήνην. Dinsmoor[176] finds that this expression

174) II,28.

175) It is the annular eclipse of -430 VIII 3 in the first year of the Peloponesian war.

176) Archons p.314.

must characterize the first of the month in order to obtain agreement with other elements of the Athenian calendar; Meritt[177], on the contrary, thinks

177) Calendar p.104.

that the unusual apposition κατὰ σελήνην ("according to the moon") intends to emphasize the astronomical new moon in contrast to the new moon of the civil calendar. This view seems to be supported by the fact that Plutarch gives the 30th as the date of ~~an sun~~ solar eclipse[178] and by the use of the same expression in a papyrus where the meaning "conjunction" is certain.[179] (But This is further ~~attested~~ confirmed by datings like "1st of Daisios (=Maked. VIII) κατὰ σελήνην 30th" which sets the civil calendar into contrast to the real lunar calendar[179a]).

178) Plutarchus, Romulus 12 (the 30th = τριακάς). The solar eclipse in connection with the horoscope of Romulus is fictitious.

179) ~~P. Mag. Par. I~~ Wessely [1] p. 64 f. 787, p. 104 f. 2389 (ἡ κατὰ θεὸν νουμηνία; that κατὰ θεὸν means the same as κατὰ σελήνην is explicitly proved e.g. by Kleomedes II,4 ed. Ziegler p.190, 21 f.).

179a) Dura, Rep. IV p. 95 No. 220; p. 107 No. 232 (= p. 115 f. No. 236).

here again local influences might give a different meaning to the same expression.

The attempt to regulate the lunar calendar according to cycles goes back at least to Eudoxos, the great mathematician of Plato's academy (ca. 375 B.C.), or even to Cleostratos (ca. 525 B.C.).[180] We find all the

180) This according to Censorinus in his "De die natali" (chap. 18; written 238 A.D.) Nilsson [1] and PTR p. 364 assumes a Delphian origin of the eight-years cycle, necessitated by the cult and the periodically repeated games.

three lunar cycles in use which we found by comparing the length of year and lunar month, namely the 8-years', the 11-years' and 19-years' cycle.[181] An exhaustive description of their history has been given by Dinsmoor.[182]

181) Cf. above p.###.

182) Archons p.297 ff.

Here it may be sufficient to show that the 8-years' cacle, the so-called octoeteris, can easily be derived from much more approximative values than used in our modern treatment of the luni-solar cycles. Let us assume simply $365\frac{1}{4}$ days as the length of the year and $29\frac{1}{2}$ days as the length of the month. A year is then $11\frac{1}{4}$ days longer than 12 months. If we now start from a New Year's date 1, the following New Year's Day becames $12\frac{1}{4}$, the next $23\frac{1}{2}$ and the third $34\frac{3}{4} \equiv 4\frac{3}{4}$ mod.30. Proceeding in the same manner we obtain the dates

date:	1	$12\frac{1}{4}$	$23\frac{1}{2}$	$4\frac{3}{4}$	16	$27\frac{1}{4}$	$8\frac{1}{2}$	$19\frac{3}{4}$	1
year:	1	2	3*	4	5	6*	7	8*	

This shows that after 8 years the dates will be repeated in the same order if we intercalate three times a third month, separated from each other twic by two ordinary years and once by only one year. This is exactly the rule of the octoeteris.[183] The possibility of deriving this cycle so directly

183) Cf. Geminus VIII, 28 ff. (ed. Manitius p.110 ff.).

from the round numbers $365\frac{1}{4}$ and $29\frac{1}{2}$ may be a reason for the large extent of its use even at times when the much better 19-years' cycle was known.[184] The 19-years' cycle itself is usually called the "Metonic cycle" because of its introduction into the Athenian calendar by the astronomer Methon in 432 B.C.[185]

184) Cf. Dinsmoor, Archons p.360 ff. (the assumption of Egyptian influence, however, made by Dinsmoor, is unfounded).

185) Cf. Dinsmoor, Archons p.309 ff.

23. Babylonia.

The history of the 19-years' cycle is not yet fully known, but it seems very probable that it originated in the Babylonian lunar calendar and not in Greece. The priority of the Babylonian 19-years' cycle, however, is not yet clearly established. What we know is its exclusive use in Hellenistic times according to the rule[186] that all years which are mod.19

186) Discovered by Kugler SSB I p.212 (1907) from his investigation of astronomical tablets; cf. also SSB II p.425.

congruent to

(35) Sel.era 1 4 7 10 12 15 18

are leap years intercalating a second 12-th month, except the years $\equiv 10$ mod.19 which intercalate a second 6th month.[186a] The same cycle can be shown in uninterupted use at least since -382, but the earlier method of intercalation is not yet clear. Kugler assumed an 8-years' cycle since -530, followed by a 27-years' cycle from -505 onwards, perhaps soon replaced by the 19-years' cycle. Parker[187], from the investigation of all available

186a) Which year in such a cycle is called "first year" is, of course, absolutly arbitrary and of no importance.

187) Parker [1] p.292 note 22.

material, assumes the use of the 19-years' cycle since -545 but occasionaly disturbed by empirical corrections. This would bring us in direct contact with the reign of Nabonidus (from -554 to -537) from which we have evidence of explicitly ordered intercalations.[188] On the other hand, already the series "mul apin" contains intercalation rules[189] which might belong to a period around 700 B.C.

188) Meissner BA II p.397.

189) Weidner [1] p.187. Lists of attested intercalations are given in Sidersky [1]. Weinbach [4] and V.

It is definitely clear that no intercalation rule existed during the First Dynasty of Babylon (1900 to 1600 B.C.). The Intercalations during the 21 years of King Ammizaduga[190] illustrate this fact. If we indicate

190) Langdon - F. - S. p.61 ff.

leapyears with XII_2 by *, with VI_2 by ** we have

1 2 3 4* 5** 6 7 8 9 10** 11**

12 13 14* 15 16 17 18 19** 20** 21 .

No system of cyclical intercalation, devised to balance a solar year and a lunar calendar, could lead to a sequence of four ordinary years; such a sequence brings the lunar calendar $1\frac{1}{2}$ months behind the astronomical seasons.

This is a very essential point for the proper understanding of many problems of Babylonian chronology. Without discussion, the majority of the scholars working in this field made the assumption that the Babylonian calendar attempts to establish a definite relation between its lunar months and the equinoxes. This is, of course, the meaning of every cyclical intercalation; if we deal, however, with periods when only arbitrary intercalations are in use, one must first ask what purpose was served by commanding the intercalation of one month. Series of intercalations like the above-given group covering 21 years show clearly that no astronomical facts could have been decisive because the most primitive observation of equinoxes or fixed star appearances gives much better results. The only possible conclusion, therefore, is that it was not the astronomical seasons which regulated the oldest form of the lunar calendar but the agricultural seasons. This conclusion is fully confirmed by all the available evidence. We know, for instance, from the time of the Third Dynasty of Ur (about -2100) that local calendars of nearby cities used different intercalations[191] but agree only in the fact that the last month is called the harvest month.[192] Obviously, different places held conflicting opinions as to the expected time of harvest or as to the necessity of enlarging the period of harvest by a second harvest month.[193] There is, on the other hand, no evidence for any astronomical interest in this calendar, except, of course, that the months are real lunar months. In other words, the Old-Babylonian calendar is a real lunar calendar with no "luni-solar" intercalation at all but strongly influenced by agricultural considerations which held the harvest months as much as possible on the real harvest. The replacement of the variable climatic seasons by astronomically defined seasons was the work of a much later period.

191) Schneider ZWU p.108 ff.

192) Schneider ZW p.108.

193) This explanation has been given already by Landsberger KK p.6 and Olmstead [1] p.115. The astronomical consequences have been emphasized by the present writer ([4] 406 f.).

The Babylonian calendar was made uniform by the abandonment of the different local calendars in favor of the calendar of the city of Nippur. This happened as a result of Hammurapi's rule over all of Babylonia.

Before 1100 B.C., the Assyrian calendar differred from the Babylonian; after this date, the Babylonian calendar was adopted also in Assyria.[194] Double dates prove that the Assyrian calendar was based on real lunar months,[195] but, so far as one can conclude from the material now available,[196] the intercalation of a thirteenth month never occurs.[196]

---

194) Ehelolf - Landsberger [1] p.217 note 2 and Schott [1] p.317.

195) Example: VAT 9557 colophon gives "month XI(ass.) which corresponds IX(bab) day 19th year ..." (KAH 2 p.44 No.73 and Ehelolf - Landsberger [1] p.217).

196) Landsberger KK p.88-91; Ehelolf - Landsberger [1]; Weidner [2] and [3].

Because twelve lunar months add up to about 11 days less than one solar year, only 33 solar years are necessary to carry each of the twelve calendar months through all four seasons. Indeed, there is evidence for the identification of the same Assyrian month with seven different Babylonian months.[197] This, combined with the lack of any intercalation in the Assy-

---

197) Namely I, II, III, VII, IX, X or XII according to Weidner [3] p.29. All these synchronisms between the Assyrian and Babylonian calendar belong to the reign of Tiglathpelesar I, i.e., to the period around 1100 B.C.

---

rian calendar, clearly indicates that the Assyrian year was strictly lunar, consisting of twelve lunar months. Consequently, year numbers in Old-Assyrian king-lists must be reduced by about 3 years for each 100 Assyrian years in order to obtain the proper number of Julian years.

niform by the abandonment of
calendar of the city of
i's rule over all of Babylonia.
endar differred from the Ba-
endar was adopted also in
yrian calendar was based on
n conclude from the material
rteenth month never occurs.196)

The fact that not only the 12th but also the 6th month has been doubled in years which needed an intercalatory month reflects a duplicity of the "New Years Day" of the Babylonian calendar: there exists not only a New Years festival in the spring (Nisan) but also one in the autumn (Tishri). This duplicity existed ~~since~~ already in Sumerian times[193a] and is still reflected in the Jewish and in the Seleucid calendar.[193b]

193a) Thureau-Dangin, Rit. Accad. p. 86 f
193b) Ginzel II p. 22 ff. and I p. 136.

---

nd Schott [1] p.317.
h XI(ass.) which corresponds
and Eheloff - Landsberger [1]

' - Landsberger [1]; Weidner

---

1 days less than one solar
rry each of the twelve calen-
there is evidence for the
h seven different Babylonian
ny intercalation in the Assy-

---

ccording to Weidner [3] p.29.
nd Babylonian calendar belong
e period around 1100 B.C.

---

syrian year was strictly lunar,
ly, year numbers in Old-Assy-
ears for each 100 Assyrian
f Julian years.

24. Medieval lunar calculations.

The Mohammedan calendar is a pure lunar calendar with no reference to the seasons (except the number 12 of its months). Here the "years" of this calendar fall back 11 days in each solar year; this results in rapidly increasing differences between the era of the Hejirah (beginning in 622 A.D.) and the Christian era.[198] But even in such a strict lunar

198) Tables for the direct reduction of dates of the Mohammedan era into the Christian era and vice versa are computed by Wüstenfeld and Mahler beginning with 622 A.D. VII 16 = 1 Hejirah I 1 and ending with 2077 A.D. X 19 = 1500 Hejirah XII 1. This reduction can also be made, of course, b using Schram.

calendar, a cyclical calculation of the type of the Egyptian method discus ed above (p.###) was used. The length of the months is assumed to be 29 an 30 alternatingly, and 11 leap-days are added during a cycle of 30 lunar years.[199] This cycle of 360 months is also quite accurate because a leng of $360 \cdot 29.5306 = 10631.016$ days would result from the average length (29

199) Cf. Ginzel I p.254 f.

of one lunation while the cycle contains $30 \cdot 354 + 11 = 10631$ days.

While the Mohammedan part of the world adhered to this newly created simple lunar calendar, the Christian nations kept alive both sol and lunar elements inherited from the Hellenistic, Roman and Jewish cale dars. We have already discussed the relation of the Christian era to the era of Diocletian (and thus to the Egyptian calendar).[200] The arrangem

200) Above p.###.

of the months follows the Roman order, which completely disregarded the lunar calendar. The problems of the lunar calendar, however, were again introduced by the adoption of the paschal festival of the Jewish calendar - and herewith parts of the Babylonian calendar, i.e., in the final analysis, the calendar of Nippur. It is well known what violent controversies arose in the Christian churches over the rules of ~~XX~~ determining the Easter date. The astronomical knowledge of the leading persons in these fights was not sufficient to make use of the achievements of Hellenistic astronomy, which could easily have solved the problems in question. Cyclical calculation appeared the only way out, and it was again the 19-years' cycle which was finally accepted after the Easter tables of Cyril and Dionysius were ~~x~~ based on this scheme,[201] supposedly accepted by the fathers of the Nicene Council through inspiration from the Holy Spirit.[202] The tables of Dionysius were continued in 616 by Felix Gillitanus[203] and finally by the table of Bede for 532 years;[204] the number 532 is the product of 19 and the sola cycle 28 in order to repeat the days of the week.[205] These tables of Bede

201) Cf. above p.***.

202) Dionysius says (Migne P.L. 67 col.19) that the members of the counci "hanc regulam .... non tam peritia saeculari quam sancti Spiritus illustratione sanxerunt" (the reading saeculari according to Ideler II, 286).

203) Concerning him, see Poole [1] p.36 f.

204) Migne P.L. 90 col.859-878.

205) Cf. above p. 18 and Krusch [1] p.115.

constituted the definite victory of the method of calculating Easter by ~~x~~ means of the 19 years' cycle - ironically enough, sufficiently late to make the inaccuracy of this period yield one day too much.[206] Bede, how

206) It must be remembered that according to (32) only 235 m 19 y hol

ever, found the explanation of this discrepancy in the assumption that Adam began the counting of time on the day when he was expelled from Para dise, March 18th.[207]

207) De temp.rat. chapter 43 (Migne P.L. 90, col.481).

We have already seen the importance of the 19 years' lunar cycle for medieval chronology during our discussion of the basis for Scaliger's "Julian period" of 7980 years.[208] In this number 7980 = 15·28·19 , the

208) Cf. above p.???.

factor 19 is introduced to bring the full moon again in the same relation to the vernal equinox as at the beginning of the cycle (the order of a year within the cycle was called the "golden number" cf. above p.); ~~But we have also mentioned that this concept was introduced as late as 1200 A.D.~~ We shall now briefly explain the corresponding concept of the "epact" of a year. This word is derived from the same Greek root (ἐπάγειν) as the expression "epagomenal (days)" in the Egyptian calendar and is well chosen because the epact is an expression for the difference between the lunar and the solar year. Its calculation is based on the following simple process: starting from 0 one adds continuously 11 (i.e. the difference between 365 and 354 days) but reduces the results modulo 30. By 19 steps one obtains the following numbers "e" which we write down beside the numbers "g" which run from 1 to 19:

(36)																			
e:	0	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18
g:	1	2	3*	4	5	6*	7	8	9*	10	11*	12	13	14*	15	16	17*	18	19*

The stars in the second series indicate where a reduction mod.30 has been made in the first line except in the last place, where the next number would be only 29. Assuming however, the correctness of the 19-years' cycle, the difference between the solar and lunar year should disappear after 19 years if we started with 0

One must therefore add not only 11 days but an additional day[209] in the

209) This additional day is essentially what the medieval computers call the "saltus lunae" (the jump of the moon).

last place and then obtain correctly $18 + 12 \equiv 0$ mod.30. The years where a reduction mod.30 was made are obviously those years which contain 13 lunar months;[210] if we replace g by $g + 1$, we get (mod.19) as the numbers for leap years 4 7 10 12 15 18 1, which are the numbers (35)[211] characterising the Seleucid intercalation rule. The primitive

210) Cf. e.g. Beda, Migne PL 90 col.507 f.

211) Cf. p.▪▪▪.

method by which we obtained the numbers e therefore leads to the same intercalation rule which is usually called the Methonic cycle.[212]

212) It must not be forgotten that we used the 19-years' cycle to derive (36) by stopping after 19 steps regardless of the fact that we should proceed with 29 10 21 etc. The method employed is therefore insufficient to discover the 19-years' cycle in contrast to the case of the 8-years' cycle, which is a direct consequence of this type of calculation, as shown on p.▪▪.

The two lines of numbers in (36) show that the position of a yea in the 19-years' cycle can be equally well determined by its number g, the golden number, or by its number e, its epact. The transformation of g into e is simply performed according to the following rule:

$$\begin{array}{lllll} & \text{if } g \equiv 0 \text{ mod.}3 & \text{then} & e \equiv g + 19 & \text{mod.}30 \\ (36a) & \text{if } g \equiv 1 \text{ mod.}3 & \text{then} & e \equiv g - 1 & \text{mod.}30 \\ & \text{if } g \equiv 2 \text{ mod.}3 & \text{then} & e \equiv g + 9 & \text{mod.}30 \end{array}$$

Although the complete equivalence of golden number and epact has been clear ever since the introduction of the golden number,[213] both sets of numbers have usually been indicated by the calendars.[214]

---

213) Massa compoti verses 303 and 304 (Van Vijk p.59, p.79 f. and p.120).

214) Tables of epact, golden number and indictio are given in Ginzel III p.393 - 405.

---

§ 3. Eclipses. Astronomical concepts.

25. The moon's latitude.

Eclipses take place when sun, moon and earth are exactly (or almost exactly) on a straight line. If the orbits of the sun and moon lay in the same plane, then every conjunction or opposition would produce an eclipse. Actually, however, the plane of the moon's orbit is inclined towards the plane of the sun's orbit (the ecliptic) by slightly more than 5°. For the occurrence of an eclipse, it is therefore necessary not only that sun and moon have the same longitude but also that the moon be in the ecliptic at the same time or at least has only a very small latitude (fig. 15). The line of intersection between the planes of the two orbits is called the "nodal line" because the points where the orbit of the moon meets the ecliptic are called the "nodes" (ascending node, ☊ , where the moon passes from negative to positive latitudes, descending node, ☋ , the opposite). We can thus say that eclipses occur only when the syzygies fall into the nodal line or at least very close to it.

The time elapsing between two consecutive passages of the same node (or between two consecutive passages of points of the same latitude) is called a "nódical month". Its length is

$$(37) \qquad m_n = 27.2122^d \quad .$$

The nodal line does not maintain an invariable position on the ecliptic but moves backwards, i.e., in a direction apposite to the movement of the sun in the ecliptic. This means that the nodes are not always projec ed in to the same fixed star but occupy all possible positions in the eclip tic. Because the period of this rotation of the nodal line is only 18.6 years the moon reaches again the same latitude at an appreciably earlier time than the same longitude; this second period is called a "tropical month" and amounts to

$$(38) \qquad m_t = 27.3216^d \quad .$$

Obviously only the length of the nodical month is essential for the occurrence of eclipses.

Let us suppose that we start from a certain eclipse, i.e., from a moment when the line of syzygies coincides with the nodal line. From (37) it follows that the moon more than two days earlier comes again to the same node than to the same longitude with the sun. The moon during these two days gained so much in latitude that an eclipse is excluded. The same holds during the next month but in the third month nodal line and line of syzygie will be about at right angle and from now on the latitude decreases and makes possible an eclipse at the opposite node after six months. This poin to a periodic repetition of eclipses after 6, 12, etc. months.

This rough consideration must now be brought into a precise form We must ask how many times the difference

$$m - m_n = 29.5306 - 27.2122 = 2.3184$$

is contained in the nodal period. An approximate solution is obviously given by

$$(39a) \qquad \frac{1}{12} < \frac{23184}{272122} = \frac{1}{11 + \frac{17098}{23184}} < \frac{1}{11}$$

which confirms our preceding estimate that eclipses will be repeated after 6 + 6 months but shows at the same time that also intervals of 6 + 5 months can be expected. As a matter of fact, this rule was well known to ancient astronomers.[215)]

215) E.g. Heron, Dioptra 35 (ed. Schöne p.302, 21 f. and Rome [1]) or Ptolemy Almagest VI, 6.

We can now refine the inequalities (39) by the same process which we used in determining the common periods of the tropical year and the synodical month.[216)] As the next step we obtain from (39)

216) Above p.###.

$$(39b) \qquad \frac{23184}{272122} = \frac{1}{11 + \frac{1}{1 + \frac{6086}{17098}}} < \frac{1}{11 + \frac{1}{1 + \frac{1}{2}}} = \frac{3}{35}$$

This further sharpens our preceding result by telling us that a combination of one 5-months' and five 6-months' intervals bring a repretition of eclipses. If we proceed in the same way, we get

$$\frac{m - m_n}{m_n} = \frac{1}{11 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{286}{4640}}}}}} < \frac{19}{223}$$

This approximation is obviously so good that we comit only a very minute error in replacing the $<$ -sign by the $=$-sign and write

$$\frac{m - m_n}{m_n} = \frac{19}{223} \qquad \text{or} \qquad \frac{m}{m_n} = 1 + \frac{19}{223} = \frac{242}{223} \quad .$$

The relation thus obtained

(40) 223 lunations = 242 nodical months

is the famous ecliptic period usually called the "saros". Because 223 synodic months are very nearly $6585\frac{1}{3}$ days, the threefold time of

(40a) 669 lunations = 19756 days

is also called "saros" sometimes. On the other hand, $223 = 235 - 12$, an 235 months = 19 years, hence

(40b) 223 months $\approx$ 18 years

(more accurately 18 years + 11 days), which is another period called sometimes "saros".

It might be remarked that the name "saros" for the cycle (40) is not of Babylonian origin but was introduced by Halley in 1691.[217)]

---

217) Cf. Neugebauer [2] p.241-247 and [3] p.407-410. Already Ideler I p. 213 doubted the correctness of Halley's etymology.

---

26. Lunar eclipses.

In order to describe the details of the appearance of an ecli we cannot restrict ourselves to a consideration of the movement of the centers of the three bodies, but must take into consideration their fina diameter. Fig.16 illustrates the typical situation of the eclipsed moon, the diameters of S, E and M of course exaggerated as compared with thei distances. The distinction between umbra and penumbra is in practice of little value because the appearance of the progress of the eclipse depends largely on the physiology of our eye. Moreover, the atmosphere of the earth exercises a considerable influence on the distribiution of the light on the moon's surface. One therefore usually considers in moon eclipses only <u>one</u> shadow cone, whose diameter is 1/50 greater than the geometrically determined umbra cone. The angular diameter of this shadow cone, seen from the center of the earth, varies from $1\frac{1}{4}^\circ$ to $1\frac{1}{2}^\circ$ at the moon's distance, which varies because of the excentricity of the moon's orbit. The disk of the moon appears under an angle of 29 ½ to 34 arc minutes, again depending upon its distance from the earth. Because the sun moves about 1° per day, the moon about 13° a day, the elongation of the moon from the sun is about 12° a day or ½ degree per hour, i.e., one moon-diameter per hour. This shows that the maximal duration of a total lunar eclipse amounts to about three hours. Fig.17 illustrates how the moon passes the shadow cone. The arrwos indicate the small velocity of the sun (and of the shadow) and the much higher velocity of the moon in their orbits. The daily rotation in opposite direction determines east and west for any given horizon; this shows that the moon enters the shadow with the eastern part of its rim or, in other words, that the shadow passes over the moon from east to west.

It is clear that it is not necessary that the moon be exactly in the node at the moment of the opposition in order to produce an eclipse. As we have seen, the diameter of the shadow is always more than twice as large as the disk of the moon and will therefore eclipse at least parts of the moon even if the center of the moon does not coincide with the center of the shadow. Let us assume that the center M of the moon (fig.18) is in the node when the center of the shadow already lies at S_0' farther east in the ecliptic. One then calls "ecliptic limit" the maximal possible distance MS_0' such that the moon still achieves at least one moment of contact with the shadow at S_1'. These ecliptic limits again vary according to the special circumstances (distance and relative velocity) between $9\frac{1}{2}^{\circ}$ and 12°. From this it follows that if the moon is in the node when the syzygial line deviates from the nodal line by less than $9\frac{1}{2}^{\circ}$, then at least a partial lunar eclipse must occur. By the same type of reasoning it can be shown that a total eclipse is certain if MS_0' is less than $4\frac{1}{2}^{\circ}$.

From fig.18 it follows that our previous statement that the shadow passes the moon from east to west only gives the main direction but that the actual contact might deviate very considerably from the eastern point of the limb. In order to describe the details of a lunar eclipse precisely, one must therefore indicate how the shadow passes over the disc of the full moon. If the eclipse is total, four characteristic points can be distinguished: (1) the point of the first contact with the shadow, (2) the point into which the still illuminated area converges with approaching totality, (3) the point where partiality again begins, and (4) the point of last contact. In the case of a partial eclipse, the two inner contacts do not exist because there always remains a certain part outside the shadow. We now define as the "north point" of the moon the northern point of intersection of the moon's disc with the great circle which passes through the moon's center and the northpole.[218] The four (or two) points of contact

218) Every such circle intersects the celestial equator at right angle and is called hour circle or circle of declination. The circle in question is therefore the hour circle of the moon.

mentioned above are determined by giving their "angles of position", i.e., the angles NM(1), ..., NM(4) of the points of contact, counted eastwards from the north point (fig.19). The north point is the highest point of the limb of the moon above the horizon only if the moon stands in the meridian;

before midnight the radius MN inclines towards the east, after midnight towards the west. This element is of great importance for the identification of an eclipse if a historical report gives details about the progress of the eclipse.[219] We shall discuss below p.!!! an example from cuneiform tablets.

219) Neugebauer (P.V.) AChr. I p.129 gives a table to compute the inclination of NM towards the horizon for the horizon of Babylon.

The "magnitude" of an eclipse is measured by the proportion of the maximum eclipsed diameter of the moon. Using this diameter as unity, all partial eclipses have a magnitude less than one; the magnitude of total eclipses, however, is equal or greater than one. A magnitude 1.5 means that the shadow covers $1\frac{1}{2}$ times the diameter of the moon (fig.20). Magnitudes are frequently also expressed in "digits", assuming the diameter equal to 12 digits. The maximal possible magnitude of a lunar eclipse is about 1.7 or 23 digits.

The definition of the "magnitude" of a lunar eclipse given here is already adopted by Ptolemy. We know, however, from him[220] that magnitudes were usually expressed by the eclipsed area, also called "digits" i.e twelfths, which must be kept in mind when using ancient eclipse reports.

220) Almagest VII, 7 (ed. Heiberg p.512 and 522).

27. Solar eclipses.

The existence of solar eclipses is one of the most peculiar and accidental facts in our planetary system. It is a pure accident that the distance of the moon from the earth is exactly such that the moon's diamete as seen from the earth appears under the same angle as the diameter of the sun. Furthermore, the distances between sun, moon, and earth are so nicely balanced that their small variations sometimes bring the moon into such a position that its disc covers only 14/15ths of the sun, thus leaving a ring of 1 arc-minute width visible („annular eclipse") for a moment. The limits between annular appearance and totality are so narrow that the same solar eclipse can be annular at some places of its path over the earth, total in the remaining part.

Because of the enormous brightness of the sun, partial eclipses are only striking if considerably more than one half of the sun is covered by the moon. In order to measure "magnitudes", we now define the diameter of the sun as "one" (or 12 digits) and count the eclipsed proportion. Without advance knowledge that a solar eclipse will take place, one will hardly recognize partial eclipses of magnitudes below 0.75 (or 9 digits), except when the sun is very close to the horizon, where even small partial eclipse become visible. This is very important for chronological problems because it rules out many eclipses which would be visible according to calculation but only by careful watching of the sun with foreknowledge of the time of the event. This is one of the great differences between moon and sun eclipses. Even a small partial lunar eclipse will be recognized as a clearly visible dark sector on the full moon, a fact which thus makes practically all lunar eclipses equally likely to be recorded. Lunar eclipses are therefore much less signeficant for historical problems than solar eclipses.

But the main reason that sun eclipses are so much more important for absolute chronology lies in the fact that moon eclipses are visible from all places on the dark half of the earth; solar eclipses, however, only in places which lie in the narrow strip described by the umbra on the surface of the earth and the parallel zones of sufficiently high partiality In regions of a geographical latitude like the Mediterrenean Sea the strip of totality reaches only about 4° width in geographical latitude. From the existence of annular eclipses it follows that this width can diminish completely; on the other hand, 10 or more degrees of geographical latitude can be in the zone of totality near the poles (cf. fig.22 and 23).

We turn now to describe the main traces of the process of a sola eclipse on the earth. Because the movement of the moon from west towards east is much faster than the corresponding movement of the sun, the moon's shadow first touches the western side of the earth (fig.24) and then moves eastwards (with a velocity between 1000 and 5000 miles per hour !), leaving the earth about 4 hours later on its eastern side. On the eastern side of the earth, there are therefore places (A_1 in fig.25) where the solar eclips begins exactly with sunrise; shortly thereafter, the umbral cone reaches th earth and creates the spectacle of a total eclipse for a place like A_2 at the moment of sunrise. Places which pass from the night half of the earth to the day half later than A_2 only see a partially eclipsed sun-rise, until A_3, where the eclipse is over at the moment of the appearance of the sun above the horizon of A_3. Analogous considerations hold for places at the western side. The distribution of these points in a typical case is illustrated by fig.26. The situation of this eclipse corresponds closely to the simple scheme assumed in fig.25: the path of the umbral cone nearly coinciding with the equator, or better, the shadow at noon falling almost vertically on the earth. If the umbra strikes the earth even at noon more tangentially curves like those given in fig.27 originate.

The preceding description of the progress of a solar eclipse on the earth are, as a whole, only of interest for the calculation of eclipses of the sun. An observer at a given place, however, does not see anything of the path of the shadow over the earth but he sees only that the dark disc of the moon passes before the sun. It follows from fig.24 that the eclipse moves from west towards east over the sun. The progress of eclipses of the sun therefore follows the opposite direction as the progress of lunar eclipses.

The first step in using eclipse reports for historical purposes will always be to exclude as many cases as possible and to restrict the detailed investigation to a small number of remaining possible cases. For the majority of cases, a knowledge of the approximate path of the zone of totality will be sufficient. The following considerations lead to the construction of approximately determined curves of totality. Let us assume the earth to be perfectly spherical and the sun to stand exactly in the nodal line. From the second assumption, it follows that all straight lines drawn from the center of the sun to the center of the moon lie in the plane of the moon's orbit. Hence the axis of the umbral cone during the eclipse describes an exact plane; consequently, the intersection of this plane with the spherical earth, i.e. the central line of the eclipse, is a circle. Actually, none of these conditions are fulfilled. The earth is not a sphere but an ellipsoid; the sun need not be exactly in the nodal line in order to produce an eclipse and, at any rate, during the eclipse moves by an angle which subtends about 10 arc minutes at the center of the earth. But it is evident that these deviations from the ideal case are sufficiently small to justify the following approximation. One determines the two points reached by umbral cone at sunrise and at sunset; furthermore, one calculates the point where the eclipse is total just at noon (point C in fig.25). These three points uniquely define a circle on the earth which can be considered as nearly identical with the path of the total eclipse.

This method was followed by Oppolzer in laying out the maps in his famous "Canon der Finsternisse". Fig.28 shows an example of the maps thus obtained. In using these maps, however, one must keep in mind that deviations which are large when seen from the small areas which are the scene of ancient history are possible. Especially eclipses whose path deviates much from a pure west-east direction are likely to be incorrectly represented by the approximatively drawn central line. Fig.29 illustrates one of these cases.[221] The curve A is a part of the path of the annular

221) This example is taken from Ginzel, Kanon, p.5 and is of course selected as an extreme case.

eclipse of -880 V 1 , the total progress of which is given on fig.28. Calculation, however, using the same elements but determining the path not only by the approximative circle, gives the strip B.

28. Frequency of eclipses.

Solar eclipses, like lunar eclipses, do not require exact coincidence of nodal line and syzygies. The "ecliptic limits" of solar eclipses are even greater than the corresponding limits for moon eclipses,[222] name-

222) Cf. above p.[illegible].

ly about $15\frac{1}{2}^{\circ}$ and $18\frac{1}{2}^{\circ}$ as compared with $9\frac{1}{2}^{\circ}$ and 12° . This means that if the moon is in the ascending node and the sun less than $15\frac{1}{2}$ degrees beyond the node, an eclipse must take place; an eclipse is, however, certainly excluded if the sun already gained more than $18\frac{1}{2}^{\circ}$ from the node.

Simple considerations, based on these values for the ecliptic limits give the following result. The least possible number of eclipses during one year is two, and both are solar eclipses. The maximum number, however, is 7, of which either 4 are solar and 3 lunar or 5 solar and 2 lunar. Hence, solar eclipses are more frequent than lunar eclipses; during 1000 years only 1543 lunar eclipses, but 2375 solar eclipses occur. Among these solar eclipses, 1537 are total or annular, the remaining 838 partial.[223]

223) Oppolzer, Kanon, p.VII.

Numbers like 1500 total solar eclipses and 1500 lunar eclipses during 1000 years would be very significant for moon dwellers. For a given place on the earth, however, the majority of sun eclipses which take place somewhere on the globe pass without notice. Among approximately 900 solar eclipses, only 224 were visible in Rome from 0 to 600 A.D.; only 13 of these 224 reached more than 11 digits in Rome, and of these, only 2 were total, one annular. Between -200 and 350 A.D., no total sun eclipse was visible in Rome.[224] During the 6th century B.C. Babylon saw 2 total sun eclipses, but 87 lunar eclipses, of which 28 were total.

224) Neugebauer (P.V.) AChr. I p.95.

The frequently repeated story that centuries of observations must have led the Babylonian to discover the saros cycle can be illustrated by the list of the 8 total solar eclipses visible at Babylon during the last 900 years before our era.[225]

225) The visibility in Babylon according to Ginzel's Kanon. The differences in days can be directly derived from Oppolzer Kanon, where the Julian day of each eclipse is given.

Dates			Differences in Days
-880	V	1	
-558	I	14	117326
-556	V	19	856
-401	I	18	56492
-177	XII	22	82154
-135	IV	15	15090
- 21	VIII	11	41756
- 9	VI	30	4341

It is hard to see how such a sequence of numbers could possibly suggest the idea of cyclic repetition of solar eclipses, a periodicity which would be difficult enough to detect even from a complete list of all solar eclipses.

29. Tables.

We give below a list of the tables which can be used to determine historical eclipses.[226)]

226) More detailed information can be found in Neugebauer (P.V.) AChr. § 13 (p.95 ff.).

canon	sun			moon			region
P.V.Neugebauer (1931/34)	-4200	to	-900	-3450	to	0	Near East
Ginzel (1899)	-900	to	+600	-900	to	+600	Mediterrenean
Schröter (1923)	+ 600	to	+1800	+600	to	+1800[227)]	Europe & NearEas
Oppolzer (1887)	-1207	to	+2161	-1200	to	+2163	World

227) Only total eclipses are listed.

This list obviously covers all periods for ancient and medieval history. There exists, however, a very serious gap of tables for solar eclipses from -900 to -400 because Ginzel's tables are not fully reliable for this perio (the reasons will be discussed in § 5, cf. below p. ff.).

§ 4. Ancient Moon Eclipses.

30. Egypt.

It is a remarkable fact that not a single record of an eclipse has been found among the countless inscriptions and papyri which we possess from all the periods of ancient Egyptian history. There is only one vague remark in a Karnak inscription of a king of the XXII^nd dynasty (Takelot II, about -850) which might possibly refer to an eclipse but could just as well mean invisibility of the moon at newmoon or due to any other cause. The text reads:[228] "Now, afterwards, in the year 15, month XII(e), day 25,

228) Following the translation of Breasted AR IV p.382 and note d, where further literature is quoted. See, moreover, Breasted AR I p.21 and the most recent discussion of this inscription by Borchardt [3] p.3 ff.

under the majesty of his angust father, the divine ruler of Thebes, heaven not having devoured the moon, great wrath arose in this land ...". But even assuming that a lunar eclipse is meant, this report is without practical value for chronological purposes. Even if we find a year in which a lunar eclipse falls in the season in question, eclipses would also have occured 11 or 12 months earlier and later, and it would take many years to bring the ecliptic months and the calendar into total disaccord.[229]

229) The situation, of course, is different if one already knows the time into which the reign of this king falls (cf. Borchardt [3] p.4 f.).

To make up for this complete lack of Egyptian eclipse reports, modern books sometimes make the statement that Diogenes Laertius (third cent.A.D.) says that the Egyptians had recorded 373 solar eclipses and 832 lunar eclipses. Actually, the passage in question[230] is by no means clear

230) Diog.Laertius, proemium .

Diogenes only says that the Egyptians consider their god Hephaistion as the creator of philosophy; since his time, Diogenes adds, 48863 years had elapsed, during which the above-mentioned number of eclipses are said to have happened. But there is no special reference to Egypt in connection with these numbers, and the continuation of this passage includes other chronological data, all of which clearly have nothing to do with Egypt at all.

Although there is, accordingly, no definite reason to conclude that Diogenes had any information about Egyptian records of eclipses, it still might be possible, of course, that such records did exist. Our knowledge of Egyptian culture in general and of Egyptian astronomy in particular is sufficient, however, to permit us to say that the development and importance of astronomy in Egypt certainly never reached the level of Babylonian astronomy; we can never expect to find many hundreds of reports of all kind of astronomical observations in Egypt such as we actually possess from Mesopotamia. The obvious explanation of this fact is the very late introduction of astrological concepts into Egypt. Astrological ideas can not be discovered in Egypt before the fourth century B.C. This reduces very considerably the probability of finding Egyptian astronomical reports; for the present, at any rate, Egyptian absolute chronology must be based on considerations of a different sort than the chronology of the Greco-Roman period, which is manly based on the lunar eclipses mentioned in the Almagest.

31. Mesopotamia.

The era of Nabonassar is chronologically determined by the eclipse reports contained in the Almagest and dated according to this era in Egyptian years. From the era of Nabonassar the chronology of the king-list of the "Ptolemaic canon" is established, thus linking together oriental and Greco - Roman chronology. There are 19 eclipses which form in this way the basis of ancient Greco - Roman chronology, all of them lunar. The elements of these eclipses are given with astronomical details which are more than sufficient to permit them to be determined uniquely by modern calculations.[231] Of these 19 eclipses, 10 are taken from Babylonian re-

231) These eclipses are listed and discussed in Ginzel, Kanon, p.229-234.

ports. They are as follows:

-720	III	19	(18.2)	-501	XI	19	(2.1)
-719	III	8	(1.5)	-490	IV	25	(1.7)
-719	IX	1	(6.1)	-382	XII	23	(3.0)
-620	IV	22	(2.1)	-381	VI	18	(5.9)
-522	VII	16	(6.1)	-381	XII	12	(18.2)

Only the first[232] and the last eclipse are total. as is shown by the magnitudes given in parenthesis.[233] The last three eclipses were used by Hipparchus from Babylonian sources.[234] The eclipse of -522 is of special interest because we have not only Ptolemy's elements but an original cuneiform report as well.[235]

232) Detailed information about the progress of this eclipse in Neugebauer (P.V.) AChr. I p.126 f.

233) According to Neugebauer (P.V.) Kanon (expressed in digits).

234) Almagest IV, 11 (ed. Heiberg p.340). Hipparchus gives the years in t Athenian eponymic fashion by mentioning the Archons. This makes these ecli ses of interest for Athenian chronology (see Dinsmoor, Archons, p.350 and p.391).

235) This was discovered by Oppert [1].

Ptolemy's description of this eclipse is as follows:[236] Cambys year 7 - Nabonassar 225 X(e) 17/18, 1^h before midnight, reaching $\frac{1}{2}$ of the moon's diameter from the north.

236) Almagest V,14 (ed. Heiberg p.419).

The cuneiform text gives:[237] year 7 IV(b) 14 $1\frac{2}{3}$ bēru after dark, total progress of the eclipse visible, covering at its maximum half of the disc from the north. In order to convert these numbers into their modern equivalents, we must note that sunset took place on the given date at 19.25^h and that $1\frac{2}{3}$ bēru correspond[238] to 3.33^h; hence, 22.6^h is the time given by the text, or about ½ hour earlier than according to Ptolemy.

237) Published in Strassmaier, Kambyses No.400. Cf. Kugler SSB I p.70/71.

238) Cf. above p.▮▮▮.

Modern calculation[239] gives as magnitude 6.1 digits from the north,[240] in agreement both with the cuneiform text and Ptolemy. As to the

239) Neugebauer (P.V.) Kanon p.40.

240) Angle of position of first contact 33° towards east, of last contact 57° towards west.

time, the beginning was 22.3^h, the end 1.0^h. Assuming that the ancient reports refer to the middle of the eclipse, we would obtain the following hours: cuneiform text 22.6, Ptolemy 23, calculation 23.7 .

We shall later discuss the problem of discrepancies between modern calculation and texts.[241] The divergences between Ptolemy and the

241) Cf. below p.▮▮▮.

other dates can be explained by the former's loose expressions in round numbers. But it is perfectly absurd to assume, as has been done occasionally, that Ptolemy used elements gained by observations from other places (say by Hipparchus in Rhodes) and recalculated the elements as he thought they should have been for observations at Babylon. This assumption exhibits misunderstanding of Ptolemy's goal, which consisted in selecting eclipses of a certain type in order to determine the basic constants of his theory. One has doubted,[242] e.g., the Babylonian origin of the elements of the eclipse of -382 XII 23 although Ptolemy says explicitly that Hipparchus received the elements from Babylon.[243] The reason for the doubts is that Ptolemy

242) Oppolzer in Ginzel, Kanon p.233.

243) Almagest IV,11 (ed. Heiberg p.340): "Hipparchus says that he uses these three eclipses for comparison among those eclipses conveyed from Babylon as observed there". One should, moreover, keep in mind how careful Ptolemy was in selecting his material. In the case of the three oldest eclipses, e.g., he mentions (ed. Heiberg p.301) explicitly that the reports concerning these eclipses give the impression of especially careful recording.

gives ½ hour before sunrise for the beginning while calculation gives 7.1^h, i.e., almost exactly at sunrise (which is at 7.2^h). Actually, the only conclusion left is that here again modern calculation gives a slightly later beginning than the facts attested by the ancient reports.

We know from Ptolemy that he selected the eclipses from more extensive material at his disposal.[244] This is confirmed by the existence of numerous astronomical reports from the same period,[245] e.g., three lunar eclipses from the reign of Šamaš-šum-ukīn namely -661 I 28,[246] -652 VII 13[247] and -652 I 18.[248] The partial lunar eclipse of -200 IX 22 had been noticed at Babylon[249] but was used by Ptolemy from Hipparchus' own observation made at Alexandria.[250]

244) Almagest, introduction to IV,6 (ed. Heiberg p.301 f.).

245) Collected in Thompson, Reports, and Harper, Letters (cf. the index to Waterman's translation of the Harper Letters).

246) Kugler SSB II p.372-380.

247) Discussed by J.Mayr in Piepkorn [1] p.105-109 (previously by Ginzel, Kanon p.252 ff. and others).

248) Harper, Letters No.137. Cf. Weissbach [1] and [2] p.65.

249) Schaumberger Erg. pl.X and p.368 f. and note 1 on these pages.

For the period in question, however, one difficulty which rises in using cuneiform material for chronological purposes must be mentioned.[251]

251) This material has not yet been fully investigated, although easily accessible in the publications Thompson Rep.; Harper, Letters; Waterman Royal corr.; Pfeiffer, Letters.

One must in each specific case exclude the possibility that the given elements result from calculation instead of observation. Although systematic calculation of the moon's movement, predicting its position with high accuracy for a long period in advance, do not exist before the third century B.C., more primitive and short-termed predictions were undoubtedly made some centuries earlier.[252] And there are cases where the decision between observed or calculated is not easy to make, and the best policy consists in not using the text for further conclusions.[253]

252) For examples see Kugler SSB II p.62 ff. or Olmstead [1] p.118 f. Such predictions are, e.g., possible by watching the moon's latitude in the middle of the month. This is at least sufficient to preclude in many cases a lunar eclipse at the next conjunction. This negative method is much more in accord with our knowledge about Babylonian astronomy than the usual assumption of cyclic prediction.

253) No agreement has been reached in the case of the lunar eclipse of -424 X 9 (mentioned in CBS 11901) declared by Kugler (SSB Erg. p.233 ff.) to have been calculated but considered by Schoch and Fotheringham (Schoch [1] p.3, Langdon -F.-S. p.50 note 4) as actually observed; cf. Schaumberger Erg. p.243 and Kugler [1]. It seems to me methodically wrong that Fotheringham ([3]) and De Sitter ([1]) also based their own calculations on this eclipse.

Almost no material exists from the period before 750 B.C. One report about a lunar eclipse has been found in the archives of Mari from the

18th cent. B.C. but is still unpublished.[254] It is possible, however, that

254) Dossin [1] p.125.

the astrological omens collected in the great series "Enūma Anu Enlil" contain metrial which can still be identified chronologically. This has been attempted in different cases but without very convincing results.[255] The

255) E.g. by Schoch [1] p.6 ff.

most detailed information is contained in one omen which concludes the destruction of Ur from the following conditions: an eclipse took place on the 14th of XII(b), beginning in the south, ending in the north, beginning in the first watch, ending in the third.[256] From these data it follows that

256) The text is contained in Virroleaud Sin XXX 79-82, translated by Jastrow, RBA II p.558.

a lunar eclipse is involved, not too distant from the vernal equinox (say ± 2 months). The length of a watch at this time of the year is about 4^h, regardless of whether one assumes seasonal watches or not. Beginning in the first, ending in the third watch therefore implies a very freat total eclipse with its middle near midnight. Finally, the angle of position of the first contact must by as much as possible be greater than 90° in order to justify the expression "beginning from south"; correspondingly, the last contact should be as near as possible to the northpoint. The best possible solution seems to be the eclipse of -2015 IV 24/25 proposed by S.Smith.[257] The date is late so far as the month is concerned (XII(b)),

257) Smith, Alalakh p.31. Schoch proposed the eclipse -2282 III 8/9 which hardly reaches totality (magn.= 12) and lasts only for 3.3 hours. The angles of position are 132° towards east (which could well enough have been called "south") but more than 100° towards west, which is not "north" at all. There are about seven eclipses in the period under consideration which fit the elements of the text no worse.

but by no means impossible within the arbitrary calendar of the Third Dynasty of Ur.[258] This eclipse reaches the magnitude 22.1 and lasts from 23.7 to 3.3, i.e., 3.6 hours, the middle being 1.5^h after midnight. The first contact took place 120° from the north-point, the last only 64° to the west of the north-point.[259] One can say that all elements fit the conditions

258) According to p. 10 the vernal equinox falls about III 21 + 18 = IV 8 at -2000.

259) One must not forget, however, that the north-point is inclined with respect to the meridian. Assuming that the eclipse actually happened 1 ½ hours earlier than according to calculation, one would obtain as point of first contact a point 152° east of the meridian, as last contact, however, 96° west (instead of "north") as fig.30 shows. Assuming the time as given by calculation, one would obtain 127° as angle of position of the beginning, 110° towards west as angle of position of the end of the eclipse.

of the text fairly well, and there is no better eclipse from the period -2300 to -1900. Assuming that the elements mentioned in the omen reflect real facts, then can hardly remain any doubt that the destruction of Ur has to be dated -2015.

32. Moon eclipses and geographical longitude.

The value of lunar eclipses in Greek and Roman times is not only restricted to the occasional checking of dates within the framework of the general chronology established by means of the eclipses in the Almagest. The difference in local time recognized in observing the same lunar eclipse yielded one of the main arguments for the sphericity of the earth[260] and

260) Cf. e.g. Theon Smyrnaeus (ed. Martin p.140 ff.) and Cleomedes I,8 (ed. Ziegler p.76), both(?) writing in the 2nd cent. A.D.

at the same time opened the only existing possibility to determine exactly the differences in longitude at far distant places. The simultaneous observation of lunar eclipses plays for antiquity the same rôle as the time - signals today; solar eclipses, on the contrary, furnish no information about the difference in space or time of different places because not only the direction of the path but also the velocity of the shadow would have to be known.

Fig.31 shows how one can directly conclude geographical longitude from the difference in local time upon the difference in ; differences in latitude make no difference because all places on the same meridian have the same local time. Each difference of one hour corresponds to 15° difference in longitude. When Pliny[261] tells us that the moon eclipse which pre-

261) Plinius NH II,180 (= p.196 2 ff. ed. Jan-Mayhoff or Ginzel, Kanon, p.184).

ceded the battle ar Arbela by 11 days happened at the second hour of the night bat at moonrise in Sicily, we have the following elements. Since the eclipse occured on -330 IX 20 , seasonal hours are therefore practically the same as equinoctial hours. Moreover, moonrise and sunset are simultaneous at a lunar eclipse; hence, the time difference is 2 hours, which is correct since Arbela lies 30° east from Sicily. The same eclipse is mentioned by Ptolemy in his Geography[262] as having taken place at the 5th

262) Geogr. I,4 (Mžik p.21 or Ginzel, Kanon, p.184).

hour in Arbela, at the second in Carthage. Here everything is wrong; Carthage lies not $3^{h} = 45^{\circ}$ west from Arbela but only 36°. Moreover, the eclipse in question, having begun at the 2nd hour, was just over at the 5th hour.[263]

263) Because Arbela and Babylon lay on the same meridian, the times from Neugebauer (P.V.), Kanon, are the same, namely, beginnung 19.8, totality ... 21.2, and 23.0

The erroneous values given by Ptolemy may have been caused by an simple error in his source; the essential point, however, is that he obviously had no other elements at his disposal because he actually based his maps on this erroneous time difference.[264] This reflects undoubtedly one of

264) According to Geogr. VI,1,5 (Nobbe I p.83,9) Arbela has the longitude 80°, according to IV,3,7 (Nobbe I p.236,9) Carthage (Karchedon) the longitude 34°50'.

the greatest difficulties in ancient mathematical geography and astronomy: the complete lack of a scientific organisation. An astronomer in Alexandria had no means at his disposal of regularly obtaining results of observations made at far-distant places. When Heron described the method of using simultaneous observations of a lunar eclipse in chapter 35 of his "dioptra", he could only use in his example elements as observed in Alexandria (partial lunar eclipse of +62 III 13 [265]) at the 5th hour of the night, as is confirmed by calculation.[266] For Rome, however, Heron assumes an observation two hours later - which is almost twice the true time difference (1^h and 10 minutes).[267]

265) Heron says 10 days before equinox which actually took place +62 III 22 21.7^h.

266) Neugebauer (O.) [5] p.23.

267) A list of the geographical coordinates of important places in antiquity is given in Neugebauer (P.V.) HAChr. III p.71.

It is, therefore, perhaps not quite accidental that the number of eclipses, recorded in Greek literature, is so small. If we do not count the 19 lunar eclipses from the Almagest, only 18 lunar eclipses (total and partial) and 36 solar eclipses (only 2 partial !) are mentioned in Greek sources between 700 B.C. and 300 A.D.[268] And these numbers would be even

268) These numbers according to Boll's list in RE 6, 2352 ff. (1909).

smaller had it not been for the importance of eclipses for superstition and astrology. The material for exact calculation was therefore restricted to a few traditional values and personal observations - a fact which should not be forgotten in evaluating the results of ancient astronomers.

§ 5. Ancient Solar Eclipses.

33. Sun eclipses of chronological interest.

Sun eclipses are of much higher interest for chronological purposes than eclipses of the moon because of their great rareness at a particular place. The mere mention of the visibility of a solar eclipse during a given period is usually sufficient to determine uniquely the date in question by simple looking at maps such as are given in Ginzel's, Oppolzer's or one of the other "canons". Knowing, on the contrary, that during the reign of a king a lunar eclipse was visible is chronologically valueless because almost every year will suit this description.

Unfortunately, the number of recorded solar eclipses is still far lower than it must be by purely astronomical reasons. One could, for example, easily get the impression from modern literature, especially from books of an expository character, that countless solar eclipses were recorded since in ancient Babylonia. Actually only a single report about a solar eclipse can be used fro chronological purposes,[269] namely, the

269) A few additional solar eclipses are mentioned in the literature, but all of them require too many additional assumptions to be considered as chronologically useful. The oldest eclipse in this group is the total eclipse

of -1062 VIII 31. Fotheringham [2] takes this eclipse in consideration, although the date given in the text is the 26th (no misreading as explicitly admitted by Fotheringham [2] p.105), not to mention the very doubtful description of the phenomenon (cf. Kugler SSB II p.372 note 2 and Ginzel [1] p.39 f.). The eclipse mentioned in Harper Letters 276, identified in Ginzel Kanon 245 ff. with the eclipse of -699 VIII 6, is also contested by Kugler [1] p.64-66. Wesson [1] attempted to identify the eclipse mentioned in a badly preserved tablet (Thompson Reports II p. 110 No. 277 R.) with the eclipse of .
We already mentioned (p.###) the text CBS 11901, which not only refers to the lunar eclipse of -424 X 9 but also the solar eclipse of -424 X 23 invisible at Babylon (Kugler SSB Erg. p.236 f.).

remark in the list of the Assyrian eponyms that during the eponymate Bur-Sagale an eclipse of the sun happened in the third month.[270] The eponymic canon alone is sufficient to determine the century of Bur-Sagale as the 8th century B.C. As fig.23 shows, only two eclipses were visible in Mesopotamia from -800 to -700, specifically -762 VI 15 and -764 II 10. The second is excluded because a third Babylonian month cannot fall near February but only near June. Hence, there remains only -762 VI 15, which is not only in best agreement with the given month but also reached almost complete totality in Nineveh; the magnitude reached for this place was 11.89 digits.[271] This eclipse is therefore the real fixed point in Assyrian chronology. For older times we must rely mainly on relative chronology until a thousand years earlier, when we get some help from observations of Venus (to be discussed in the following chapter) and from the moon

270) The text is K 51, published in II R 52, the passage in question reproduced in Ginzel Kanon p.243. Cf. Reallex. II p.413.

271) This according to Schoch [1] p.24. Fig.23 gives the zone of totality according to Ginzel, Kanon, about 100 km too far north.

eclipse connected with the destruction of Ur, the chronological value of

the very latest centuries of Babylonian history, we have records of a few additional eclipses of the sun,[273] but the chronology of this period is already exactly determined by the Babylonian lunar calculations of Seleucid times.

273) Listed in Ginzel Kanon p.259 f.; but only one of them was visible at Babylon (−122 I 23).

The situation is only slightly better in the Greek sources. It should be remarked that Ptolemy does not give a single date of a sun eclipse, obviously because he had no material,at his disposal which seemed to him to be sufficiently trustworthy. It is also of interest that Cleomedes[274] denies the existence of annular eclipses, although, e.g., the eclipse of −430 VIII 3, mentioned by Thucydides[275], was annular.[276] Actually, all

274) Cleomedes II,4 (ed. Ziegler p.190,23).

275) Thucydides II,28. Cf. above p.[illegible].

276) Cf. about their frequency above p.[illegible].

chronologically important reports of eclipses of the sun in Greek sources belong to the period of the Peloponnesian war and the time shortly thereafter. The just previously mentioned eclipse of −430 falls in the first

271a) The solar eclipse of −1331 XII 30 has been assumed to be recorded in Hittite sources and would constitute an important chronological element (cf. e.g. F.Bilabel, Geschichte Vorderasiens und Ägyptens vom 16.-11.Jahrhundert v.Chr. [Heidelberg, Winter, 1927] p.291). It has been shown, however, that this assumption is based on an erroneous translation of the text (cf. A.Götze, Nochmals sakiiah(h)-. Kleinasiatische Forschungen 1 (1930) p.401-413). It might be added that this eclipse only reached 5 digits for the region in question and was therefore hardly recognized at all (cf. Neugebauer (P.V.) KS p.24).

year of this great conflict, the partial eclipse of -423 III 21 in the eighth year.[277] Then follows the eclipses of -403 IX 3, establishing the date of the defeat of Larissa by Lycophron,[278] -393 VIII 14 the invasion of Bocotia by Agesilaos[279] and -363 VII 13 the war between Pelopidas and Alexander of Pherae.[280]

---

277) Thucydides IV,52. The eclipse of the moon which caused the Sicilian catastrophy of the Athenean expeditionary force falls ten years later (-412 VIII 27).(CAH V p.306).

278) CAH VI p.36.

279) CAH VI p.47.

280) CAH VI p.86.

The last eclipse in connection with Greek history is the "Agathocles" eclipse of -309 VIII 15, which will be discussed below in greater detail.[281] The earliest Roman solar eclipse is -216 II 11 which is followed by four additional recordings in the year before the beginning of our era.[282] The latest Greek reports on eclipses do not refer to political history; the total eclipse of 71 A.D. III 20, seen by Plutarch, is of astronomical interest,[283] while the eclipse of +320 X 18 is used by Pappus,[284] that of +364 VI 16 by Theon[285] in their astronomical treatises.

---

281) Cf. p.###.

282) Cf. the list given by Boll RE 6, 2357 f.

283) Cf. below p.###.

284) Rome p.X.

285) Boll RE 6, col. 2363.

Altogether, one may say that no more than about 20 solar eclipses which are useful for establishing the fixed points of absolute chronology are known from all periods of ancient history.

34. Secular acceleration.

In 1698 there appeared in the Philosophical Transactions[286)]

286) No.218 for 1695.

"An exctract of the journals of two several voyages of the English merchants of the factory of Aleppo, to Tadmor, anciently call'd Palmyra". In an appendix to this report, E.Halley wrote[287)]: "And if any curious Traveller, or

287) Halley [2] p.174 f.

Merchand residing there, would please to observe, with due care, the Phases of the Moons Eclipses at Bagdat, Aleppo and Alexandria, thereby to determine their Longitudes, they could not do the Science of Astronomy a greater Service: For in and near these Places were made all the Observations whereby the Middle Motions of the Sun and Moon are limited: And I could then pronounce in what Proportion the Moon's Motion does Accelerate; which that it does, I think I can demonstrate, and shall (God willing) one day, make it appear to the Publick."

In order to explain what Halley meant in speaking of an acceleration of the moon's motion we must briefly return to the description of the movement of the moon given in § 1. For the sake of simplicity, we disregard the moon's latitude and consider only its longitude as depending on time. We disregard, moreover, all irregularities in the moon's velocity by considering only the average velocity as obtained from, say, one or two centuries of observations. Let us suppose λ_0 to be the longitude of the moon at a certain moment (called the "epoch") from which we begin to count the time t. This means that we suppose that λ_0 is the moon's longitude at $t=0$ and that the future corresponds to positive values of t, the past to negative values, "future" and "past" as understood from the epoch chosen. According to our assumption, the moon moves with constant velocity α_1 i.e., its longitude increases by α_1 degrees in each unit of time. The longitude at the time t will hence be given by

$$\lambda = \lambda_0 + \alpha_1 t \tag{41}$$

(we count here, of course, longitudes not mod.360^0 but admit continuously increasing values like distances on a straight line). What Halley discovered was the following: in discussing four eclipses[288] observed by Al Battani (the solar eclipses of 891 VIII 8 and 901 I 22 and the lunar eclipses of 883 VII 23 and 901 VIII 2[289]) and comparing them on the one hand with the eclipses described in the Almagest and with the elements of his own time in the other, he found that these three groups of observations could not be reconciled with each other by the simple formula (41). Although the eclipses of Al Battani were just in the middle of the time between Halley and Ptolemy corrections appeared to be necessary, different for the first and for the second half of this time interval. Therefore a formula like (41) does not describe the mean longitude of the moon but we need formula like

$$\lambda = \lambda_0 + \alpha_1 t + \alpha_2 t^2 \tag{42}$$

where the littmle coefficient α_2 measures the incrdase of velocity, i.e., the acceleration of the movement.

288) Halley [1].

289) The passages in question are now published in Al Battani p.56 f. and discussed by Schiaparelli on p.226-234.

The existence of such a slow increase in the moon's mean longitude, although only visible after many centuries, constitutes a permanent challenge to astronomers,[290] as expressed in the following sentence of Laplace[291]: "la correspondance des autres phénomènes célestes avec la

290) Cf. Lalande [1].

291) Laplace [1] p.244 f.

théorie de la pesanteur est si parfaite et si satisfaisante que l'on ne peut voir sans regret l'équation séculaire de la Lune se refuser à cette théorie et faire seule exception à une loi générale et simple dont la découverte, par la grandeur et la variété des objects qu'elle embrasse, fait tant d'honneur à l'esprit humain". It is one of the great achievements of Laplace to have been able to show that such an acceleration is the consequence of a slow decrease in the excentricity of the orbit of the earth caused by the perturbation of the other planets.[292] It turned out, how-

292) Laplace [1], published 1788 in the Mémoires de l'académie royale des Sciences de Paris, année 1786. For the modern theory, see Tisserand MC III chapter XIII and Brown Lth. p.243 and 267 ff.

ever, that the numerical value of the coefficient α_2, as deduced from the comparison of the ancient eclipses with modern elements, is about twice as much as anticipated by the theory (although numerically very small, about 10" if one Julian century is chosen as the unit of t.

This discrepancy between empirical values and the results of celestial mechanics gives a new interest to the investigation of far remote eclipses, especially eclipses of the sun. A small difference in the moon's longitude of course affects the time of the syzygies, and hence the beginning or end of an eclipse. Such small differences are, however, below the limits of accuracy of ancient time measurement or, at least, of the accuracy and reliability of ancient reports; they can therefore not be detected in reports on lunar eclipses. A small difference in the time of conjunction will not only affect the moment of recording of a solar eclipse but also influences the position of the zone of totality on the earth, because different parts of the earth will be in the shadow at different times. Very small changes in the moon's longitude therefore might move the path of totality by 50 or 100 miles and make the eclipse total for a certain place, where it was only partial according to slightly different elements of calculation. The totality of an eclipse is so different a phenomenon from merely partial solar eclipses that there can be no doubt as to the reliability of ancient reports in this respect. Ancient solar eclipses are therefore a very important element for determining the empirical value of the constant of secular acceleration in the theory of the moon.

Before we go on to describe in some special cases the influence of the value of the secular acceleration on the appearance of ancient solar eclipses, we must first discuss the modern explanation of that part of the secular acceleration which is not yet covered by the Laplacian arguments. This will lead us once more to the necessity of analysing the concept of "time". We saw how the apparently "natural" concepts like year, month, and day are actually of a very complex nature and deserve much analysis before they can become scientifically useful. The basis for all these definitions is the assumption of a unit of time of definite length. The instruments for checking the invariability of time intervals are our clocks. The accuracy of clocks must be controlled, and this is done by using one revolution of the earth, the "sidereal day", as the fundamental unit of all time measurement. The sidereal day is thus the time which elapses between two consecutive meridian transits of the same fixed star, or more accurately, of the vernal point, which is not quite the same, because of the precession of the equinoxes. All previously introduced units like hour, mean solar day, lunation, etc., are in this way eventually defined by means of the sidereal day.[293] We must now consider the consequences of the assumption of a slow reduction of the rotational speed of the earth.

---

293) The numerical relation between sidereal day and mean solar day can easily be derived as follows. Because the sun is delayed with respect to the daily rotation of the fixed stars, a given point ♈ on the sky will again reach the meridian earlier than the sun. After one tropical year, the sun will have fallen behind the vernal point one complete circuit; or in other words, the vernal point will have gained one passage of the meridian. Hence, if $y = 365.2422$ is the number of days in a tropical year, the corresponding number of sidereal days will be $y + 1$. There fore

$$(43) \qquad 1 \text{ mean solar day} = \left(1 + \frac{1}{y}\right) \text{ sidereal days}$$

from which follows

$$(44) \qquad 1 \text{ mean sol.d.} = 1.0027 \text{ sid.d}$$

XXXXXXXXXX

$$1 \text{ sid.day} = 0.9973 \text{ mean sol.d.}$$

or approximately 1 sidereal day = 1 mean solar day, - 4 minutes.

Let us suppose that two vehicles, M and S, travel with uniform velocity on a road (cf. fig.32), but M much faster than S. Their movement shall be observed by an observer on a rotating disc E, which defines the "velocity" of M and S by counting the numbers of miles covered during one revolution of E. If E rotates uniformly, the observer's definition of velocity will yield the same result as the measurement of the actual velocity, of the vehicles on the road. If, however, the angular velocity of the disc slowly diminishes, the observer on E will realize that M and S cover more miles during one rotation than before, and he will consequently speak about an "acceleration" of their movement. Moreover, he will detect this acceleration more easily by observing the faster vehicle M than the slower vehicle S because M covers much more distance than S in the same time.

The application of these considerations to our present problem is obvious. A slow increase of the length of the sidereal day must be interpreted, according to our definitions, as an acceleration of the movement of sun, moon and planets, and these accelerations must be much more visible in the case of the moon than in the case of the sun because this kind of secular acceleration will be proportional to the actual mean velocity of the body. This part of the secular acceleration of the moon is therefore not a consequence of the interaction of forces in our planetary system but is merely due to our employment of the rotation of the earth as our supreme clock. Reasons why this clock does not move in an absolutely regular manner have been given; among them, tidal friction is evidently the main cause.[294)]

294) Cf. Jeffery [1] where more references are given.

The method of detecting a secular acceleration of the moon is closely related to the chronology of eclipses. If the present length of the sidereal day is used as the unit of time, all longitudes calculated for past times will be too great if we apply simply the formula (41). This error will be in proportion to the coefficient α_1 , i.e., to the mean velocity. Therefore the moon's longitude will be much more affected than the sun's longitude; in other words, the elongation of the moon from the sun will increase. Too great an elongation, however, means that the conjunction is assumed to be later than was really the case. We showed on p.### that such a delay can affect the strip of totality of solar eclipses, which is thus at present the most sensitive instrument to measure the amount of the secular acceleration caused by the slow decrease of the earth's rotation.

A few general remarks may be added. We assumed in our example in fig.32 that the two vehicles M and S "actually" moved with constant velocity. This supposes the existence of some method of measuring time independent of the rotation of E. The same problem now rises on the earth after one has good reason to distract the basic assumption of the unvariability of the earth's rotation. This problem can be solved only by finding a new method of measuring time intervals absolutely independent of the phenomena in question. Pendulum clocks cannot be the solution because their frequency depends upon the constney of gravity, which is, as is well known, subject to changes in the distribution of mass inside the body of the earth. The only way open today is therefore the use of atomic processes which are, in all our present knowledge, absolutely independent of influences connected with the earth's rotation. This principle is the basis of the so-called "crystal clocks", which encourage our expectation of the possibility of controlling the change in the velocity of the daily rotation with the same degree of accuracy resulting from detection by accumulated effects during 20 or 25 centuries.[295)]

295) A report of the development of these clocks from 1929 (W.A.Marrison) to 1936 is given in Scheibe [1].

It will be clear from the preceding discussion that two different causes become visible in the moon's "secular acceleration": one completely determined by means of celestial mechanics, the second explained in principle but to be determined numerically only by empirical means. The first part is of course taken into account in all modern lunar tables. The second part, however, largely depends upon the selection of recorded eclipses which are supposed to be so reliably recorded that modern calculation can be tested on them. This explains why modern eclipse tables are not alike in their results in representing all ancient eclipses. Ginzel's Kanon, for example, is based mainly on elements chosen to agree with medieval eclipses Fotheringham and Schoch, on the contrary, required higher accuracy of the representation of ancient eclipses. This difference in procedure results in not quite negligible differences in the localization of the totality of solar eclipses for the period before 400 B.C. Around 1500 B.C., the difference amounts to about 600 Kilometers; accordingly, Ginzel's curves lie farther east than Schoch's.[296] There can be little doubt that Schoch's ele-

296) Neugebauer (P.V.) AChr. I p.131.

ments are at present the best solution of the representation of ancient eclipses. The following examples will show the effect of these new corrections on Ginzel's elements.

a. The Agathocles and Hipparchus eclipse.

Sicily was fpr centuries the battlefield between Carthaginian and Greek colonization; in one of these wars, Agathocles, the tyrant of Syracuse escaped with the fleet from besieged Syracuse in order to attack the Phoenicians in Africa.[297] Now Diodorus (first cent. B.C.) reports[298] that

297) Cf. CAH VII p.625 and RE I col.752.

298) Diodorus XX 5,5.

"on the following day (after Agathocles had left Syracuse) there occurred an eclipse of the sun such that it became night and stars were visible". From the period in question, it follows that the eclipse in question was the total solar eclipse of -309 VIII 15, visible in Sicily. Furthermore, the eclipse must have been total wherever the fleet happened to be at the moment because of the appearance of stars (which refers, at least, to Venus). We know, moreover, that Agathocles landed six days later at Cape Hermaeum,[299] not far from Carthage; but it is not explicily stated whether

299) Now Cape Bon. Cf. the map in CAH VII facing p.617.

Agathocles sailed around the northern shore of Sicily or took the shorter way southwards, although the comparatively long sailing time makes the first assumption more probable.

It is a peculiar fact that the two possibilities, northern or southern route, are equivalent to placing the zône of totality according to Ginzel's elements or according to Schoch's. Fig.33 shows how Schoch's correction for secular acceleration moves the zone of totality so far towards north that only the northern route around Sicily remains possible. Ginzel's elements, on the contrary, would clearly speak in favor of the direct way from Syracuse southwards. Fig.33 shows, moreover, the southern limit of the zone of totality according to Stockwell and the zone according to Hansen in order to show how sensitive these areas are to changing elements, which all represent almost equally well the present-day appearances. It is obvious that both Stockwell and Hansen are incompatible with Diodorus' report.

The decision whether Ginzel or Stockwell are right seems to come from a notice in Justinus,[300] who tells us that Agathocles kept secret his real goal, the attack on Africa, and told his officers and soldiers that he planned to sail to Sardinia, which was also in Phoenician hands. This makes it almost certain that he left Syracuse in the northern direction and that the eclipse reached the fleet when it was in the strait of Messina, just in the central line of totality according to Schoch's calculation.[301]

300) Justinus XXII,6.

301) The details of the progress of this eclipse are given in Neugebauer (P.V.) AChr. I p.112-121.

The same eclipse plays a rôle in the discussion of the material used by Hipparchus in the First Book of his work on the sizes and distances of sun and moon. Pappus in his commentary on the Fifth Book of the Almagest[302]

302) Now edited by A.Rome.

says that Hipparchus used the fact that a solar eclipse, total at the Helles pont, was seen in Alexandria as only partial, covering at its maximum only 4/5 of the sun's diameter. The same phenomenon is quoted by Cleomedes[303] and alluded to by Ptolemy.[304] The possible eclipses can easily be selected: Alexandria was founded in −331[305] and Hipparchus' observations fall in the years −160 to −125.[306] During this period, the only four eclipses

303) Cleomedes II,3 (ed. Ziegler p.174 and p.178).

304) Almagest V,11 (ed. Heiberg p.402).

305) RE I, 1277.

306) Berger, GFrH p.6 note 4 and RE 8, 1666.

visible at the Hellespont and simultaneously partial at Alexandria were the following:

−309	VIII	15	magn. at Alexandria	9.3 digits[307]
−262	II	9		9.1
−189	III	14		11.0
−128	XI	20		9.4

307) The magnitudes according to Hultsch [1] p.197 f.

The first is the Agathocles eclipse, the last is usually accepted as the Hipparchus eclipse. The second is ruled out because it was an annular eclipse which would hardly have been called a total eclipse by Hipparchus. The third eclipse is too great because 4/5 are only 9.6 digits, and Hipparchus must have had good reasons to trust the accepted magnitude as accurate if he based his calculations on this essential element. Hence only the two first and the last date remain as very likely. Fig.34 gives the zones of totality according to Ginzel and according to Schoch. The eclipse of −128 is barely total and does not quite cover the Bosphorus according to Ginzel or Schoch.[308] The Agothocles eclipse, however, covers the whole

308) Concerning the rôle of this eclipse for Schoch's elements cf. Schoch [1], Neugebauer (P.V.) AChr. I p.132 and Pogo [2] p.164.

Hellespontic region according to Schoch but passes south of this area according to Ginzel. If one therefore accepts Schoch's elements as the best representation of the Agathocles eclipse in the description of Agathocles' manoeuvre, it becomes very likely that Hipparchus did not use an eclipse observed by himself in -128 but relied on an older report of the Agathocles eclipse recorded for the Bosphorus and Alexandria. The final decision would be obtained by some knowledge of the arrangement of Hipparchus' numerous writings.[309]

309) An attempt to date the work of Hipparchus is made by Rehm RE 8, 1668 -1671, but for the date of the work in question the eclipse discussed here is the main argument.

b. The eclipse of -321 IX 26.

Fotheringham found a convincing argument in cuneiform sources for the accuracy with which Schoch's elements represent ancient eclipses.[310]

310) Fotheringham [3].

It seems to me useful to discuss this case because it exhibits typical difficulties involved in such arguments.

The text in question is an unpublished tablet,[311] mentioned by

311) Now in the British Museum (Sp.I, 192).

Kugler,[312] according to which a solar eclipse began on the 28th of VI(b) 4 us before sun-set, i.e. 16 minutes,[313] in the second year of Philip.

312) Kugler SSB I p.259 and pl.XXIV bottom; SSB II p.385 gives the passage in question.

313) Because one uš corresponds to one degree (cf. p.▮▮▮).

Fotheringham calculated the time of sun set for Sippar (about $\frac{1}{2}^{\circ}$ north of Babylon) and the moment of the first contact with Schoch's elements and found that the Babylonian report is only 3 minutes earlier than the resulting calculation. This would seem to be a very close coincidence indeed.

It is only a very minor argument against Fotheringham's conclusions that the text scarcely comes from Sippar.[314] The difference in longitude between the two sites is negligible and the difference in latitude has practically no effect on the time of sunset because the date of the eclipse is almost equinox. Very serious, however, is the objection that Fotheringham treats the time given in the text as if it were a modern, highly precise observation of the very moment of the first contact. It is hard to understand what means the Babylonians should have used to be able to recognize this moment in view of the fact that the sun was above the horizon by about six times its diameter. The clipse reached only a magnitude of 2.4 digits at Babylon[315] and it is much more likely that the

314) Fotheringham quotes Langdon in supposing that the Spartoli collection came from Sippar. This contradicts not only the statement of Babylon provenience made by Bezold Lit. p.149 (ad 18) but also various arguments which can be derived from the astronomical texts of the Seleucid period.

315) This according to Ginzel, Kanon, p.63. Fotheringham does not give the magnitude according to Schoch's elements, but the difference cannot be essential.

eclipse was not discovered until it had reached, say, 2 digits, which might have been interpreted as the "beginning" of a total eclipse whose further progress was invisible because of sunset. Such an argument cannot be proved but it should be strictly excluded before the statement of the text can be used for such minute discussions as the determination of the correction of the secular constants. The only conclusion which seems justified to me is that any calculated eclipse which starts later than a text reports must be corrected; however, a time of first contact, say, twice as early could easily be explained by the insufficient accuracy of the ancient means of observation.

35. The Thales eclipse.

The most famous eclipse of antiquity is undoubtedly the Thales eclipse. Virtually no textbook on ancient history omits a reference to it even when no more than a few pages are devoted to ancient science. This eclipse has become some kind of symbol of the glorious early rise of Greek science and philosophy in Ionia in the sixth century B.C.

Optimistically expressed, the actual foundations of this story are rather weak. The main source is Herodotus I,74 who tells about the war between the Lydians and the Medes: its cruel beginning with the slaughter and cooking of a son of the Median king and the happy end of the revolting war in its sixth year when a solar eclipse brought a halt to a running battle and the new alliance of the enemies was confirmed by a marriage.

The story about the eclipse is amplified by the famous notice "and this change (from day to night) had been predicted to the Ionians by Thales, who gave as time the year in which this change actually happened". Obviously, Pliny has the same event in mind when he says[316] that Thales

316) NH II,53 (ed. Ian-Meyhoff p.143 No.12 (9)). This and all other relevant passages are collected in Diels VS$^{(5)}$ 11 [1] A 5 (p.74f.).

"investigated" the cause of an eclipse during the reign of Alyattes in Ol. 48,4 = a.u.c. 170[317] (which would mean[318] -584/3 or -583/2).[319]

317) Some manuscripts offer variants for the a.u.c.-years, namely, 120, 160, 180 - all excluded by the Olympiad.

318) Cf. equations (15) and (16a) p.[illegible].

319) Clemens Alex. Stromata I, 354 gives Ol. 50,1 ≈ -579/8.

After very much discussion, modern historians seem to agree that the battle in question was fought near the Halys River and that the eclipse in question was the eclipse of -584 V 28.[320] As a matter of facts, modern elements, particularly Schoch's elements, yield totality of this eclipse in the region in question (cf. fig.35).[321] It can be considered

320) Cf. CAH III p.512.

321) Schoch [1] p.25; for details see Neugebauer (P.V.) AChr. p.122-125.

at least as very unlikely that it is a pure accident that the calculated place and date, the date of Pliny and the report of Herodotus agree so well.

Absolutely contradictory to all our modern knowledge of both Greek and Babylonian astronomy, however, is the story of the <u>prediction</u> of the solar eclipse by Thales. The wording of Herodotus' remark is alone sufficient to arouse suspicion: to predict an eclipse " for a certain year" is astronomically meaningless; and Pliny (i.e. his source) speaks much more modestly only about the "investigation" of the eclipse.[322] An-

322) About prediction speaks Eudemus (pupil of Aristotle; according to Clemens Alex.Atrom. I p.354) and Cicero, De div. I, 49.

cient reports notwithstanding, the fact remains that no means existed at

genes Laertius, on the contrary, tells us that Thales predicted (!) not only a solar eclipse but the solstices,[322a] ~~[illegible]~~ problems of very different level, indeed.

the time of Thales to predict the visibility of a solar eclipse at a given place.[323] The usual "explanation" of historians that Thales used Babylon-

323) When Darius, 70 years later, ordered the Ionian commanders to watch the bridge across the Danube for 60 days, he had to give the Greek tyrants a leather thong with 60 knots in it, in order to untie one of them every day (Herodotus IV, 98). This is sufficient to characterize the astronomical level of this period, and is fully supported by all our information about the geographical concepts of the early Ionians (cf. Heidel AGM).

ian cycles collapses by virtue of the fact that no cycles of solar eclipses can be discovered at a given place.[324] Even assuming a complete knowledge

324) Cf. the examples given above or Ginzel, Kanon p.15.

of the mechanism of solar eclipses which would lead to the establishment of periods for the whole world, predictions made for Babylon would be of very restricted value for Asia Minor. We are today so well informed about Babylonian astronomy of the latest period that we can fully confirm the statement of Diodorus[325] that "regarding solar eclipses the Chaldeans are very inefficient and do not dare to predict them nor to give exact time limits".[326]

325) Diodorus II, 31 (ed. Vogel p.222, 6-9).
326) Cf. Neugebauer (O.) UAA III p.294 ff.

The only methodical approach to the problem id the discussion of the passage as an illustration of purely literary technique according to which the prediction of extraordinary events is one of the characteristics of superior wisdom. We need only quote Diodorus' remarks about the Egyptians:[327] "they fortell destructions of the crops or, on the other hand

327) Diodorus I, 81 (Loeb p.279).

abundant yields, and pestilences that are to attack men or beasts, and as a result of their long observations they have prior knowledge of earthquakes and floods, of the rising of the comets, and of all things which the ordinary men look upon as beyond all finding out". Exactly the same kind of abilities are assumed for the Ionian sages: Anaximandes predicted an earthquake,[328] Anaxagoras the fall of an aërolite[329] and Thales a rich olive-crop.[330] It is clear that a famous eclipse happenning in the lifetime of

328) Pliny NH II,191.

329) Aristotle, Meteor. I,7; Pliny NH II, 149. Plutarch, Lys. 12 (= Diels VS(5) A 12 vol. II p. 9)

330) Aristotle, Polit. I, V; Diogenes Laertius I,26 (= Diels VS(5) p.68,25)

such an extraordinary man must have been predicted by him just as he was made the inventor of various astronomical and mathematical theorems whose origin was unknown to later centuries.[331]

331) It is interesting to notice that practically no effect was exercised on later writers by Rawlinson's commentary on Herodotus (I p.163 note 6): "The prediction of this eclipse by Thales may fairly be classed with the prediction of a good olive-crop or of the fall of an aerolite".

The high estimation in which eclipses were held in this milieu is shown by another story of Herodotus[332] relating in great detail the ocurrance of a solar eclipse when Xerxes left Sardis for the invasion of Greece. No eclipse happenned anywhere at this time; but eclipses are greatly respected by modern historians, so an annular eclipse of -477 II 17 has been found[333] which Herodotus should have quoted in "sagenhafter

332) Herodotus VII, 37.

333) Cf. RE 6, 2354.

Rückübertragung". The only eclipse which has, to my knowledge, escaped dating until now is the eclipse predicted by the wise shepherd Chrisostom[334]: "he knew the science of the stars, and what the sun and moon are doing up there in the sky, for he told us exactly of the clipse of the sun and moon" - "Eclipse it is called, friend, and not clipse ..." said Don Quixote.[335]

334) The analogy reaches still farther: both Thales and the shepherd knew in advance whether or not the next year would be a good year for oil and both could have grown rich if they wanted.

335) Cervantes, Don Quixote I ch.12.

§ 6. Bibliography of Chapter II.

36. General.

The works quoted in the following are not intended for the astronomer but are specifically written for the use of historians.[336] Best

336) For a reader who wants an introduction to astronomy in general, the "Spherical astronomy" of W.M.Smart can be mentioned.

fitted for this purpose is P.V.Neugebauer's "Astronomische Chronologie", Vol.I, which not only gives the explanation of the general concepts but contains examples of all calculations which might be of interest in historical investigations. This work, moreover, contains a critical bibliography for each section telling the reader which tables are best fitted for use in a special problem. No historian who has to deal with astronomical problems should neglect to consult this work and the tables given in HTChr.

General astronomical introductions are, of course, also given in most chronological works, e.g., in Ginzel I or separately in Wislecenus AChr. but they usually contain more details than the historian needs in practice and do not help him to solve a specific problem.

So far as the moon in particular is concerned, much depends on the specific type of problem. Necessary advice can again be found in P.V. Neugebauer's AChr. It might be mentioned that frequently only very approximate information about the moon's position is necessary, e.g., in dealing with horoscopes. A horoscope was usually cast years after the date of birth and is therefore based on ancient tables for sun, moon and planets. These tables were certainly not accurate enough to give positions of the moon within an accuracy of say $\pm 10^\circ$ or even more; one must not forget that the moon's mean motion amounts to more than 13° per day so that already small errors in time or epoch result in very considerable deviations from the truth. Moreover, most horoscopes disregard the movement in latitude completely, not to mention the difficulties involved for us by the use of different points of origin in the ecliptic.[337] It is therefore useless

337) Cf. above p.***.

to compute lunar positions with great accuracy when an equal accuracy of the ancient records con not be assumed. Such approximate determinations of the lunar longitude can be easily calculated by a few additions from the tables in P.V.Neugebauer's TAChr.II according to rules given there p.XXIV f. combined with AChr.I p.59/60. In many cases an even rougher procedure is possible, e.g., if only the zodiacal sign of the moon is given. Ginzel I and II contain tables for new moon and full moon for the following periods

New moons	-604 to -99	vol.I	p.547-562
	-99 to +308	vol.II	p.544-556
Full moons	-499 to +100	vol.II	p.557-575 .

These tables give the chronologically arranged Julian dates of the syzygies from which the position of the sun can be derived very simply (e.g., by the tables in P.V.Neugebauer TAChr.III p.67). Positions of sun and moon are therefore known for dates about 15 days apart - which is sufficient to determine by simple interpolation the approximate position at any intervening date. In order to avoid errors, one must not forget that the Ginzel tables are based on B.C.-years and Greenwich time with noon-epoch. In order to get civil time for the longitude of Babylon, one must therefore add 15 hours = 0.62 days, for Alexandria only 14 hours = 0.58 days.

As an example of the other extreme, namely, where calculations with high accuracy are involved, the determination of the times of the new crescent may be mentioned. This problem is of special importance for the Babylonian calendar; Schoch therefore computed special tables, published in Langdon F.-S., for Babylon. Some additions to these tables are given in Schoch [1] p.20.

37. Eclipses.

In the overwhelming majority of cases, historical problems connected with eclipses can be solved by consulting the existing lists and maps already mentioned in § 3 No.28. Methods of computing more detailed elements of the progress or the magnitude of an eclipse are described in P.V.Neugebauer AChr.I p.109-133.

Discussion of the eclipses from classical sources[338] are given

338) The discussion of the Babylonian material given on p.234-260 (through collaboration with C.F.Lehmann[-Haupt]) are now antiquated.

in Ginzel, Kanon, p.167-234. Ginzel gives not only all astronomical commentaries but also the passages of the sources and older literature. The list of these eclipses is given by Boll RE <u>6</u>, 2352-2364 who added a few more ~~XXX~~ dates, most of them of very doubtful character (e.g., an alleged eclipse at the date of the foundation of Rome).339) Eclipses which are of importance

339) The only serious new date is the presumed Hipparchus eclipse which probably must again be eliminated for reasons given above p.###.

for the problem of secular acceleration are discussed by P.V.Neugebauer in an appendix to Schoch's collected publications (1930, Schoch [1] p.24). Their list is

-762	VI	15	+29	XI	24	1147	X	26
-660	VI	27	71	III	20	1239	VI	3
-647	IV	6	484	I	14	1241	X	6
-584	V	28	693	X	5	1267	V	25
-556	V	19	878	X	29	1339	VII	7
-309	VIII	15	1033	VI	29	1912	IV	17
-128	XI	20	1133	VIII	2			

It follows from these dates that this series of solar eclipses includes all periods of history from which eclipse reports are available and therefore represents the most complete account of the consequences of Schoch's theory of secular acceleration.340)

340) It might be remarked that Schoch himself made it difficult for historians to accept his results because he frequently printed them in a very peculiar form and added baroque historical comments. It is therefore necessary to emphasize the seriousness of his astronomical work which is of greatest value for ancient chronology.

More information about the modern discussion of the problem of secular acceleration by De Sitter, Fotheringham, Schoch and others can be found in the bibliographies Fotheringham [B] and Schoch [B].